SPECIAL AND GENERAL RELATIVITY

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Summary

The introduction by Einstein of the idea that space and time are not absolute concepts in physics translated into a revolution in physics. In particular we have a new theory of gravity and general relativity, which predicts the existence of gravitational waves, black holes and the expansion of the universe.

1. Relativity in Physics

Many scientists consider that modern physics starts in the 17th century when Newton used differential calculus to formalize the laws of classical mechanics. Such laws describe the motion of particles and systems and introduce the basic concepts of mass, force, velocity and acceleration. Newton's fundamental law is that the acceleration of a particle is proportional to its velocity. Underlying the framework of classical mechanics is the choice of a frame of reference with respect to which to define the motion of particles. Newton's laws (as originally written) hold only in frames of reference called inertial. In fact the latter are defined as those frames where Newton's laws hold, and experimentally it is observed that inertial frames appear to move with constant velocity with respect to the frame defined by the position of distant stars in the universe. Different observers associated with different frames of reference that move at constant speed with respect to the distant stars (and therefore with respect to one another) all observe experimentally the same form for Newton's law of motion. The velocity of a system is a frame dependent object. If one measures the velocity with respect to a given frame and then considers another frame moving with respect to the first one the velocity with respect to the second one will be the sum of the velocity of the particle with respect to the first frame and the velocity of the second frame with respect to the first one (if the particle and the frame move in opposite directions; if they move in the same direction, it would be the difference). Throughout these considerations it is assumed that time is universal and invariant from one frame of reference to another. Changes of reference frame therefore only change velocities (and positions), not time.

The fact that Newton's theory is invariant under change of inertial reference frames constitutes what is called as "Galilean relativity". In this context, Newton's 2nd law that states that the acceleration is proportional to the mass is an invariant law. The velocities of the system under study are "co-variant" in the sense that they change from one frame to another in a well defined way. In Galilean relativity any value (in particular an arbitrarily large value) of the velocity is possible. Given a certain system, one can always encounter an inertial frame where it is moving at any conceivable speed. For all inertial systems there is a universally defined time that takes the same value in all systems.

Galilean relativity is a completely consistent logical framework. It is therefore not entirely surprising that for close to three centuries it was the accepted paradigm for transformations of reference frames in physics. Conceptually, problems started to arise with the introduction of Maxwell's theory of electromagnetic phenomena. Maxwell's equations are not invariant under Galilean relativity. Worse, they are not covariant either. Maxwell's equations are covariant under a different type of transformation called Lorentz transformations.

Lorentz transformations between two frames moving with respect to each other have important differences when compared to Galilean transformations. To begin with, they are transformations that mix space and time. That is, every inertial reference frame has a different notion of time. Moreover, the velocity of a system in a new frame that moves at a given speed with respect to another frame is not just given by a simple addition or subtraction of the frame relative speed. In fact, the transformation law is non-linear in such a way that speeds have a maximum value: nothing can exceed the speed of light. This is very counter-intuitive; one could imagine an object traveling at the speed of light and consider such object with respect to a moving reference frame. Wouldn't the object move faster or slower with respect to the new frame? In Lorentzian relativity the answer is no: an object moving at the speed of light moves at such speed with respect to all coordinate frames.

The reformulation of Newtonian mechanics in such a way that it is co-variant with respect to Lorentz transformations was carried out by Einstein although a revolution of this magnitude was obviously the buildup of many ideas contributed by various authors (see the reference by Whittaker for a historical account). The resulting theory is loosely referred to as "special relativity". The theory is most naturally formulated mathematically if one considers that space and time form a four dimensional space endowed with a metric structure that is not positive definite (the "distances" defined by the metric can be positive, negative or vanishing). In fact, the element of distance between two points in space-time is given by,

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \qquad (1)$$

Where we have chosen units such that the speed of light is unity (c=1), otherwise there would have been present a factor of c^2 in the first term on the right hand side.

Just like ordinary rotations in three dimensional space keep invariant the distance between two points, Lorentz transformations keep invariant the four dimensional element of distance defined above. In fact they can be viewed as "rotations" in four dimensions, except for the fact that due to the minus sign in the temporal direction of the line element the transformations are not really rotations. Notice that given a trajectory of a physical system in space-time, the "distance" Δs would correspond to the time measured by a clock at rest with respect to the system under study (for such a clock the spatial separation as the system evolve will remain zero). Therefore two physical systems whose trajectories intersect multiple times as they evolve will note when they compare their respective clocks that they have lost synchronism. This is known as "*the twin paradox*". That is, every reference frame has a clock associated with it that keeps a different time from that in other coordinate systems. Many apparent paradoxes can be constructed due to the removal of the lack of simultaneity from what appears our intuitive understanding of physics. A law similar to Newton's second law can be constructed in relativistic dynamics, stating that the four-dimensional acceleration of a particle is proportional to a four dimensional force. Force four dimensional vectors can be associated with a charge living in a Maxwell theory, for example, and the resulting mechanical theory yields usual electrodynamics of charges, in a Lorentz invariant fashion.

The fact that physical quantities are now associated with vectors and scalars that live in a four dimensional space-time rather than to vectors living in space as in ordinary physics implies that many "physical quantities" we are accustomed to view as invariant are not.

For instance, the energy is the time component of a four vector (the energy-momentum vector) and therefore changes from one frame to another. Particles have energy even in their rest frame, proportional to their mass (the famous $E=mc^2$ formula). Spatial lengths and time lapses are *not* invariant and objects viewed from moving frames appear to *contract* (effect known as *Lorentz contraction*). Electric and magnetic fields are not vectors anymore but the components of a four dimensional tensor (matrix). When one changes to a moving reference frame, a linear combination of electric and magnetic fields is the new electric field and similarly for the magnetic field.

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Biographical Sketch

Jorge Pullin, was born in Buenos Aires, Argentina in 1963. He studied at the University of Cordoba and got his Ph.D. in physics from the Balseiro Institute in Bariloche, having done research for his thesis at the University of Cordoba. He was a postdoctoral researcher at Syracuse University and the University of Utah and on the faculty at PennState. He currently holds the Horace Hearne Chair in Theoretical Physics at the Louisiana State University. He has received Alfred P. Sloan, John S. Guggenheim and Fulbright fellowships. He is a fellow of the American Physical Society and fellow and Chartered Physicist of the Institute of Physics (UK). He was the recipient of the Edward Bouchet award of the American Physical Society. He is currently the president-elect of the Topical Group in Gravitation of the American Physical Society. His research has been funded by the National Science Foundation of the US, NASA and NATO.