MATHEMATICAL MODELS OF MANAGEMENT OF THE ENVIRONMENT AND ITS NATURAL RESOURCES

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Contents

Summary

The purpose of this chapter is to provide a basic overview of the main mathematical models for the management of the environment and its natural resources. The models attempt to be a fundamental underpinning towards the understanding and management of current important environmental problems such as:

a) The phenomenon of the excessive provision of environmental bads in modern industrialized countries;
b) The clear insufficiency of the market mechanisms in order to achieve a socially optimum provision of environmental public goods;
c) The risk of biological collapse of many natural resources when managed without adhering to certain conditions.

In all the cases analyzed, the strategy of presentation is as follows. Once the economic logic underlying each situation is fully understood, several mechanisms and policy instruments devised to address the corresponding problem are presented; that is, in spite of the analytical orientation of the chapter, some of the most important environmental implications are highlighted.

In short, the mathematical models presented attempt to be tools towards the understanding of some of the basic problems related to the possible compatibility between the environment as a life support system and as support of the economic activity.

1. Introduction

The purpose of this chapter is to present, at an introductory level, the mathematical models that support the field of environmental and natural resources management. The variety of models, as well as the number of recent contributions is quite impressive. Consequently, in order to keep the length of the chapter within sensible limits, the presentation will focus on the basic mathematical models that provide support to more advanced approaches. The bibliography at the end of the chapter provides the reader with guidelines for further reading.

The chapter is organized as follows. Section 2 explains how most production processes provide society not only goods sold in markets but also environmental public "bads" or negative externalities (pollution) and environmental public goods or positive externalities without markets. Externalities generate a "market failure" which makes it impossible for market mechanisms to provide a socially efficient allocation of resources. Section 3 presents an overview of the basic models devised to determine the socially optimal level of negative externality. Section 4 presents the main policy instruments designed for achieving of the optimal levels of externality previously defined. Section 5 is devoted to the presentation of the basic analytical for the determination of the socially optimum level of environmental public goods. Finally, Section 6 presents a unified framework for the optimal management of natural resources.

2. Positive and Negative Externalities

An externality exists when the output of a firm or the satisfaction of a consumer is affected not only by the values achieved for the variables controlled by the producer or consumer but also by variables controlled by another economic agent. Let us introduce the following production function:

$$y = y(v_1, v_2, ...v_i, ..., v_n, w)\,, \qquad (1)$$

where y represents the amount of output, $v_1, v_2, ...v_i, ..., v_n$ are the levels of the inputs under the control of the entrepreneur and w the amount of another input controlled by another economic agent (an environmental input in our context). There are two possibilities. First, an increase in the value of the environmental input w implies an increase in the amount of output y (i.e., $\partial y / \partial w > 0$).

In this case, we are in the presence of a positive externality. Second, an increment in the value of the environmental input w implies a decrease in the amount of output y (i.e., $\partial y / \partial w < 0$). In this case, we are in the presence of a negative externality.

We have illustrated the analytical structure for the generation of externalities (positive and negative) when the generators (polluters) as well as the sufferers of the externality are producers (i.e., an externality of a production process into another production process). To illustrate the concept of externality on consumption processes, let us now introduce now the following utility function:

$$u = u(x_1, x_2, ..., x_m, z), \qquad (2)$$

where u represents the utility or satisfaction derived from the consumption of the commodity basket $x_1, x_2, ..., x_m$ and z represents a variable, again of environmental nature in our context, controlled by another economic agent. Once gain, there are two possibilities. First, an increase in the value of the environmental variable z implies an increase of utility or satisfaction (i.e., $\partial u / \partial z > 0$). In this case, we are in the presence of a positive externality. Second, an increase in the value of the environmental variable z implies a decrease of utility or satisfaction (i.e., $\partial u / \partial z < 0$). In this case, we are in the presence of a negative externality. Although, we have illustrated the concept of externalities when both the generator (polluter) and the sufferer are producers or consumers it is rather obvious that the concept can be easily extended to a mixed context of producers and consumers. Thus, in a broad sense, we can say that an externality exists when the activity of an economic agent positively or negatively affects to the economic activity of another agent.

The ideas outlined above are crucial for our work for the following reasons:

a) Most of the production processes undertaken in a modern economy not only provide goods sold in the markets but also environmental public bads (pollution) that are, from an economic point of view, negative externalities.
b) Many production processes provide outputs sold in markets as well as environmental goods that are, from an economic point of view, positive externalities. These environmental goods are public goods produced in an unintentional way by entrepreneurs that do not receive any economic compensation for their provision.
c) In the presence of externalities (positive or negative), there is the existence of what is known as a “market failure”. It is well known that a “market failure” avoids the existence of competitive market equilibrium and/or an efficient allocation of resources.

In line with the ideas presented above, the mathematical models of management of the environment and its natural resources should address the task of determining the socially optimal levels of environmental public bads (negative externalities) as well as of environmental public goods (positive externalities). Finally, we will need models to support policy instruments to achieve these social optima. These problems will be addressed in the following sections.

3. Socially Optimum Provision of Environmental Bads

This section presents the basic theoretical framework for establishing the optimal level of negative externality (pollution) generated by a polluter firm. We will undertake the task by referring the analysis to a perfectly competitive polluter firm (i.e., a price-taker), although the basic analysis presented is extensible to other market structures. As a first step in our analysis, let us determine the transformation curve or production possibility frontier in the private benefits-external costs space. The private benefit function PB for the polluter or externality generator is given by:

$$PB = px - C(x)\,, \qquad (3)$$

where p is the market price, x the amount of output produced and $C(x)$ the internal cost function.
Let us now represent the external cost of the sufferer as:

$$EC = g(x)\ . \qquad (4)$$

From Eq. (4) we obtain:

$$x = g^{-1}(EC)\,. \qquad (5)$$

By substituting Eq.(5) in Eq.(3) we have:

$$PB = pg^{-1}(EC) - f[g^{-1}(EC)] = \Phi(EC)\ . \qquad (6)$$

Expression (6) represents the transformation curve or production possibility frontier in the *PB-EC* space shown in Figure 1. This frontier function can be represented in implicit equations such as:

$$T(PB, EC) = K\ . \qquad (7)$$

The next ingredient that we need is a social welfare function representing the preferences of the society for the private benefits *PB* and for the external costs *EC*, such as:

$$U = U(PB, EC)\,. \qquad (8)$$

It is now assumed that the social welfare function (8) holds the usual properties (i.e., $U_{PB} \geq 0$, $U_{EC} \leq 0$ $U_{PBPB} \leq 0$, $U_{ECEC} \geq 0$). From this setting the following three optima can be defined.

Environmental optimum

This point does not have an economic meaning. It corresponds to the origin of co-ordinates that implies nil production of the output considered. For this solution there is no externality, indeed, there is no production! This solution can be justified only when the damage to the environment caused by the economic activity is considered irreversible.

Private optimum

This point corresponds to the point where the private benefit PB achieves a maximum value PB^{MAX}. It is represented by the point $Z(PB^{MAX}, EC^{MAX})$ of Figure 1. Without any kind of intervention or negotiation process this point Z represents the market equilibrium or solution.

Social optimum.

The social optimum corresponds to the point where the social welfare function $U(PB, EC)$ achieves a maximum value over the production possibility frontier $T(PB, EC) = K$. This optimum point will be obtained by solving the following optimization problem:

max $U(PB, EC)$
Subject to: $T(PB, EC) = K$.

Graphically the social optimum corresponds to the point of tangency $Z_1(PB^*, EC^*)$ between the family of iso-utility contours $U(PB,EC) = \lambda$ and the frontier, as shown in Figure 1. This simple analysis clarifies an important result. In presence of negative externalities (pollution), the simple functioning of the market mechanisms provides an inefficient allocation of resources.

In other words, there is a divergence between the private optimum and the social optimum. Thus, in the private optimum (market solution) there is an excess of private benefits equal to PB^{MAX}-PB^* and an excess of externality (environmental damage) equal to EC^{MAX}-EC^* (see Figure 1).

In other words, there is a socially optimal level of externality equal to EC^*. The rest of the externality EC^{MAX}-EC^* is socially unwarranted and has to be removed by a market intervention or any other policy instrument.

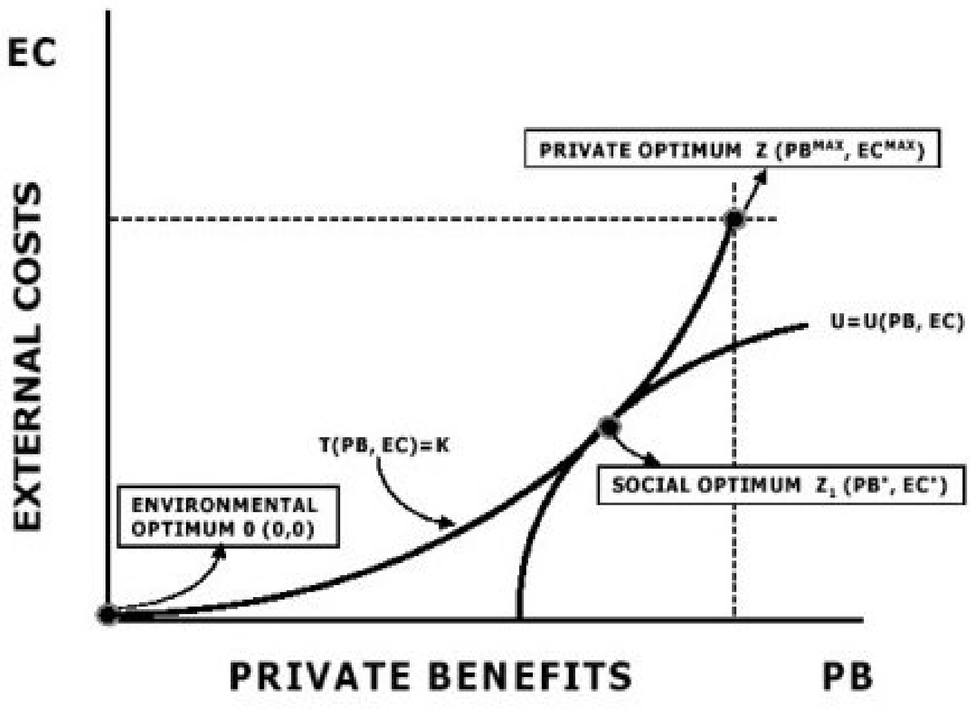

Figure 1. Transformation Curve, Environmental Optimum, Private Optimum and Social Optimum (Environmental Bads).

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Biographical Sketch

Carlos Romero is a Professor of Economics at the Technical University of Madrid. Formerly he was Professor of Agricultural Economics at Córdoba University and Visiting Professor at Reading University. He is the author of nine books in Spanish on agricultural economics, environmental economics and decision analysis. He is also the author of *Handbook of Critical Issues in Goal Programming (*Pergamon Press, 1991) and co-author of *Multiple Criteria Analysis for Agricultural Decisions* (Elsevier, 1989) and *Multiple Criteria Decision Making and its Applications to Economic Problems* (Kluwer, 1998). Dr. Romero has published more than 130 papers, about 60 of them in international refereed journals such as: *Agricultural Systems, American Journal of Agricultural Economics, Applied Mathematics Letters, Computers and Operations Research, Engineering Optimization, European Journal of Operational Research, European Review of Agricultural Economics, Forest Ecology and Management, Forest Science, Journal of Agricultural Economics, Journal of Environmental Management, Journal of Multi-Criteria Decision Analysis, Journal of Optimization Theory and Applications, Journal of the Operational Research Society, Lecture Notes in Economics and Mathematical Systems, Omega, Operations Research Letters, Theory and Decision* and others. He has co-edited a special issue of *Agricultural Systems* and recently two volumes of *Annals of Operations Research* devoted *to* mathematical models in the management of natural resources. Dr. Romero was a member of the EURO Gold Medal Jury He received in 1994 the Research Award of the Technical University of Madrid and in 2001 the National Prize of "Economics and the Environment". He is a Fellow of the Operational Research Society and of the World Academy of Productivity Science and Member of the Executive Committee of the International Society on Multiple Criteria Decision Making. Professor Romero is chiefly interested in the development of goal programming and another MCDM methods and its connection with applied and theoretical problems related to the management of the environment and its natural resources.