

# PLANETARY RING DYNAMICS

**Matthew Hedman**

*University of Idaho, Moscow ID, U.S.A.*

**Keywords:** celestial mechanics, disks, dust, orbits, perturbations, planets, resonances, rings, solar system dynamics

## Contents

1. Introduction to Planetary Ring Systems
  2. Orbital Perturbations on Individual Ring Particles
  3. Inter-particle Interactions
  4. External Perturbations on Dense Rings
  5. Conclusions
- Acknowledgements  
Glossary  
Bibliography  
Biographical Sketch

## Summary

Planetary rings provide a natural laboratory for investigating dynamical phenomena. Thanks to their proximity to Earth, the rings surrounding the giant planets can be studied at high resolution and in great detail. Indeed, Earth-based observations and spacecraft missions have documented a diverse array of structures in planetary rings produced by both inter-particle interactions and various external perturbations. These features provide numerous opportunities to examine the detailed dynamics of particle-rich disks, and can potentially provide insights into other astrophysical disk systems like galaxies and proto-planetary disks.

This chapter provides a heuristic introduction to the dynamics of the known planetary rings. We begin by reviewing the basic architecture of the four known ring systems surrounding the giant planets. Then we turn our attention to the types of dynamical phenomena observed in the various rings. First, we consider how different forces can modify the orbital properties of individual ring particles. Next, we investigate the patterns and textures generated within a ring by the interactions among the ring particles. Finally, we discuss how these two types of processes can interact to produce structures in dense planetary rings.

## 1. Introduction to Planetary Ring Systems

Ring systems surround all four of the giant planets in the outer solar system. While all these rings consist of many small particles orbiting their respective planets, the known rings exhibit a wide diversity of structures, and occupy a broad range of dynamical environments. Furthermore, different rings exhibit patterns and features generated by such diverse processes as inter-particle interactions, gravitational perturbations from various satellites, and a number of non-gravitational forces. Hence, before we consider

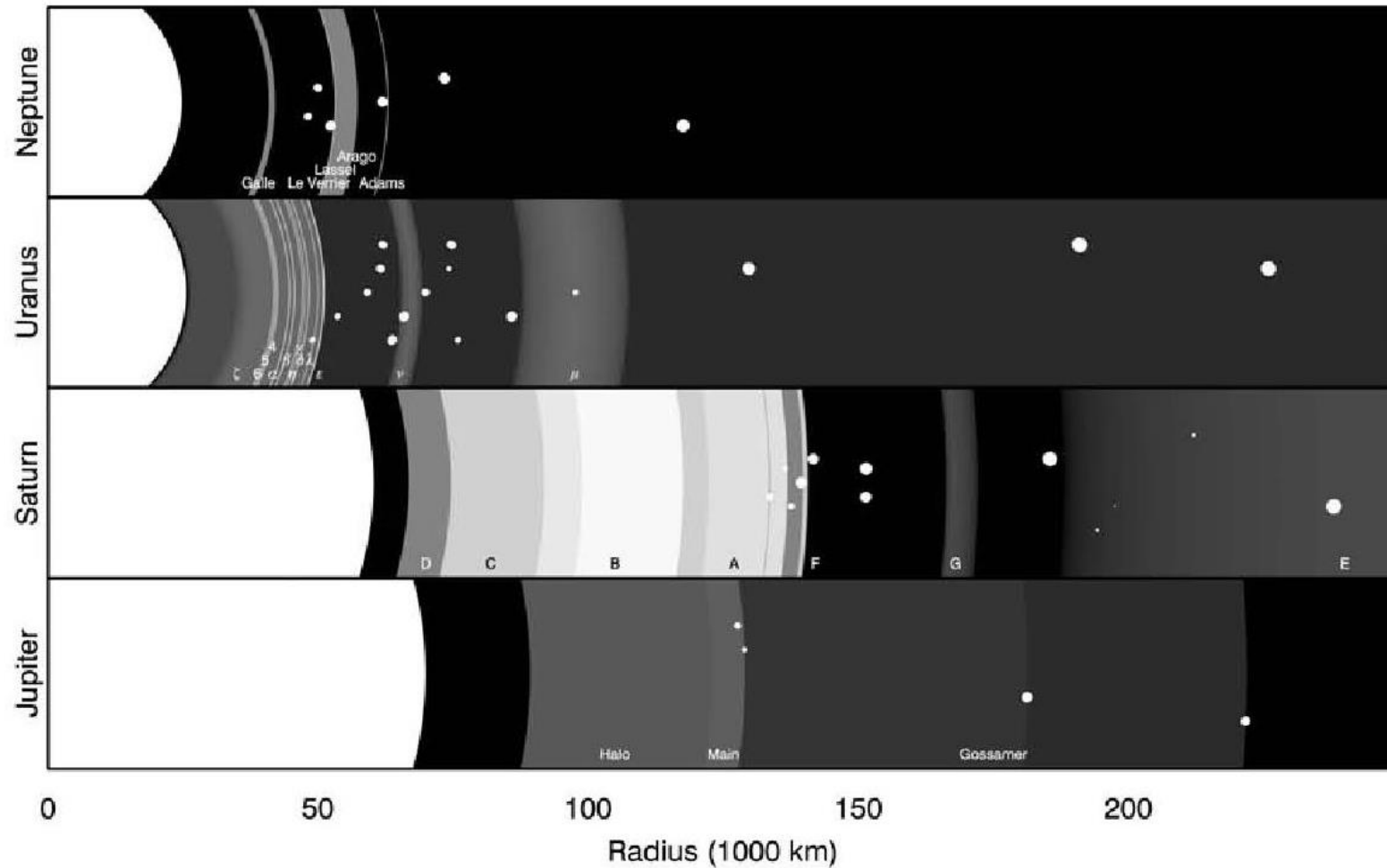


Figure 1. The ring systems of the giant planets, shown to scale. Each grey-scale level corresponds to a decade in ring optical depth. The small white dots correspond to the various moons, with the size of the dots being proportional to the logarithm of their true size. None of these moons are shown to scale with the rings.

the dynamical phenomena operating in various rings, it is useful to briefly review the properties of the known planetary rings.

As shown in Figure 1, the rings of the giant planets vary dramatically in their structure and their opacity. Opacity is a particularly useful parameter for describing and categorizing rings because it is correlated with such fundamental ring parameters as surface mass density, and because it can be directly measured by observing the amount of light transmitted through or scattered by the rings. Ring opacity is typically quantified using either a transmission coefficient  $T$  or an optical depth  $\tau = -\ln(T)$ . Such parameters depend on the exact path the light takes through the rings, but they can be used to estimate a viewer-independent parameter called the normal optical depth; the optical depth of the ring observed when the light passes perpendicularly through the rings. The normal optical depth of the known planetary rings ranges over more than eight orders of magnitude.

The most extensive and complex ring system belongs to Saturn. Furthermore, thanks to the vast amount of data returned by the Cassini spacecraft, Saturn's ring system is now the best-studied in the outer solar system. The most familiar of Saturn's rings are the so-called "Main Rings", which include the A, B and C rings, as well as the Cassini Division between the A and B rings. These are all broad rings with substantial optical depths ( $\tau$  ranging from around 0.1 to over 5) composed of millimeter-to-meter-sized ice-rich particles. Their high opacities and reflectivities make these rings the easiest ones to see with Earth-based telescopes. Still, it is important to realize that none of these rings are completely homogeneous. Instead, they possess structures on a wide range of scales, and there are even several nearly empty gaps in the A ring, the C ring, and the Cassini Division. Some of these patterns can be attributed to particle-particle interactions or various gravitational perturbations from Saturn's various moons, and therefore provide illustrative examples of the dynamical phenomena that can operate in such dense rings. However, the processes responsible for producing many other structures are still not well understood. Detailed reviews of Saturn's main rings are provided by Colwell *et al.* (2009) and Cuzzi *et al.* (2009).

In addition to these main rings, Saturn also possesses a diverse suite of (mostly) fainter rings, including the D E, F and G rings, some narrow ringlets occupying gaps in the main rings, material in the orbits of various small moons, and the enormous but tenuous disk of debris extending between the orbits of Phoebe and Iapetus. The normal optical depth of these rings ranges from 0.1 for the core of the F ring, to about  $10^{-3}$  for parts of the D ring, to  $10^{-6}$  for both the E and G rings, to as low as  $10^{-8}$  for the extensive Phoebe ring (Horanyi *et al.* 2009, Verbiscer *et al.* 2009). Unlike the dense main rings, the visible appearance of these fainter rings is dominated by dust-sized particles less than 100 microns across. Such small particles are especially sensitive to non-gravitational forces, and so the dynamics of these dusty rings are quite different from those of Saturn's main rings. Horanyi *et al.* (2009) provides a recent review of Saturn's dusty rings.

After Saturn, Uranus possesses the most substantial ring system, which is dominated by an array of dense, narrow rings. These rings are designated using either numbers (the 4, 5 and 6 rings) or Greek letters (such as the  $\alpha, \beta, \gamma, \delta, \epsilon$  and  $\eta$  rings). Most of these

rings have optical depths between 0.1 and 1.0 and are between 1 and 10 km wide. The exception is the  $\epsilon$  ring, whose width ranges between 20 and 100 km, and whose optical depth ranges between 0.5 and 2.5. These narrow, dense rings are surrounded by dusty material that has numerous fine-scale structures, including some narrow dusty ringlets like the  $\lambda$  ring. A sheet of dusty material, known as the  $\zeta$  ring, extends inwards of the dense rings, and two diffuse dusty rings, called the  $\mu$  and  $\nu$  rings, have been recently discovered further from the planet (de Pater *et al.* 2006a, b). French *et al.* (1991) and Esposito *et al.* (1991) provide the most recent comprehensive discussions of Uranus' rings, and while the data are still rather limited, several interesting dynamical phenomena have been observed in this system. For example, many of the narrow rings exhibit variations in their widths and radial locations that are due to a combination of perturbations from nearby moons and excited normal modes.

Most of Neptune's dusty rings are rather tenuous, with optical depths around  $10^{-3}$ . The exception is the Adams ring, which contains a series of longitudinally confined arcs with optical depths as high as 0.1. These arcs are of special interest here because they may be confined by a co-rotation resonance with Neptune's small moon Galatea. Porco *et al.* (1995) provide a detailed review of Neptune's ring system.

Jupiter's ring system is the most tenuous of all, and appears to be formed primarily of fine debris knocked loose from Jupiter's small moons (see Burns *et al.* 2004 for detailed discussions of this system). The so-called Main ring extends interior to the small moons Metis and Adrastea, which are likely to be important sources for the ring material. At the inner edge of the Main ring is the Ring Halo, a vertically extended distribution of debris that likely represents particles whose inclinations have been excited by their interactions with Jupiter's magnetic field. Exterior to the main ring are two extremely faint Gossamer Rings, which appear to be composed of debris knocked off from Amalthea and Thebe. Both these moons are on inclined orbits, and the vertical extent of these structures is consistent with those inclinations.

The structure and dynamics of these planetary rings are due to a diverse array of processes, including interactions among the particles within the rings and external perturbations on the orbits of individual ring particles. Furthermore, many features in the ring actually reflect interactions among multiple dynamical processes. For example, the density waves in Saturn's Main Rings are generated by resonant gravitational perturbations from Saturn's moons, but their propagation through the rings is controlled by collisions and gravitational interactions among the ring particles. Fortunately, there are also features in planetary rings that more clearly document individual dynamical phenomena. In particular, various patterns in the low optical depth rings allow us to demonstrate how external forces can perturb the orbits of individual ring particles, while the small-scale textures of the main rings provide insights into how ring particles interact with each other.

## 2. Orbital Perturbations on Individual Ring Particles

Here we consider how the orbit of an individual ring particle can be perturbed by outside forces. After reviewing the nomenclature for the orbital parameters that will be used in this chapter, we present the generic perturbation equations for these parameters.

We will then use these equations to quantify how the ring particles should respond to steady and time-variable (i.e. resonant) perturbations.

## 2.1. Orbital Elements

In the following discussions of ring dynamics, we will use both physical coordinates of ring features and orbital parameters of the component ring particles. For a planetary ring, a cylindrical coordinate system is most natural, so we will designate the location of ring features by their radius  $r$ , vertical offset  $z$ , and longitude  $\lambda$ . Note that  $\lambda$  is measured relative to a fixed direction in inertial space, so orbiting particles cycle through all possible values of  $\lambda$  once each orbit.

While cylindrical coordinates are a natural basis for describing ring features, the dynamics of these systems are best described in terms of the six classical orbital elements: the semi-major axis  $a$ , the eccentricity  $e$ , the orbital inclination  $i$ , the longitude of ascending node  $\Omega$ , the argument of pericenter  $\omega$  and the particles true anomaly  $f$  (Figure 2). The inclination of the ring particles will be measured from the planet's equatorial plane, and  $\Omega$  is measured from the same inertial direction used to define  $\lambda$ .

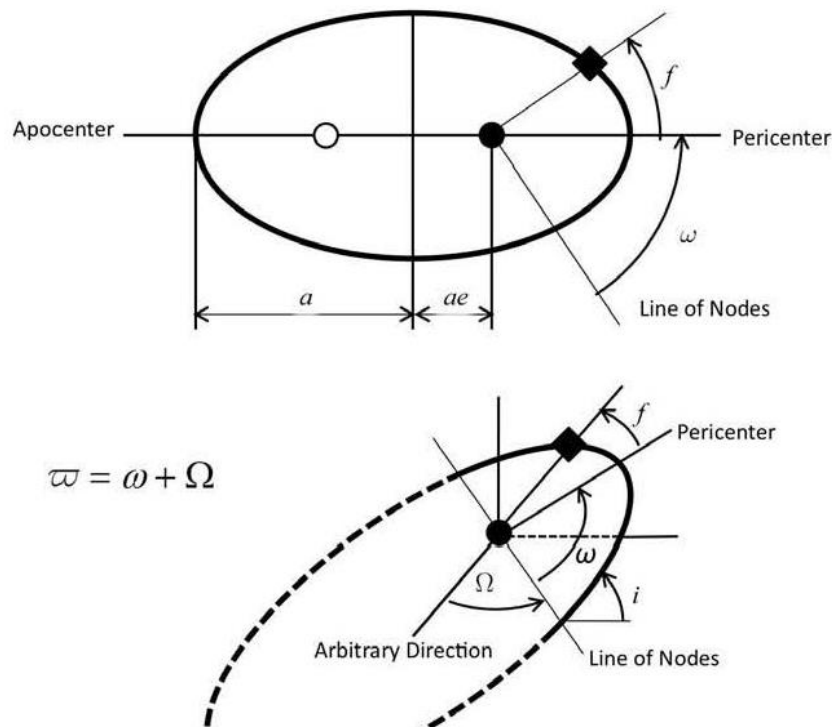


Figure 2. The orbital element nomenclature used in this chapter.

Most planetary rings are on very low inclination orbits in this coordinate system, so the longitude of pericenter  $\varpi = \Omega + \omega$  is often a more useful parameter than either  $\Omega$  or  $\omega$ . In terms of this parameter, the longitude of a ring particle can be written as  $\lambda \approx \Omega + \omega + f = \varpi + f$ . Many planetary rings also consist of particles on low-eccentricity orbits, so in the absence of other perturbations, these particles will move

around the planet at a rate  $n$  that is approximately equal to  $(GM_p/a^3)^{1/2}$  where  $G$  is the universal gravitational constant,  $M_p$  is the planet's mass and  $a$  is the orbital semi-major axis.

## 2.2. Perturbation Equations

In addition to the central force from the planet's gravity, ring particles feel various (small) perturbing forces. In general, such a force can be written as:  $\vec{F} = F_r \hat{r} + F_\lambda \hat{\lambda} + F_z \hat{z}$ , where  $\hat{r}$  is a unit vector pointing in the radial direction,  $\hat{\lambda}$  is a unit vector pointing in the azimuthal direction, and  $\hat{z}$  points normal to the orbit plane. Including such a perturbing force in the appropriate equations of motion yields a series of perturbation equations, which specify how the particles orbital elements should change over time in response to this force. A heuristic derivation of these equations can be found in Burns (1976), and we will not attempt to derive these equations here. Instead, we will simply present the equations, illustrate how they can be simplified for ring particles on nearly circular, low-inclination orbits, and briefly discuss why the resulting expressions are intuitively sensible.

First, let us consider the perturbation equation for the particles semi-major axis:

$$\frac{da}{dt} = \frac{2an}{(1-e^2)^{1/2}} \left[ \frac{F_r}{F_G} e \sin f + \frac{F_\lambda}{F_G} (1+e \cos f) \right] \quad (1)$$

where  $F_G = GM_p m/a^2$  is the central force from the planet. The dominant term in this expression is proportional to  $F_\lambda/F_G$ . This makes sense, since a steady force applied along the direction of motion will cause the particle to accelerate, gain energy, and move away from the planet. The other terms involve  $e$ , and therefore will be small corrections for most ring particles. However, these terms can still be important in situations where the average azimuthal force applied to the particle over the course of one orbit is zero (e.g. the direction of the force is fixed in inertial space). Thus, even in the limit of a nearly circular orbit ( $e \ll 1$ ), this equation can only be slightly simplified by approximating  $(1-e^2)^{1/2}$  as simply unity:

$$\frac{da}{dt} = 2an \left[ \frac{F_r}{F_G} e \sin f + \frac{F_\lambda}{F_G} (1+e \cos f) \right], \quad (2)$$

Next, consider the perturbation for the particles eccentricity. The general expression is:

$$\frac{de}{dt} = n(1-e^2)^{1/2} \left[ \frac{F_r}{F_G} \sin f + \frac{F_\lambda}{F_G} (\cos f + \cos \epsilon) \right], \quad (3)$$

where  $\epsilon$  is the eccentric anomaly, a quantity that is approximately equal to the true anomaly  $f$  for particles on nearly circular orbits considered here. Thus, for typical ring particles we may approximate the above expression as:

$$\frac{de}{dt} = n \left[ \frac{F_r}{F_G} \sin f + 2 \frac{F_\lambda}{F_G} \cos f \right], \quad (4)$$

In order to verify that this expression is sensible, consider the following: A force that accelerates a particle along the direction of motion near pericenter (i.e.  $F_\lambda > 0$  when  $f = 0$ ) causes the eccentricity to grow, while a similar force applied near apocenter (i.e.  $F_\lambda > 0$  when  $f = 180^\circ$ ) causes the eccentricity to shrink. Both of these results are consistent with standard orbital dynamics.

The perturbation equation for the orbital inclination is:

$$\frac{di}{dt} = n (1 - e^2)^{1/2} \frac{F_z}{F_G} \frac{\cos(\omega + f)}{1 + e \cos f}, \quad (5)$$

or, in the limit of nearly circular, low-inclination prograde orbits:

$$\frac{di}{dt} = n \left[ \frac{F_z}{F_G} \cos(\lambda - \Omega) \right], \quad (6)$$

where we have used the approximate relationship  $\lambda \simeq f + \omega + \Omega$ , which is valid for low inclinations. Again, we can verify this equation gives sensible results in various simple situations. For example, a vertical force  $F_z > 0$  applied when the particle is near its ascending node ( $\lambda \simeq \Omega$ ) and thus already moving northwards, will cause the inclination to grow, while the same force applied near the descending node ( $\lambda \simeq \Omega + 180^\circ$ ) will cause the inclination to shrink.

The perturbation equation for the longitude of ascending node is:

$$\frac{d\Omega}{dt} = \frac{n(1 - e^2)^{1/2}}{\sin i} \left[ \frac{F_z}{F_G} \frac{\sin(\omega + f)}{1 + e \cos f} \right], \quad (7)$$

For nearly circular, low inclination, prograde orbits, this expression becomes:

$$\frac{d\Omega}{dt} = \frac{n}{\sin i} \left[ \frac{F_z}{F_G} \sin(\lambda - \Omega) \right], \quad (8)$$

which is a plausible complement to the above perturbation equation for the inclination. For example, a positive vertical force applied when the particle is near its maximum vertical excursion ( $\lambda \approx \Omega + 90^\circ$ ) will delay the particles' return to the ring-plane and thus move the node longitude forward around the planet.

Finally, we have the following expression for the argument of pericenter:

$$\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} = \frac{n(1-e^2)^{1/2}}{e} \left[ -\frac{F_r}{F_G} \cos f + \frac{F_\lambda}{F_G} \sin f \frac{(2+e \cos f)}{(1+e \cos f)} \right] \quad (9)$$

For orbits with small  $e$  and  $i$ , this equation can be transformed into a simpler perturbation equation for the longitude of pericenter  $\varpi = \omega + \Omega$ :

$$\frac{d\varpi}{dt} = \frac{n}{e} \left[ -\frac{F_r}{F_G} \cos f + 2 \frac{F_\lambda}{F_G} \sin f \right] \quad (10)$$

This is a reasonable complement to the perturbation equation for the orbital eccentricity. For example, an outward radial force applied near apocenter ( $f \approx 180^\circ$ ) will delay the particles' inward motion and thus cause the pericenter to shift forward in longitude.

-  
-  
-

TO ACCESS ALL THE 44 PAGES OF THIS CHAPTER,  
Visit: <http://www.eolss.net/Eolss-sampleAllChapter.aspx>

### Bibliography

Borderies, N. P. Goldreich and S. Tremaine. (1982). Sharp edges of planetary rings. *Nature* Vol 299 pp 209-211. [Describes the basic physics behind resonant confinement of ring edges]

Borderies, N. P. Goldreich and S. Tremaine. (1989). The formation of sharp edges in planetary rings by nearby satellites. *Icarus* Vol 80 pp 344-360. [Describes the physics behind gravitational shepherding]

Bridges, F.G. A. Hatzes and D. Lin. (1984). Structure, stability and evolution of Saturn's rings. *Nature* Vol 309, pp. 333-335. [Describes experimental measurements of the restitution coefficients between icy objects]

Burns, J.A. (1976). Elementary derivation of the perturbation equations of celestial mechanics. *American Journal of Physics*. Vol. 44 pp 944-949 (plus erratum in Vol 45 pp 1230) [Provides a good heuristic derivation of the perturbation equations for the various orbital elements.]

Burns, J.A., P.L. Lamy and S. Soter. (1979). Radiation forces on small particles in the Solar System. *Icarus*. Vol 40, pp 1-48. [A useful overview of some of the non-gravitational forces that can influence the orbital dynamics of small particles in planetary rings.]



- Burns, J.A. L.E. Schaffer R.J. Greenberg and M.R. Showalter. (1985). Lorentz resonances and the structure of the Jovian ring. *Nature*. Vol 316 pp 115-119. [Describes how the vertical structure of Jupiter's rings can be produced by vertical resonances with the planet's magnetic field]
- Burns J.A. M.R. Showalter D.P. Hamilton P.D. Nicholson I. de Pater M.E. Ockert-Bell and P.C. Thomas. (1999). The formation of Jupiter's faint rings. *Science*. Vol 284 pp 1146-1150. [Describes how Jupiter's Gossamer rings can be generated from impact debris onto two of Jupiter's moons.]
- Burns, J.A. D.P. Simonelli M.R. Showalter D.P. Hamilton C.C. Porco and H. Throop. (2004). Jupiter's ring-moon system. Bagenal F., T.E. Dowling and W.B. McKinnon (eds). *Jupiter: The Planet Satellites and Magnetosphere*. Cambridge, Cambridge Univ. Press. pp 241-262. [A recent overview of the structure, composition and dynamics of Jupiter's ring system.]
- Charnoz, S. A. Brahic P.C. Thomas and C.C. Porco. (2007). The equatorial ridges of Pan and Atlas: Terminal accretionary ornaments? *Science*. Vol 318 pp 1622-1624. [Discusses the possibility of ring material accreting onto Pan and Atlas]
- Charnoz, S. L. Dones, L.W. Esposito P.R. Estrada and M.M. Hedman. (2009). The origin and evolution of Saturn's ring system. Dougherty, M.K., L.W. Esposito, and S.M. Krimigis (eds). *Saturn from Cassini Huygens*. New York, Springer. pp 375-413. [A good review of processes relevant to the long-term evolution of dense ring systems.]
- Charnoz, S. J. Salmon and A. Crida. (2010). The recent formation of Saturn's rings from viscous spreading of the main rings. *Nature*. Vol 465. pp 752-754. [Presents calculations that show how moons might coagulate out of the spreading main rings.]
- Colwell, J.E. L.W. Esposito M. Sremečević and G.R. McClintock. (2007). Self-gravity wakes and radial structure of Saturn's B ring *Icarus*. Vol 200 p 574-580. [Describes occultation observations of self-gravity wakes and overstable structures obtained by the ultraviolet spectrometer onboard Cassini.]
- Colwell, J.E. P.D. Nicholson M.S. Tiscareno C.D. Murray R.G. French and E.A. Marouf. (2009). The structure of Saturn's rings. Dougherty, M.K., L.W. Esposito, and S.M. Krimigis (eds). *Saturn from Cassini Huygens*. New York, Springer. pp 375-413. [A recent overview of the observed structures in Saturn's dense rings, including the A, B, C and F rings.]
- Cuzzi, J. R. Clark G. Filacchione R. French R. Johnson E. Marouf and L. Spilker. (2009). Ring particle composition and size distribution. Dougherty, M.K., L.W. Esposito, and S.M. Krimigis (eds). *Saturn from Cassini Huygens*. New York, Springer. pp 459-509. [A good recent overview of the available constraints on the particle size distribution and material composition of Saturn's rings.]
- Dermott, S.F. (1984). Dynamics of Narrow Rings. R. Greenberg and A. Brahic (eds) *Planetary Rings*. Tucson, University of Arizona Press. pp. 589-639. [General review of the physics of narrow ring systems, including resonances and local perturbations.]
- de Pater, I. S.G. Gibbard E. Chiang H.B. Hammel B. Macintosh F. Marchis S.C. Martin H.G. Roe and M. Showalter. (2005). The dynamic Neptunian ring arcs: Evidence for a gradual disappearance of Liberté and resonant jump of Courage. *Icarus*. Vol 174 pp 446-454. [Describes observations showing temporal changes in Neptune's Adams ring.]
- de Pater, I. S.G. Gibbard H.B. Hammel. (2006a). Evolution of the dusty rings of Uranus. *Icarus*. Vol. 180 pp 186-200. [Describes observations hinting at temporal changes in the dusty rings around Uranus.]
- de Pater, I. H.B. Hammel S.G. Gibbard M.R. Showalter. (2006b). New dust belts of Uranus: One ring, two ring, red ring, blue ring. *Science*. Vol. 317 pp 1888-1890. [Describes the discovery of two faint Uranian rings, including one that appears to be generated by collisional debris from a small moon.]
- Dougherty, M.K. L.W. Esposito, and S.M. Krimigis. (2009). *Saturn from Cassini Huygens*. New York, Springer. [A book that provides a general overview of the current state of knowledge of many aspects of the Saturn system, including Saturn's rings.]
- Dumas, C. R.J. Terrile B.A. Smith G. Schneider and E.E. Becklin. (1999). Stability of Neptune's ring arcs in question. *Nature*, Vol 400 pp 733-735. [Discusses complications associated with potential resonant confinement mechanism for Neptune's ring arcs.]
- Esposito, L.W. A. Brahic J.A. Burns and E.A. Marouf. (1991). Particle properties and processes in Uranus' rings. Bergstralh, J.T. E.D. Miner and M.S. Matthews. *Uranus*. Tucson, University of Arizona

Press. pp. 410-465. [The most recent overview of the particle size distribution and material composition of Uranus' rings.]

French, R.G. P.D. Nicholson, C.C. Porco and E.A. Marouf. (1991). Dynamics and structure of the Uranian rings. Bergstralh, J.T. E.D. Miner and M.S. Matthews. *Uranus*. Tucson, University of Arizona Press. pp. 327-409. [The most recent overview of the dynamical processes operating in Uranus' rings.]

Goldreich, P. and S. Tremaine. (1982). Dynamics of Planetary rings. *Ann. Rev. Astron. Astrophys.* Vol 241. pp 425-441. [A good overview of some of the important dynamical processes that can operate in dense rings, including discussions of spiral waves.]

Greenberg, R. and A. Brahic. (1984). *Planetary Rings*. Tucson, University of Arizona Press. [A good book containing many chapters discussing the dynamics of planetary ring systems.]

Hahn, J.M. J.N. Spitale and C.C. Porco. (2009). Dynamics of the sharp edges of broad planetary rings. *ApJ*. Vol 699 pp 686-710. [Discusses simulations of sharp ring edges maintained by resonances.]

Hedman, M.M. J.A. Burns M.S. Tiscareno C.C. Porco G.H. Jones E. Roussos N. Krupp C. Paranicas and S. Kempf. (2007). The source of Saturn's G ring. *Science*. Vol 317 pp 653-656. [Describes observational evidence that a resonantly-confined arc of debris is the source of the G-ring]

Hedman M.M. J.A. Burns M.S. Tiscareno and C.C. Porco. (2009a). Organizing some very tenuous things, Lindblad resonances in Saturn's faint rings. *Icarus*. Vol 202 pp 260-279. [Describes structures in Saturn's tenuous rings that appear to be generated by Lindblad resonances.]

Hedman M.M. C.D. Murray, N.J. Cooper, M.S. Tiscareno, K. Buerle, M.W. Evans, J.A. Burns. (2009b). Three tenuous rings/arcs for three tiny moons *Icarus*. Vol 199 pp 378-286. [Describes resonantly-confined ring arcs associated with several of Saturn's moons.]

Hedman, M.M. J.A. Burt, J.A. Burns, M.S. Tiscareno. (2010a). The shape and dynamics of a heliotropic dusty ringlet in the Cassini Division. *Icarus*. Vol 210 pp 284-297. [Describes a ringlet in Saturn's rings that exhibits heliotropic behavior.]

Hedman, M.M. P.D. Nicholson, K.H. Baines B.J. Buratti C. Sotin R.N. Clark R.H. Brown R.G. French and E.A. Marouf. (2010b). The architecture of the Cassini Division. *AJ*. Vol 139 pp 228-251. [Describes occultation measurements of edge positions for the B-ring edge and various gaps and ringlets in the Cassini Division.]

Horanyi, M. J.A. Burns M.M. Hedman G.H. Jones and S. Kempf (2009). Diffuse rings. Dougherty, M.K., L.W. Esposito, and S.M. Krimigis (eds). *Saturn from Cassini Huygens*. New York, Springer. pp 511-536. [The most recent review of the structure and dynamics of Saturn's faint, dusty rings, including the D, E and G rings.]

Julian, W.H. and A. Toomre. (1966). Non-axisymmetric responses of differentially rotating disks of stars. *ApJ*. Vol 146 pp 810. [The paper where the critical wavelength of self-gravity wakes in dense disks was first derived (in a galactic context)]

Murray, C.D. and S.F. Dermott, (1999). *Solar System Dynamics*. New York, Cambridge University Press. [A very useful textbook on dynamical processes, which includes treatments of many dynamical phenomena that occur in dense rings.]

Namouni, F. and C.C. Porco. (2002). The confinement of Neptune's ring arcs by the moon Galatea. *Nature*. Vol 417 pp 45-47. [Describes a mixed resonance that could be responsible for confining Neptune's ring arcs]

Nicholson, P.D. I. Mosquera and K. Matthews. (1995). Stellar occultation observations of Neptune's rings: 1984-1988. *Icarus*. Vol 113 pp 295-330. [Describes observations of Neptune's ring arcs that complicate simple resonant models of their confinement.]

Porco, C.C. (1991). An explanation for Neptune's ring arcs. *Science*. Vol 253 pp 995-1001. [Presents a model in which Neptune's ring arcs are confined by a resonance with Galatea.]

Porco, C.C. P.D. Nicholson J.N. Cuzzi J.J. Lissauer and L.W. Esposito. (1995). Neptune's ring system. Cruikshank, D.P. (ed). *Neptune and Triton* Tuscon, University of Arizona Press. pp 703-804. [The most recent review of the structure and dynamics of Neptune's ring system.]

- Porco, C.C. P.C. Thomas J.W. Weiss and D.C. Richardson. (2007). Saturn's small satellites: Clues to their origins. *Science*. Vol. 318 pp 1602-1607. [Presents shapes, sizes and densities of several of Saturn's small moons that are closely associated with the rings.]
- Robbins, S.J. G.R. Stewart M.C. Lewis J.E. Colwell and M. Sremečević. (2010). Estimating the masses of Saturn's A and B rings from high-optical depth N-body simulations and stellar occultations. *Icarus*. Vol 206 pp 431-445. [Presents simulations showing a strongly non-linear relationship between a rings' opacity and its surface mass density]
- Salo, H. (2001). Numerical simulations of the collisional dynamics of planetary rings. *Lecture Notes in Physics* Vol 564 pp 330. [Paper discussing the dynamics of inter-particle interactions within a dense ring.]
- Salo, H. R. Karjalainen and R.G. French. (2004). Photometric modeling of Saturn's rings II. Azimuthal asymmetry in reflected and transmitted light. *Icarus*. Vol 170 pp 70-90. [Paper discussing simulations and observations of self-gravity wake structures in Saturn's rings.]
- Schmidt, J. K. Ohtsuki N. Rappaport H. Salo and F. Sphan (2009). Dynamics of Saturn's dense rings. Dougherty, M.K., L.W. Esposito, and S.M. Krimigis (eds). *Saturn from Cassini Huygens*. New York, Springer. pp 413-458. [A good discussion of many aspects of the dynamics of dense rings, especially inter-particle interactions]
- Shu, F. 1984. Waves in planetary rings. R. Greenberg and A. Brahic (eds) *Planetary Rings*. Tucson, University of Arizona Press. pp. 513-561. [The classic reference on the dynamics of spiral waves in dense rings.]
- Sicardy, B. F. Roddier C. Roddier E. Perozzi J.E. Graves O. Guyon and M.J. Northcott. (1999). Images of Neptune's ring arcs obtained by a ground-based telescope. *Nature*. Vol 400 pp 731-733. [Describes observations of Neptune's ring arcs that complicate simple resonant models of their confinement.]
- Spitale, J.N. and C.C. Porco. (2009). Time variability in the outer edge of Saturn's A-ring revealed by Cassini imaging. *AJ*. Vol 138 pp 1520-1523. [Describes observations of the resonantly-confined outer edge of Saturn's A ring.]
- Spitale, J.N. and C.C. Porco (2010). Detection of free unstable modes and massive bodies in Saturn's outer B ring. *AJ*. Vol 140 pp 1747-1757. [Describes observations of the resonantly-confined outer edge of Saturn's B ring.]
- Sremečević, M. J.Schmidt, H. Salo, M. Siess, F. Spahn and N. Albers. (2007). A belt of moonlets in Saturn's A ring. *Nature*. Vol 449 pp 1019-1021. [Describes observations of propellers in Saturn's A ring]
- Thomson, F.S. E.A. Marouf G.L. Tyler R.G. French and N.J. Rappaport. (2007). Periodic microstructures in Saturn's rings A and B. *GRL*. Vol 34 L24, 203. [Describes radio occultation observations of possible overstable structures in Saturn's rings.]
- Tiscareno, M. S. (2012) Planetary Rings. L. French and P. Kalas (ed.), *Solar and Planetary Systems*. Springer (arXiv 1112.3305) [A recent chapter reviewing the current state of knowledge of various planetary rings. Includes a particularly interesting discussion of the Roche limit in these systems.]
- Tiscareno M.S. J.A. Burns M.M. Hedman C.C. Porco J.W. Weiss L. Dones D.C. Richardson and C.D. Murray. (2006). 100-meter-diameter moonlets in Saturn's A ring from observations of "propeller" structures. *Nature*. Vol 440 pp 648-650. [Describes discovery of propellers in Saturn's rings]
- Tiscareno, M.S. J.A. Burns P.D. Nicholson M.M. Hedman and C.C. Porco. (2007). Cassini imaging of Saturn's rings II. A wavelet technique for analysis of density waves and other radial structures in the rings. *Icarus*. Vol 189, 14-34. [Describes technique for quantifying and characterizing density waves and moonlet wakes in Saturn's rings.]
- Tiscareno, M.S. J.A. Burns M.M. Hedman and C.C. Porco. (2008). The population of propellers in Saturn's A ring. *AJ*. Vol 135 pp 1983-1091. [Describes observations of propellers in Saturn's A ring]
- Tiscareno, M.S. *et al.* (2010). Physical characteristics and non-Keplerian orbital motion of "propeller" moons embedded in Saturn's rings. *ApJL*. Vol 718 pp L92-96. [Describes observations of propellers that demonstrate these objects do not follow simple Keplerian orbits.]
- Tremaine, S. (2003). On the origin of irregular structure in Saturn's rings. *AJ*. Vol 125 pp 894-901. [Describes a theoretical model of extremely dense ring systems where particles become locked into rigid aggregates.]

Verbiscer A.J. M.F. Skrutskie and D.P. Hamilton. (2009). Saturn's largest ring. *Science*. Vol 461 pp 1098-1100. [Describes the discovery of an enormous, tenuous ring around Saturn that was produced by debris knocked off of distant moons like Phoebe.]

Weiss, J.W. C.C. Porco and M.S. Tiscareno. (2009). Ring edge waves and the masses of nearby satellites. *AJ*. Vol 138 pp 272-286. [Describes observations and simulations of perturbations on gap edges produced by nearby moons.]

### **Biographical Sketch**

**Matthew Hedman** (born in 1974 in St. Paul, Minnesota, USA) is an assistant professor at the University of Idaho. For the last 12 years, he has been working for the Cassini Mission to Saturn on a variety of efforts to understand the structure, composition and dynamics of Saturn's rings.