THE DESIGN OF EXPERIMENTS

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Summary

Experimental design as a statistical methodology started in agriculture but today is applicable to virtually all areas of scientific endeavor. This chapter looks at the key features of a basic design, how the experiment itself should be conducted and how the subsequent data are analyzed. The chapter then addresses two major designs, split-plot designs and repeated measures designs, which frequently arise when the underlying conditions for a basic design do not pertain. In a split-plot design, there are restrictions imposed on the randomization process, often for sheer practical reasons; while in a repeated measures design, any one experimental unit (subject) is used for several treatment levels rather than for one level only as in a standard design. Although different designs produce data that can be organized into seemingly similar tables, the subsequent analysis differs according to the design used and so can produce different results and conclusions. Some key principles are discussed and illustrated.

1. Introduction

The statistical design of experiments has its historical roots firmly planted in the agricultural sciences dating back to the early days of statistics itself in the first third or so of the twentieth century. Work in the United Kingdom tended to revolve around researchers connected to the Rothamsted Experimental Station, while that in the United States tended to emanate out of the corresponding Research Laboratory at what is now called the Iowa State University in Ames. Both places served as hubs for a plethora of activity. Over time, key personnel moved to other locations to establish new hives of activity. Today, decades later, the subject is well advanced and quite sophisticated; and, likewise, areas of application cover an extensive list of scientific fields, e.g., agriculture, meteorology, environmental sciences, social sciences, business, earth sciences, engineering of all kinds, in fact almost every area of scientific endeavor imaginable.

Since this chapter cannot possibly cover the entire breadth of this rich field, attention herein will focus on a few key principles instead.

One key component of any experiment is its actual design. A particular basic design is a factorial design (and/or variations thereof). For example, suppose it is desired to study the impact of fertilizer on the yields of several varieties of a crop (sorghum, corn, wheat, apples, etc.). While the collected data may be organized systematically into a table (such as Table 1), the experimental units to which the treatments (here fertilizers and varieties) were applied would not be systematically arranged. How these designs are constructed and the experiment run is discussed in Section 2. In a very real sense, the factorial design represents the ideal design. Unfortunately, however, there may be real practical considerations that in effect impose restrictions which limit designing a pure factorial design. Consequently, since the substantive scientific questions and answers are still demanded, the experimenter is compelled to design an experiment around those enforced restrictions. Each of the various restrictions leads to various particular experimental designs. There are endless possibilities. Two broad classes of designs constructed to accommodate two frequently occurring restrictive situations are the class of split-plot designs and the class of repeated measures designs, covered in Sections 3 and 4, respectively.

Once the design is constructed and the data collected, the analysis and interpretation of the results proceeds. This is also covered in the respective Sections 2-4. What becomes clear is the fact that how the experiment was designed dictates how it is to be analyzed. Further, using an inappropriate analysis on data from a particular design will give incorrect answers; see Section 5.

Preceding the design and subsequent analysis, is the initial stage. This is where the scientist (the agriculturalist, meteorologist, environmentalist, biologist and others) must determine what information is being sought from the experiment. In the example on crop yield, is it the influence of only fertilizers on the various varieties of the crop (sorghum, say)? Or, do soil types play a role? Or, is it both? Do these interact in some way, e.g., do some varieties grow better in some soils than in others? Perhaps, it is the level of irrigation (water) that is the important factor either singly or in combination with other factors. Again there are many potential sources of variation which can influence the actual value of what is being measured (here, the yield). Once it has been determined which factors are to be studied, the scientist then has to consider which levels of each factor are to be used. For example, should fertilizer be merely absent or present? And, if present, how much fertilizer should be used? Should the levels be 0, 1, 2, (say) oz per unit area, or should they be 10, 20, 40 (say) oz, or whatever? Is there interest in all the varieties (of sorghum), or in only certain specific varieties? The questions are many. Answers have to be found before the experiment can be designed. This chapter assumes these questions have been asked and answered. It also assumes practical limitations have been explained to the (statistical) expert designing the experiment. However, it is most likely that it is only in the give-and-take of the collaborations between the field scientist and the statistical scientist that these restrictions and the design resolutions are effectuated in a proper manner.

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Bibliography

There are literally hundreds of texts and reference papers covering design issues. Some focus on application issues; some tend to be solely theoretical in nature; some are oriented to particular discipline specific areas; with others covering various other possibilities across this extensive range. The following list presents two texts that have been "classics" and as such are still widely available, plus one recent text targeted for the nonmathematical reader. Also, listed are three classic journal articles which together cover important basic concepts and which are as relevant today as when published originally.

Bartlett, M. S. (1947). The use of transformations. *Biometrics* 3, 39-52. [Describes the importance of what transformations can be used for various types of data to ensure underlying model assumptions hold.]

Cochran, W. G. and Cox, G. M. (1957). *Experimental Designs* (2nd ed.), John Wiley, New York. [Has an extensive collection of well illustrated examples, mostly from agriculture. Is a universal text, and a beloved classic.]

Cochran, W. G. (1947). Some consequences when the assumptions for the analysis of variance are not satisfied. *Biometrics* 3, 22-38. [Describes the impact of departures from model assumptions; this involves robustness issues.]

Cox, D. R. (1958). *Planning of Experiments*. [A classic text, elegantly describes the basic principles and philosophical issues underlying the design and planning of experiments.]

Dean, A. and Voss, D. (1999). *Design and Analysis of Experiments*, Springer-Verlag, New York. [Written for the applied user, this text presents a nice coverage of basic designs and includes the use of the SAS statistical package as an adjunct to the many examples examined.]

Eisenhart, C. (1947). The assumptions underlying the analysis of variance. *Biometrics* 3, 1-21. [Looks at model assumptions and their importance, including especially the distinctions between fixed and random effects.]

Biographical Sketch

Lynne Billard was born in Toowoomba, Australia; she obtained her BS First Class Honors degree (1966) and PhD degree (1969) in Statistics from the University of New South Wales Australia. She has spent 13 years in administration including 9 years as Head of Statistics and 2 years as Associate to the Dean. Has held academic positions in Australia, Britain, Canada and the USA. Currently, she is University Professor and Professor of Statistics at the University of Georgia where she has taught design of experiments to graduate students since 1980; she has also taught a wide range of courses (including time series, introductory statistics, theoretical statistics) to both undergraduate and graduate students. She has over 150 publications mostly in the major journals including 6 books edited or co-edited, in sequential analysis, AIDS and epidemics, time series, and inference, with applications in agriculture, biology, epidemiology, education and social sciences. She has been accorded many honors and awards including the 1990 American Statistical Association's (ASA) Award for Outstanding Statistical Application paper (shared), the ASA 1999 Wilks Award and the ASA's 2003 Founders Award, and the University of Georgia Creative Research Award. She has held numerous professional offices including International President 1994 and 1995 of the International Biometric Society and was the 1996 President of the American Statistical Association. She has served on the International Council of both the International Biometric Society and of the International Statistical Institute, and was the 1985 President of the Eastern

North American Region of the International Biometric Society, and served on the executive committee as Program Secretary of the Institute of Mathematical Statistics. She has served on many US national committees including the Advisory Committee for DMS National Science Foundation, Panel on AIDS and Panel on Microsimulation Modeling of Social Welfare Policy both for the National Research Council, the National Academy of Sciences' Board of Mathematical Sciences, was Chair of the Conference Board of Mathematical Sciences, and numerous review panels for the National Institute of Health and the National Science Foundation as well as the UK Research Council. She currently serves on the US Secretary of Commerce Census Advisory Committee. She is a Fellow of the American Statistical Association and the Institute of Mathematical Statistics and an elected Member of the International Statistical Institute.