COMPUTATIONAL METHODS AND ALGORITHMS

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Summary

So, let us look first at the existing trends. The simplest prediction consists in exponential extrapolation of the growth of computer capacity. Of course, this growth will give more and more ability to solve the same computational class of problems. But the classes of problems will be widened. Other side of this growth consists in the change of paradigm of algorithmic realization. Nowadays hot time comes to parallelize computational algorithms because of parallel architecture of computers, computers clusters, and computer nets. And on horizon the quantum computers begin to manifested. In some type of problems the amount of data for simulation and after it becames enormous. It takes special tools for visualization which became important mean for monitoring and adaptation of complex computational processes.

Second, let us look at those existing challenges where new development ripens enough. To get more accurate models, one involves more and more different properties and effects. Therefore models became interdisciplinary and multiscale ones. In such a multiscale models the smallest scales are resolved over the largest scales. It takes a huge of unknowns. Therefore it need to develop new algorithms. One of way to resolve a small scales is a posteriori adaptation and computational zooming not only to diminish discretization error but to locally detail model. For this purpose one need some kind of estimators. Any mathematical model contains some simplifications. Therefore it initially contains error of model. In principle we need to estimate this error in addition to discretization one. Thus, in ideal situation the computations have to give not only the necessary data but an interval between upper and lower bounds of these data.

And of course, computational algorithms as sagnificant part with mathematical modelling, numerical analysis, computers, and computer science will serves more and more complicated problems: new materials, chemical and physical processes, weather prediction, environmental flows, computer model of human body, etc.

1. Introduction

Mathematics has arisen from daily practical needs of human being early in the development of the human society in the course of its struggle for survival. But having once arisen, abstract ideas started to develop independently. Nowadays the development of mathematics is due to the development of productive forces of society, the development of industry, transport, financial and insurance business, information providing for society. In the last decades none of the knowledge areas has changed as much as mathematics. The major reason is the invention of a computer. In the opinion of N.N. Moiseev [2], this invention can be compared only with getting control over fire and the invention of a steam-engine. Due to getting control over fire, great individual resources of a human organism, that were expended on digesting raw food, were released and turned to the intellectual evolution; this enabled the mankind to survive hard thousand years of the glacial period. Similarly, a steam-engine allowed a man to overcome the energy crisis. It permitted the use of energy resources that had been accumulated on the planet in milliards years of its existence. Besides, it was a starting mechanism of the chain reaction which is called the technical progress. Computers enable us to get out of the information crisis. The extreme complication of science, technology, and industry has generated the need to deal with a great body of information, with more and more efforts being required to process and assimilate it. But physiological resources of a man remain the same. The use of a computer increases the power of our tools of knowledge and control. At first, a computer was used by physicists and engineers as a calculator. At that time a new field of mathematics, namely, computational one, began to develop. In this branch of mathematics attention was concentrated on solving well-defined mathematical problems and the task was reduced to the construction of a method for solving a problem with the help of a computer. During this period of the science development a number of problems arised, such as the stability of difference schemes, the convergence of iterative processes, the influence of errors etc. Since 1950s, an increasing number of computers (of the second generation) was coming into use in economics and management. Problems arising in these fields involve the processing of a great body of information. At that period new branches of mathematics were developed. Among them the theory of algorithmic languages stands out. It is a new type of formalization which is suitable for the formulation of problems of various kinds.

Then a dialog with a computer during the process of computation was made possible with systems of vizualization of results. This provides a new tool for analysis that enables to test hypotheses and to perform computer experiments.

Finally, parallelizm, i.e., solving a problem with the use of tens, hundreds, or thousands of processors, becomes the means for acceleration of computations. This gives impetus to revising the efficiency of algorithms employed.

Consider a scheme on Figure 1 of solving some physical problem [1]. We mark the objects and the results of operations by rectangles and the operations by arrows. From here on we shall refer to this scheme.

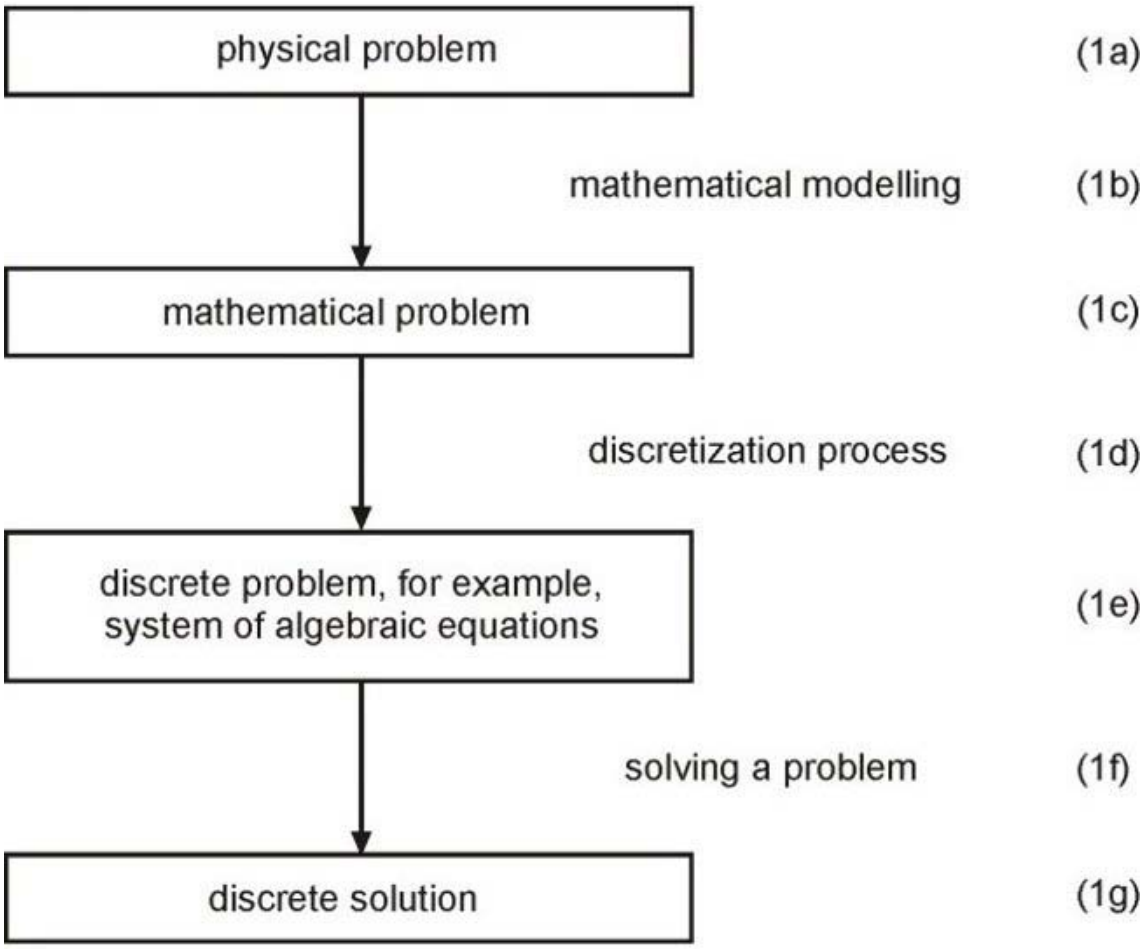

Figure 1. Standard scheme of solving a physical problem

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