MATHEMATICAL MODELING IN AGRICULTURAL ECONOMICS

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Contents

Summary

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The discipline of agricultural economics has played a pioneering role in the application of mathematical models in economics. Originally, farm management specialists developed enterprise budgets to help farmers identify their best farming activities. These budgets, which specified the amounts of inputs required and outputs generated by allocating a unit of land to various crop or livestock production activities, facilitated the application of mathematical programming techniques as they were developed to offer normative advice to farmers. Eventually, programming models were expanded to offer advice at regional and national levels on issues as diverse as commercial agricultural policy, environmental policy, water and soil conservation policy, and public investment in infrastructure such as waterway and irrigation development. Agriculture was a natural application of stochastic and dynamic programming because of the stochastic nature of production and prices and the dynamic issues of capital investment and grain stock accumulation.

On another front, agricultural economics played a key role in the development and application of econometrics for positive modeling purposes. With a wide variety of reasonably competitive product markets and abundant public data, agriculture presented a ready field of application for various statistical regression techniques as they were developed. Simultaneous equation models were easily expanded to address issues of international trade as they became important. In many cases, agricultural production characteristics such as heterogeneity, imperfect anticipation of product prices, and production risk motivated development and refinement of econometric techniques. With perceived competitive conditions, agriculture served as a ready laboratory for application of techniques that increasingly incorporated microeconomic theory in model specification, first with primal and then with dual approaches. In more recent times, as contracting has replaced open market transactions, agriculture has become a ready laboratory for application of game theory and the principal-agent models of mechanism design that now dominate microeconomic analysis.

1. Introduction: The Pioneering Role of Agricultural Economics in Mathematical Modeling

The discipline of agricultural economics has played a pioneering role in the application of mathematical models in economics. The discipline began with the study of farm management in the nineteenth century, when a majority of all families were involved in some form of farming. In the United States, The Morrill Act of 1862 set up land grant colleges and was soon followed by the Hatch Act of 1887, which set up agricultural experiment stations connected with land grant colleges. This led to the development of academic disciplines including agricultural economics, and research efforts intended to improve productivity and management of family farms. Later, the Smith-Lever Act of 1914 set up cooperative extension services attached to these institutions of higher learning, which had the purpose of extending new research knowledge to farmers to promote its early implementation. This system of combining teaching, research, and extension in institutions of higher learning was so successful that it has been emulated throughout the world.

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Early research in agricultural economics in these institutions focused on farm management. Enterprise budgets were developed that offered farmers guidelines on input use and costs as well as average output quantities and revenue associated with allocating a unit of land to alternative crop or livestock production activities using production practices prescribed by the production sciences. When linear programming became practical in the 1950s and 1960s with the growth of computing power, these enterprise budgets provided the data necessary to implement this new optimization tool. Further, the large number of alternative farming production activities and the wide variety of constraints associated with land, labor, capital, and machinery capacity made farming a useful demonstration of the tool. Agricultural economists applied programming tools widely, readily adopting and in some cases innovating subsequent generalizations associated with quadratic and nonlinear functions representing risk, non-continuous decision variables, stochastic constraints, and multiple objective criteria. Owing heavily to Earl O. Heady at Iowa State University, a major land grant institution, programming models were expanded during the 1960s from the farm level to various regional levels with activities and constraints numbering in the thousands.

After the initial focus on farm management, the discipline expanded to study marketing with the intent of improving farmers' well-being by more effective purchasing of inputs and selling of outputs. Accordingly, the name of the discipline was changed from farm management to agricultural economics. Because the markets for grains and livestock are so interrelated, this led to modeling market supplies and demands as systems of equations, and to the need for estimating those systems. At the same time, agricultural production economists recognized that the budgets based on production scientists' notions of optimality were based more on maximizing physical output rather than maximizing farmers' profits. This led to estimation of production functions specifying output quantities as functions of factor input levels and other characteristics of the production environment including human capital. From these, agricultural economists determined profit maximizing input and output levels and their response to changes in prices and technology. These studies were both normative (prescribing optimal choices) and positive (reflecting actual behavior). The study of farm management thus grew into the study of production economics.

The need to estimate relationships in both production and marketing led to the application of statistical methods in economics, now called econometrics. One of the early efforts occurred in the 1930s when the California land grant college at Berkeley (now the University of California) hired a young psychometrician, George M. Kuznets. He and colleague Ivan M. Lee were soon training the likes of Lawrence R. Klein, Arnold Zellner, Zvi Griliches, and Yair Mundlak, who were early leaders in the development of econometrics. Arnold Zellner was either inventor or co-inventor of two of the three most common multiple-equation estimation methods (three-stage least squares and seemingly unrelated regression). Together with the development of time series techniques, huge econometric models were developed in the 1970s and implemented commercially with the intent of predicting prices and quantities in agricultural markets.

As estimation techniques were perfected and computer capacity was expanded, and as agriculture was heavily regulated following the depression politics of the 1930s, the

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study of agricultural policy became an important sub-discipline in agricultural economics. Accordingly, the discipline matured as a social science, seeking to understand behavior in order to predict how farmers would respond to changes in agricultural policies. In turn, this led to new normative pursuits, seeking to optimize social well-being through the design of agricultural policy. Application of welfare economics (the economics of social well-being) as a policy analysis tool became a primary purpose for mathematical modeling. Agricultural economists played a major role in generalizing the methodology of welfare economics to cases involving risk behavior and interrelated markets.

Finally, with the game theory revolution in economics led by Nobel laureates John C. Harsanyi, John Nash, and Reinhard Selten, many further aspects of generalization have been incorporated into mathematical models used and applied in agricultural economics. As models have focused on bilateral agreements and contracting between individuals, firms, and countries, the lines between normative and positive pursuits have blurred, data representing actual choices in the field have been increasingly hard to obtain (because they are proprietary), and economic analysis has turned increasingly toward theoretical and conceptual mathematical models. Thus, the mathematical sophistication of the discipline continues to increase to the point that a paper devoid of substantial mathematics can hardly be found in the current academic journals of the discipline.

2. Simulation Models and Normative Modeling

The earliest and simplest matrix-oriented models in agricultural economics were input-output models as developed by Nobel laureate Wassily Leontief. His input-output model consisted of a square matrix of fixed input-output coefficients, $\mathbf{A}$, for a variety of production activities such that the vector of outputs (supplies), $\mathbf{s}$, is exactly equal to the vector of inputs (demands), $\mathbf{d}$. Equilibrium satisfies $(\mathbf{I} - \mathbf{A})\mathbf{s} = \mathbf{d}$ where $\mathbf{I}$ is the identity matrix and $\mathbf{As}$ is the production used as factor inputs in order to supply excess production $\mathbf{s}$ necessary to satisfy demand $\mathbf{d}$. The solution is the unique set of activity levels that exactly satisfies all constraints.

Such models were lacking for agricultural production applications because they assume all resources are fully exhausted and do not optimize profit or some measure of economic benefits. Consequently, the primary use of input-output models has been in regional planning and rural development where economic multipliers represent the equilibrium benefits from a project after all effects filter through various sectors of the target economy. Linear programming and other mathematical programming techniques are better suited where optimization is of interest.

Programming models have been highly attractive to applied decision makers and agricultural economists offering practical advice in farming and regional agricultural development because the models are easily understood by lay decision makers and they require little data compared to sophisticated econometric models. They represent the highly intuitive Von Liebig production specification where production follows fixed input-output coefficients but is limited by resource constraints. Programming models require explicit specification of the decision maker’s goal, usually specified as profit maximization, and require knowledge of all relevant constraints and relationships

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determining output and profit. The most basic and widely applied tool among a broad range of mathematical programming models in agricultural economics has been linear programming.

2.1. Linear Programming

Linear programming is a powerful tool for maximizing profits or expected profits when a decision maker has a collection of potential production activities that each use a variety of production resources with fixed input-output coefficients and the production resources are either available in fixed supply or can be purchased at fixed prices.

The standard linear programming problem is

$$\max_{x_1,\dots,x_n} \pi = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \\ &a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \\ &\qquad\qquad\vdots \\ &a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m \\ &x_1,\dots,x_n \ge 0, \end{aligned}$$

or, more compactly, $\max_{\mathbf{x}}(\mathbf{c}^{\mathrm{T}}\mathbf{x})$ subject to $\mathbf{Ax} \le \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$ where $\mathbf{c}$ is an *n*-vector with element c_j, $\mathbf{x}$ is an *n*-vector of activity levels x_j, $\mathbf{b}$ is an *m*-vector with element b_i, $\mathbf{0}$ is a vector of zeros, and $\mathbf{A}$ is an $m \times n$ matrix with element a_{ij} in the *i*th row and *j*th column.

Linear programming became practical with the introduction of the simplex method in the late 1940s, which allowed the problem to be solved in a small number of steps, and the development of computers in the 1950s and 1960s. The simplex method is an iterative method that introduces the next most profitable activity into the solution at each iteration until no improvements are possible. Assuming the number of constraints is less than the number of activities ($m < n$), the optimal solution has no more than m activities at positive levels.

In agricultural production applications, π is expected profit, c_j is expected profit per acre devoted to crop or livestock production activity j, x_j is the number of acres devoted to crop or livestock production activity j, a_{ij} is the number of units of resource i required by crop or livestock production activity j on one acre of land, and b_i is the maximum amount of resource i available on the farm. For example, if the first resource is land, then b_1 is total land available on the farm and each a_{1j} is 1. Alternatively, several resource constraints may be used to represent the availability of land of different

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qualities. If the second resource is family labor, then b_2 is the total availability of family labor and a_{2j} is the amount of family labor required to devote one acre to crop or livestock production activity j. Alternatively, several resource constraints may be used to represent the availability of labor in individual time periods throughout the cropping season. Other resource constraints may represent the capacity of various types of machinery or facilities, irrigation water availability, government program constraints, and capital requirements necessary to incur production expenses before harvest time.

A slight generalization can account for input purchases and output sales by letting some of the production activities represent the purchase of one unit of an input at a fixed price or selling one unit of an output at a fixed expected harvest-time price. Such buying and selling activities do not use land (the relevant a_{1j} is zero) and a constraint is added to account for each such input or output. In a constraint requiring purchases of an input to be at least as great as the amount used for production, the a_{ij} would be -1 in the activity representing purchase of the input and the a_{ij} in each production activity would be the number of units of that input required for one acre of production activity j. Likewise, in a constraint requiring sales of an output to be no more than production of that output, the a_{ij} would be 1 in the activity representing sales and the a_{ij} in each production activity producing that output would be the negative number of units of that output produced from one acre. The c_j would be the negative price of one unit of the input for each input purchasing activity or the expected price of one unit of the output for each output selling activity. The c_j on production activities would then include only costs and revenues not accounted separately by input purchasing and output selling activities.

Linear programming can also be used for a variety of other agricultural optimization problems such as finding a least-cost livestock feed mix satisfying minimal nutritional requirements (where maximization is replaced by minimization and the direction of the inequalities in the constraints are reversed). In this case, each activity represents a feed ingredient where c_j is the cost of one unit of ingredient j, each a_{ij} represents the amount of nutrient i contained in one unit of ingredient j, and b_i represents the overall feed mix requirement of nutrient i. With various generalizations and adaptations, linear programming can be used to find normative solutions for optimal farming systems in large regions as well as on individual farms. Such applications have been useful in evaluating benefits of potential public water projects that supply agricultural irrigation or potential navigable waterways that provide transportation options for agricultural output.

2.2. Integer Programming

A useful variation of linear programming is the case where the decision variables in $\mathbf{x}$ must take integer values. Introduced in 1958 by R.E. Gomory, integer programming incorporates additional constraints involving cutting planes that force the program to solve in integers. This was a useful innovation in agricultural economics because it allowed sensible representation of the adoption of technologies that require discrete

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investments in machinery or facilities and allowed optimization of transportation costs where grain must be shipped in whole truck, train, or barge loads.

2.3. Quadratic and Risk Programming

As the uses of linear programming have expanded, its shortcomings have also become apparent. When applied in modeling large regions, the assumption of fixed prices of inputs and outputs became questionable. If prices vary according to linear supplies and demands such that the price of an input increases linearly in the amount purchased and the price of an output declines linearly in the amount sold, then prices follow $c_j = c_{j0} + c_{j1}x_j$ or, more generally, $\mathbf{c} = \mathbf{c}_0 + \mathbf{C}\mathbf{x}^{\mathrm{T}}$ where $\mathbf{c}_0$ is an n-vector with element c_{j0} and $\mathbf{C}$ is an $n \times n$ matrix. Then the problem is to maximize $\mathbf{c}^{\mathrm{T}}\mathbf{x} = \mathbf{c}_0^{\mathrm{T}}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{C}\mathbf{x}$ subject to $\mathbf{A}\mathbf{x} \le \mathbf{b}$ and $\mathbf{x} \ge \mathbf{0}$, which is the quadratic programming problem. A fairly straightforward iterative generalization of the simplex algorithm made the solution of this programming problem feasible as computing power increased, but accordingly estimates of supply and demand systems are required to represent the coefficients of the objective function. In agricultural applications, these specifications represent systems of supplies of agricultural crop and livestock outputs and demands for agricultural inputs such as seed, fertilizer, pesticides and the like.

Another use of quadratic programming has been to represent risk aversion by farmers. If farmers discount expected profits in some fixed proportion to the amount of variance in profits, then the decision criterion is linear in the mean and variance of profits. If $\mathbf{c}_0$ is a vector of mean profits per unit of the various production activities represented by $\mathbf{x}$, and $\mathbf{C} = -\varphi\mathbf{\Sigma}$ where φ is the proportion by which expected profit is discounted per unit of profit variance and $\mathbf{\Sigma}$ is the covariance matrix of per unit profits of the various production activities, then $\mathbf{c}^{\mathrm{T}}\mathbf{x} = \mathbf{c}_0^{\mathrm{T}}\mathbf{x} + \mathbf{x}^{\mathrm{T}}\mathbf{C}\mathbf{x}$ is expected aggregate profit discounted for risk, typically called the certainty equivalent profit. Such models together with the technology constraints represented by $\mathbf{A}\mathbf{x} \le \mathbf{b}$ were among the earliest models used to model risk preferences in agricultural production. Later adaptations linearized the consideration of risk by replacing variance in the objective function with the mean of total absolute deviations.

2.4. Nonlinear Programming

One of the readily apparent shortcomings of linear programming was the assumption of fixed input-output coefficients. In economics, marginal productivity is widely held to be decreasing. That is, where $y = f(x_1, \ldots, x_n)$ represents a production function (x_i is the quantity used of each factor input i and y is the quantity of output produced therewith), practical assumptions are that f is increasing and concave in $\mathbf{x} = (x_1, \ldots, x_n)^{\mathrm{T}}$, i.e., the marginal productivity is positive, $\partial y / \partial \mathbf{x} > 0$, and decreasing in $\mathbf{x}$ such that $\partial^2 y / \partial \mathbf{x}^{\mathrm{T}} \partial \mathbf{x}$ is a negative definite matrix.

Theoretical development of the associated nonlinear programming problem began with the work of Harold W. Kuhn and Albert W. Tucker in 1951. Beginning with the

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problem of minimizing $f(\mathbf{x})$ subject to m constraints of the form $g_i(\mathbf{x}) \leq 0,\ i = 1, \ldots, m$, and non-negativity constraints $\mathbf{x} \geq \mathbf{0}$, they found that necessary and sufficient conditions for an optimal solution are

$$\nabla f(\mathbf{x}) = \sum_{i=1}^{m} \lambda_i \nabla g_i(\mathbf{x}),$$

$$\lambda_i g_i(\mathbf{x}) = 0, \quad i = 1, \ldots, m,$$

where $\lambda_i \geq 0,\ i = 1, \ldots, m$, are shadow values of the respective resource constraints and f and the g_i are continuously differentiable and concave functions that define a nontrivial and convex constraint set. Numerical algorithms have been developed to find such solutions, and the steady growth in computing power has made them feasible for practical application since the early 1970s. While many empirical applications have been made, the generality of the Kuhn-Tucker conditions has also proven useful for theoretical purposes.

Alternatively, because of the complexity of nonlinear programming in practice, most numerical applications in agricultural economics have relied on piece-wise linear approximations of linear functions, which can be implemented by a series of linear constraints as long as (a) weak concavity holds for constraints of the form $g_i(\mathbf{x}) \leq 0$ (weak convexity if the inequality is reversed) and (b) the objective criterion, $f(\mathbf{x})$, is weakly convex (weakly concave for minimization). Extension efforts in agricultural economics have found piecewise linearity to be a concept much easier to convey to lay audiences as well as scholars in the production sciences.

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Biographical Sketch

Richard E. Just is Distinguished University Professor of Agricultural and Resource Economics at the University of Maryland. Formerly, he was Professor of Agricultural and Resource Economics at the University of California, Berkeley, where he received both M.A. and Ph.D. degrees in 1971 and 1972, respectively. He is a fellow of the American Agricultural Economics Association and a former editor of the *American Journal of Agricultural Economics*. He is the youngest person ever named a fellow of the American Agricultural Economics Association and has received more research awards from that professional society than any other person. His research has ranged widely among fields in agricultural production, trade, policy, development, resource, and environmental issues. His work has been instrumental in modeling risk response, attitudes toward risk, the endogenous determination of risk and its welfare effects, and extending the methods of duality accordingly. He has extended the modeling of welfare economics to consider general equilibrium impacts and the role of information. A survey of the agricultural economics journals found his research to be cited more than any other author. He was listed in *Who's Who in Economics* as one of the major economists since 1700 after only 12 years of his professional career.

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