QUANTUM MECHANICAL LAWS

Bogdan Mielnik and **Oscar Rosas-Ortiz**
Departamento de Física, Centro de Investigación y de Estudios Avanzados, México

Keywords: Indeterminism, Quantum Observables, Probabilistic Interpretation, Schrödinger's cat, Quantum Control, Entangled States, Einstein-Podolski-Rosen Paradox, Canonical Quantization, Bell Inequalities, Path integral, Quantum Cryptography, Quantum Teleportation, Quantum Computing.

Contents:

Summary

The present day quantum laws unify simple empirical facts and fundamental principles describing the behavior of micro-systems. A parallel but different component is the symbolic language adequate to express the 'logic' of quantum phenomena. This report is the sum of both elements. Sections 1-7 present the decline of the classical theory and the evidence of the particle-wave duality on an elementary level. In Sections 8-9 the origin of the Schrödinger's wave mechanics and Born's statistical interpretation are

presented. The following Sections 10-23 are basically addressed to the adepts of exact sciences. They outline the formal and fundamental problems of Quantum Mechanics, interpretational paradoxes, concepts of *complementarity*, *teleportation* and the idea of *quantum computing*. Though using the symbolic language, Quantum Mechanics is not a hermetic science. It is an open domain, crucial for the present day particle physics, quantum field theory and contemporary informatics, though carrying a luggage of unsolved interpretational problems.

1. Introduction

Quantum Mechanics (QM) was one of the greatest revolutions in physics. Although it did not abolish but rather extended the former classical laws, the generalization was achieved at the cost of adopting a completely new language of concepts and a new way of thinking at phenomenological and mathematical levels.

The classical theories were *deterministic*. For the classical mechanics (Newton, Hamilton) the physical systems were collections of *material points* and *fields of force* moving (evolving) in a completely predictable way. If one only knew the present state of the system and the external forces, one could predict with arbitrary precision its future evolution, or alternatively, extrapolate the past. The entire universe seemed like an enormous, infinitely precise watch, in which the present day state is defined, up to the smallest detail, by the yesterday state, and this one in turn, at least in principle, by the state of million years ago (Laplace). On the astronomical scale, these laws permitted to integrate the planetary and stellar orbits (Newton). In the laboratory experience they granted that the initial conditions of any experiment determine uniquely its final result. The simplest illustration might be the elastic scattering of classical balls running between some fixed obstacles (Figure 1) to hit the screen behind. If one repeats the experiment sending many balls with different initial positions $\boldsymbol{x}$ and momenta $\boldsymbol{p}$, the places in which they touch the screen will differ. However, if we start to eliminate the initial differences, the discrepancy of the final results will tend to disappear. In limit, if we could repeat the experiment with the initial conditions exactly identical then, according to the classical mechanics, all the balls would hit the screen in exactly the same point.

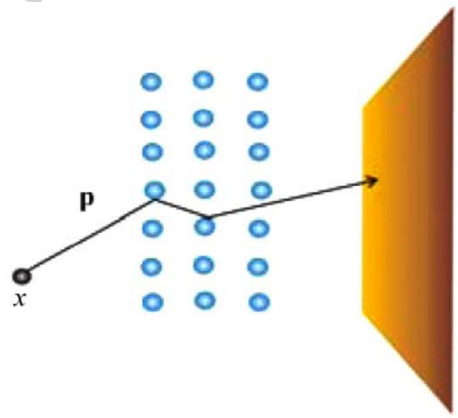

Figure 1: The deterministic character of the classical collision experiments.

The theory of micro-phenomena which emerged in the first decades of XX century had to abandon this picture, together with the realistic image of particles, as “material points”, balls, or pebbles. The objects which replaced them were the tiny corpuscle of electricity called *electron*, the corpuscles of the light (photons) and other particles. The existence of electrons was predicted quite early in the inspired 1744 letter of Benjamin Franklin (c.f. Millikan 1948); proved rigorously by the 1874 experiments of Thomson with the “cathode rays” (Watson 1947), almost simultaneously in the works of Zeeman (see Romer 1948). The photons, in a sense, were foreseen by Newton, dismissed by Huygens and Maxwell, and finally discovered by Einstein (1905). Soon, new micro particles were discovered—all of them showing the need for a basically new theory replacing the deterministic laws by probabilistic ones.

2. Black Body Radiation: The Lateral Problem Becomes Fundamental

While developing the deterministic scheme, the classical theories did not neglect the statistical laws. The scientists of XIX century knew and were applying the probabilistic descriptions in which not all physical parameters could be controlled by the experimentalist. The fundamental step was the application of Boltzmann statistics to explain the principles of Thermodynamics. One of the unsolved problems concerned an idealized physical medium (a *perfectly black body*) in thermal equilibrium with the surrounding radiation bath. According to Boltzmann, the radiation of such a body should have obeyed the Raleigh law (see Figure 2), but the experiments were showing a different pattern. Near the end of XIX century this discrepancy could seem a lateral trouble, but its fundamental character was soon recognized. In his polemic with Boltzmann, Max Planck assumed that the surface of the black body can be represented by an ensemble of harmonic oscillators. He then noticed that the false conclusions disappear if one assumes that the energies exchanged by the radiation field with this surface cannot be arbitrarily small. By comparing his hypothesis with the known empirical data he found an unexpected consistency requirement: that the energy portions exchanged with the radiation of frequency ν should be $h\nu$, where h is a natural constant $h = 6.626\times10^{-34}$ J s. Planck attributed the phenomenon to the structure of the black body material (c.f. Born 1969 Ch VII; see also Cervantes-Cota et al 2005), but soon it turned out that this hypothesis contained only a part of the truth.

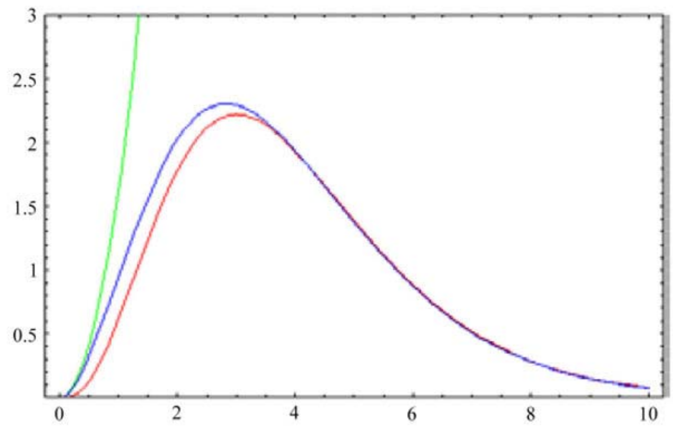

Figure 2: The Rayleigh-Jeans (green), Wien (red) and Planck (blue) distributions.

3. The Discovery of Photons

Since the late XIX century it was known that the surfaces of certain metals (e.g., silver) when illuminated by the light emit electrons. Although the existence of this phenomenon did not contradict the classical electrodynamics, the details did. The energies of the pulled out electrons did not depend on the light beam intensity. When the intensity decreased, the number of emitted electrons decreased as well, but their energies did not change; they depended only on the color of the light. For the monochromatic light beam of frequency ν, the energies absorbed by the liberated electrons could be reconstructed (c.f. Einstein 1905; Einstein and Infeld 1966; Jammer 1966, 1974; Primas 1983; Cervantes-Cota et al 2005) and turned out proportional to the frequency:

$$E = h\nu \tag{1}$$

where h is the same Planck constant which defined the black body radiation. The experiment developed as if the electrons in the metal were unable to absorb the beam energy smaller than $h\nu$, depending only on the light color but not on the properties of the metallic surface. It looked as if the light beam itself was split into indivisible portions of energy $h\nu$. Such was the hypothesis of Albert Einstein in his 1905 paper (Nobel Prize 1921).

The idea contradicted classical electrodynamics and henceforth, was not accepted without resistance. Perhaps, argued the opponents, the light is absorbed continuously, but the detectable effect (the electron emission) occurs only after enough energy is accumulated. Then, however, it would be difficult to explain why the electrons obtain the energy $h\nu$, *above* the minimum necessary to leave the metal. Despite this, the experimentalists tried to check whether the light absorption in the photoeffect, had no characteristics of a continuous accumulation. In crucial experiments (Joffe 1913; Meyer and Gerlach 1914), the light beam was falling not onto a wide metallic surface but on a metallic powder dispersed in vacuum. According to the classical electrodynamics, the beam energy falling onto each grain of the powder per time unit could be determined by the Pointing vector integrated over the exposed grain surface. The beam intensity was so low and the grains were so small that if the beam were a continuous medium, then the critical energies $E = h\nu$ needed few seconds to accumulate on each grain; henceforth the photo-effect should start with a visible delay. However, the powder response, even if weak, was immediate; an effect which could be hardly deduced from the classical theory of the continuous electromagnetic field. The fundamental discussions are still not over (c.f., e.g., Marshall and Santos 1989), thought the single photons can be already created and dropped one by one onto the screen (Kim et. al. 1999), and to be "seen without being destroyed" (Nogues et. al. 1999).

4. Compton's Effect: Collisions Confirm the Existence of Photons

The corpuscular effects of the light were independently detected for the light beams falling onto a gas of free electrons. According to the classical theory, the plane electromagnetic wave of frequency ν falling onto an electron must produce (in the electron rest frame) the spherically diffracted wave of the same frequency. The scattered wave should just show the Doppler effect in the initial, laboratory frame. This did not agree with the experiment. In 1923, Compton decided to calculate the same effect by assuming that the electron of an initial momentum $\boldsymbol{p}_{\mathrm{e}}$ collides elastically with a *light particle*, i.e., with a photon of energy $E = h\nu = c\,|\,\boldsymbol{p}\,|$ and momentum $\boldsymbol{p} = \frac{E}{c}\boldsymbol{n} = \frac{h}{\lambda}\boldsymbol{n} = \hbar\boldsymbol{k}$, where $|\boldsymbol{n}| = 1$, $\lambda = c/\nu$ is the wavelength, $\boldsymbol{k} \equiv 2\pi\boldsymbol{n}/\lambda$ is the wave-vector and $|\,\boldsymbol{k}\,|$ is the wave-number (see Figure 3). A simple argument based on the energy and momentum conservation proved that the scattered photons cannot conserve their initial frequency ν. The dispersed light must have the angle dependent frequency shift in the electron rest frame. If the scattering angle is θ, the wavelength λ' of the dispersed light is

$$\lambda' - \lambda = \lambda_{\mathrm{e}}\left(1 - \cos\theta\right) \qquad (2)$$

where $\lambda_{\mathrm{e}} = \frac{h}{m_{\mathrm{e}}c} \approx 2.43\times10^{-12}$ m is the *Compton wavelength* (Compton 1961; Born 1969 Ch V-4). The prediction fit the experiment. The photoeffect and the effect of Compton in the first part of XX century were two basic phenomena unexplainable by the Maxwell's theory of light. It is worth reminding that Compton's argument was sharply criticized by other authors, including C.V. Raman: "Compton, you are a good debater, but the truth is not in you".

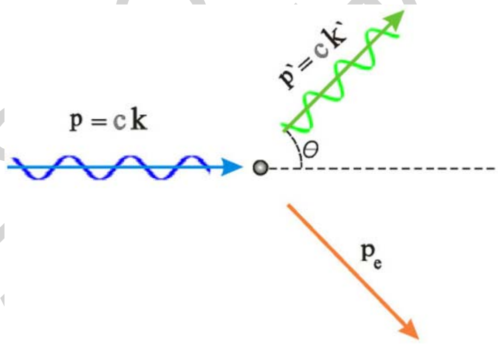

Figure 3: Compton effect: the photon scattered in the direction θ has the new frequency $\nu' < \nu$ consistent with (2).

5. Atoms: The Contradictions of the Planetary Model

The disquieting questions arose also about the nature of atoms. Though their existence was already known (see e.g. Newburgh, Peidle and Rueckner 2006), the exact composition of an atom was ignored. Since 1911, Rutherford used electrons to bombard the atoms in order to get more precise information. The results exhibited a highly inhomogeneous structure: the heavy, positively charged nuclei were surrounded by the light, negative electrons, so that the total electric charge was vanishing. The image of a planetary system in miniature, with the electrostatic attraction substituting the gravity (the "planetary model") was so suggestive, that it persists in physicists minds until today. However, the model proved wrong.

If the atoms were like the planetary systems the question would arise, why atoms of the same element (e.g. hydrogen) must have the same size (Feynman, Leighton and Sands 1963). The radius of the hydrogen atom is $\mathrm{r} \approx 5.3 \times 10^{-11}$ m; so 1 mol of the atomic hydrogen, according to this model, should be like a galaxy of 6.02252×10^{23} simple planetary systems composed of one sun and one planet, where each of 6.02252×10^{23} planets circulates at the same distance from its sun. The almost surrealistic vision contradicted the common sense and the probability calculus (the classical theory cannot restrict the size of the planetary systems!).

Even more serious was the stability problem. The electrostatic Coulomb field is not the only force acting on the electrons in the atom. Should the classical model be true, the electrons circulating on their "planetary orbits" should produce the electromagnetic waves (radiation) at the cost of the orbital energy. Taking away the energy, the emitted radiation should act back on the electrons slowing down their orbital motion. The effect of this self interaction (the *radiative damping*) is expressed by the third time derivative of the trajectory $d^3\boldsymbol{x}/dt^3$ and must transform the elliptic orbit into the spirals converging to the attraction center. The phenomenon exists also in gravitation, for the true planetary motion, but is incomparably weaker. The clean result was obtained in (Dirac 1938), though the problem was known much before. If resembling the classical point charges, the electrons would spiral down to the nuclei and the atoms should disappear in a fraction of second. Some authors tell about the "suicide of a classical theory" (see Figure 4).

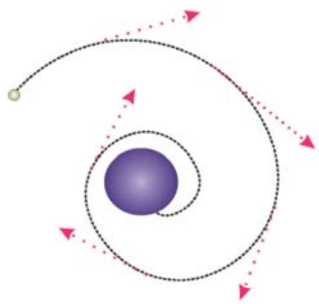

Figure 4: The suicide of the classical theory: due to the radiative dumping the atoms should collapse within a fraction of second.

6. The Mystery of the Allowed Energy Levels

The optical phenomena added an extra challenge. The atoms exposed to the oscillating fields or thermal collisions radiate, but their emission has little to do with the short time continuous burst predicted by the spiral collapse (Figure 4). The spectroscopy registered the characteristic sequences of *spectral lines* emitted without any catastrophe and allowing us to identify the atoms of each element. Thus, the sodium (Na) could be easily recognized by the dominating yellow light of frequency $\nu = 503\text{-}520 \times 10^{12}$ Hz, rubidium (Rb) by its red line $\nu = 304\text{-}482 \times 10^{12}$ Hz. The observed emission-absorption lines of the atoms typically formed double sequences. Thus, e.g., for the "hydrogen-like atoms", the distinguished frequencies were:

$$\nu_{mk} = cRZ^2\left(1+\frac{m_e}{M}\right)^{-1}\left(\frac{1}{n^2}-\frac{1}{k^2}\right), \quad n,k = 1,2,\ldots, , \tag{3}$$

where $c = 2.9972\times10^{8}$ m/s is the speed of light, $m_e = 9.109\times10^{-28}$ g is the electron's mass, M and Ze respectively are the mass $(M \gg m_e)$ and charge of the nucleus, $R = \frac{m_e c\alpha^2}{4\pi\hbar} = \frac{\alpha}{4\pi a_0} \approx 1.097\times10^{7}$ m^{-1} is the *Rydberg constant*, $\hbar \equiv \frac{h}{2\pi}$ =1.055×10^{-34} J s, $a_0 = 0.529\times10^{-10}$ m is the *Bohr radius and* $\alpha \approx 1/137$ was latter called the *fine structure constant*. The fact was extremely useful in spectroscopy, but incomprehensible for the planetary model. Moreover, due to the relation between the light color (frequency) and photon energy, the spectral formula (3) suggested that the energies of the emitted photons correspond to differences between some distinguished *energy levels*:

$$E_n \equiv h\nu_n = -\frac{1}{2}\mu\left(\frac{Zc\alpha}{n}\right)^2 \approx -Z^2\frac{13.6}{n^2}\ \text{eV}, \quad n = 1,2,\ldots \tag{4}$$

where $\mu \equiv m_e M/(m_e + M)$ is the reduced mass. Such was the famous observation of Niels Bohr in his 1913 paper (see also Pais 1991). Consistently, Bohr and Sommerfeld proposed a *modified planetary model* (Bohr 1913; Sommerfeld 1916), still for a classical charge circulating in the Coulomb field, but assuming that only a sequence of particular energies (4) was allowed for the electron orbits. The atomic emissions would correspond to discontinuous jumps of the electron between the permitted levels (4); the energy conservation law explaining the emission spectra. Strangely enough, Bohr followed the original Planck's arguments, while opposing strongly the existence of photons (see Pais 1991, Ch 11(d)), using (3-4) to explain the frequency, but still rejecting the quantization of the emitted light waves. The so formulated theory was later called the "old quantum mechanics". What it could not explain, however, was the very existence of the "allowed" and "forbidden" orbits.

7. Luis de Broglie: Particles or Waves?

Though this be madness, yet there's method in't.
W. Shakespeare, *HAMLET* (3.2)

While the new facts were arising, the old ones caused new problems. The existence of the interference pictures was no mystery for the Maxwell's theory of the light waves. However, if the light was a beam of particles, then it was hard to understand how the particle trajectories could create the wave pictures. The simplest case was the traditional double slit experiment for a monochromatic light beam (Figure 5). If the light is granular and if each photon crosses just one of the slits, how can it know that the other slit exists at all? And it must, since in absence of the second slit, the interference picture does not appear. Later on, Richard Feynman (see Feynman, Leighton and Sands 1963, p. 37) would qualify the double slit phenomenon as the "only mystery" of quantum theory (though, perhaps, he unjustly dismissed the other puzzles; c.f. Section 17).

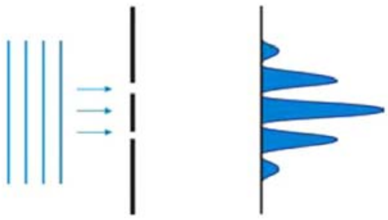

Figure 5: Double slit experiment: The greatest of all mysteries (Feynman).

A paradoxical choice of Luis de Broglie was to face the crisis by making it still deeper. If photons show a peculiar behavior, then "there is a method in it". In his 1924 paper, de Broglie assumed that the wave-like patterns cannot be limited to photons, but it must affect all physical particles. He thus proposed to associate with an electron moving with a momentum $\boldsymbol{p} = (p_1,\ p_2,\ p_3)$ and kinetic energy E, a fictitious (heuristic) image of a wave of frequency $\nu = E / h$:

$$\phi_{\boldsymbol{p}}\left(\boldsymbol{x},t\right) = Ce^{i(\boldsymbol{px}-Et)/\hbar},\ C = \text{constant} \qquad (5)$$

with the adimensional exponent $(\boldsymbol{px} - Et)/\hbar$, where , $\boldsymbol{px} = p_1x_1 + p_2x_2 + p_3x_3,\ \hbar = \dfrac{h}{2\pi}$.

The idea turned prophetic. The strange vision of de Broglie was confirmed in a sequence of experiments by Davisson and Germer (1927), Thomson and Reid (1927), and Thomson (1927). The beam of electrons from a stationary source, with the same carefully defined momenta $\boldsymbol{p}$, penetrated through a crystalline lattice (most typically, a thin sheet of metal) falling onto a photographic plate. The electron hits were visible as the clear points on the emulsion. Their localization was erratic but their accumulations formed a picture of regular concentric circles, typical for the interference effects (see Figure 6).

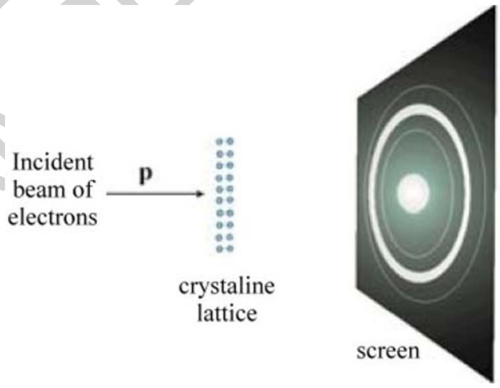

Figure 6: The scheme of Thomson experiment. The beam of electrons of the same momentum p crosses the crystalline lattice and falls onto the screen behind. Differently than for the classical scattering of Figure 1, the points marked by the single electrons are unpredictable, but their accumulation forms a regular pattern imitating the interference picture of the plane wave of length $\lambda = h / p\ (p = |\,\boldsymbol{p}\,|)$.

The diameters and the intensity of the circles (i.e., the density of hits) corresponded to the diffraction of a plane wave of the length inversely proportional to the momentum, $\lambda = h/|\boldsymbol{p}|$, where h is the same Planck constant which already appeared in the phenomenon of the black body radiation and the photoeffect energy (1). Moreover, for $p = c/\lambda$ and $E = cp$ (valid for photons), the de Broglie formula (5) reduced neatly to the Einstein's $E = h\nu$, indicating that it was a part of a universal law.

The attempts of explaining the phenomenon by the "cooperation" between the beam electrons failed as the result of the Biberman, Shushkin, and Fabricant (1949) experiments in which the beam intensities were too low to grant the existence of any ordered electron groups, yet the accumulation of the erratically dropping electrons created exactly the same diffraction pictures which did not depend on the beam intensity. The capacity of forming the interference patterns must have been encoded in each single electron; the conclusion much firmer now due to the techniques which permit the controlled emission of single electrons and photons (Ekstrom and Wineland 1980; Kim et al 1999).

Step by step it was also verified that the *corpuscular-wave duality* indeed affects a wider class of micro-objects. For neutrons it led to the useful techniques of *neutron interferometry* (Greenberger 1983). The tendency of forming the interference patterns is already checked for heavy micro-objects (Arndt et al 1999, Hackermüller et al 2003). No known modifications of the classical motion equations for the beam particles could explain the phenomenon (see also Section 18.1). It was the idea of Erwin Schrödinger to look in a different direction.

8. Schrödinger's Wave Mechanics: Wave Vibrations Explain the Energy Levels

The well known wave phenomena (the strings in guitars, air in flutes, trumpets, etc) are characterized by certain distinguished frequencies (notes) proper to each stationary oscillation pattern. An inspiration of Schrödinger was that the electron in an atom might not resemble a classical material point "on the orbit", but rather a piece of palpitating continuous medium obeying a certain wave mechanics (the hydrogen singularity could be a kind of a trumpet generating only specific frequencies). Yet, there must be a correspondence to the classical theory! Following this idea, Schrödinger turned attention to the families ("congruencies") of many simultaneous trajectories described by the generating functions $S(\boldsymbol{x},t)$ of the Hamilton-Jacobi (HJ) equation in classical mechanics. Could it be that these trajectories are equivalents of the rays in the geometric optics while the surfaces $S(\boldsymbol{x},t)$ = *const* are the wave fronts for a certain new wave phenomenon? In his 1926 quantization papers (reprinted in Schrödinger 1978), he explored new aspects of the HJ-equation $\frac{1}{2m}(\nabla S)^2 + V(\boldsymbol{x}) + \frac{\partial}{\partial t}S = 0$ for the non-relativistic Hamiltonian $H = \frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{x})$. By taking $S(\boldsymbol{x},t) = S(\boldsymbol{x}) - Et$ and $S(\boldsymbol{x}) = \hbar \ln \psi(\boldsymbol{x})$, the equation was reduced to $K(\psi) = -\frac{\hbar^2}{2m}(\nabla\psi)^2 + (V(\mathrm{x}) - E)\psi^2 \equiv 0.$ However, instead of assuming that $K(\psi)$ vanishes at every point $\boldsymbol{x}$, Schrödinger considered the integral $I = \int_{R^3} K(\psi)d^3x$ and demanded its stability under small variations $\delta\psi$ of ψ ($\delta I = 0$). By assuming that $\delta\psi$ vanish outside of finite space domains and by applying the standard variational method, he arrived at a new second order differential equation for ψ:

$$E\psi(x) = -\frac{\hbar^2}{2m}\Delta\psi(x) + V(x)\psi(x) \qquad (6)$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator and E corresponds to the system energy. In his next works, Schrödinger considered the non-stationary ψ. Looking for consistency with the dynamics of de Broglie waves (5), he replaced the energy E in (6) by $i\hbar\frac{\partial}{\partial t}$, $i = \sqrt{-1}$, obtaining the general propagation equation for the hypothetical wave $\psi(\boldsymbol{x},t)$:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\Delta\psi(x,t) + V(x,t)\psi(x,t), \qquad (7)$$

in presence of the external potential $V(\boldsymbol{x},t)$. The time independent equation (6) was then recovered by looking for the stationary (standing wave) solutions $\psi(\boldsymbol{x},t) = e^{-i\omega t}\psi(\boldsymbol{x})$, $\omega = E/\hbar$, with a time-independent *oscillation mode* $\psi(\boldsymbol{x})$. As found later, the similar heuristics could also lead to non-linear wave equations, but the simple guess of Schrödinger turned especially fortunate. His principal results concerned the static potentials $V(\boldsymbol{x},t) \equiv V(\boldsymbol{x})$. By choosing $V(\boldsymbol{x}) = -e^2/r$, where $r = |\boldsymbol{x}|$ (the Coulomb potential acting on the hydrogen electron) he could show that the stationary oscillations (6), trapped by the attraction center (with $\psi(\boldsymbol{x}) \to 0$ as $|\boldsymbol{x}| \to +\infty$), admit only a discontinuous sequence of proper energies (4) avoiding the self-destructive interference of ψ in the potential well (Figure 7). Though the physical nature of the complex wave $\psi(\boldsymbol{x})$ was still unknown, the central law of quantum mechanics was already discovered!

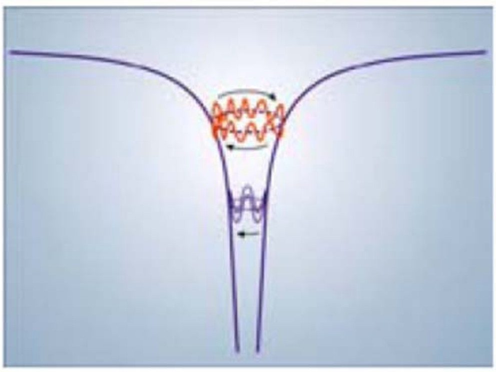

Figure 7: The mechanism of the allowed levels: only for the exceptional energies the wave (6) circulating in the Coulomb well survives the destructive interference.

The physical sense of $\psi(\boldsymbol{x},t)$ was a subject of polemic discussions (until today not entirely concluded). Schrödinger initially proposed a straightforward interpretation: that the electron *is indeed* a piece of continuous medium represented by a complex function $\psi(\boldsymbol{x},t)$, which oscillates around the atomic nucleus, producing the Bohr-Sommerfeld sequence of distinguished frequencies. Yet, this literal interpretation could not be defended. As objected in the Saclay Conferences, a single wave function propagating according to (6) can reach two distant detecting screens but each electron can be absorbed by only one of them. Moreover the characteristic property distinguishing the wave propagation (6) from the motion of a classical particle was the *tunnel effect*: the wave of de Broglie when facing an arbitrarily high potential barrier $V(x) \geq 0$ is never completely reflected but it is split into two parts, one reflected one transmitted. Yet, each single electron is never divided: In can be detected either on one or on the other side of the barrier (Fig. 8)

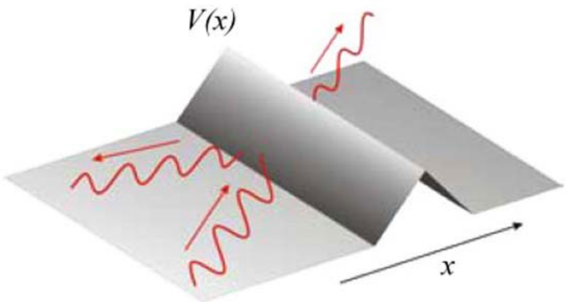

Figure 8: An electron and a potential barrier. The Schrödinger's wave function splits into two parts, but the electron can be detected just in one of them.

-
-
-

Bibliography

Abbe E. (1873) Beiträge zur theorie des mikroskops und der mikroskopischen wahrnehmung, *Arch. F. Microsc. Anat.* **9**, 413-468 [The historical paper explaining essential optical concepts for the resolution of microscopic images. Used subsequently to illustrate the quantum uncertainty principle in the "Heisenberg's microscope"]

Accardi L. (1981) Topics in quantum probability, *Phys. Rep.* **77**, 169-192 [An outstanding review of the hidden parameter theory. For readers interested in the subject all works of Accardi are relevant]

Arndt M., Nairz O., vos-Andrae J., Séller C., van der Zouw G., and Zeillinger A. (1999) Wave particle duality of C_{60} molecules, *Nature* **401**, 680-682 [The interference pictures obtained for beams of massive corpuscles C_{60} are one of the most convincing proofs that the particle-wave duality is not limited to photons, electrons and other tiny particles. Indeed, it was verified independently for nucleons (neutron interferometry) and for some atoms (e.g. sodium). Most recently it is also observed for C_{70} fullerenes]

Ashtekar A. and Geroch R. (1974) Quantum theory of gravitation, *Rep. Prog. Phys.* **37**, 1211-1256 [An essay explaining the ideas of the *loop quantum gravity*, a quickly developing domain, originated by works of Abhay Ashtekar (Center for Gravitational Physics and Geometry, USA). In his vision the space points are substituted by closed curves (loops), the surfaces and volumes are quantized, an effect which becomes essential below the Planck distance. The interested readers might consult the works of Ashtekar A., Rovelli C., Lewandowski J., Smolin L. and others. In particular, Ashtekar A, Rovelli C, (1991) Gravitations and loops, *Phys. Rev.* D **44**, 1740-1755, Rovelli C.(2004); furthermore, the study of the Big Bang from this point of view is found in Ashtekar et al., "Quantum nature of the big-bang: an analytical and numerical investigation", *Phys.Rev. D* **73**, 124038-124071 (2006). The critical discussions of the black hole entropy are found, e.g. in (Domagala M and Lewandowski J (2004) "Black hole entropy from quantum geometry", *Class .Quant. Grav*. **21**, 5233-5244]

Attewischer E., van Exter M.P., and Woerdmann J.P. (2002) Plasmon-assisted transmission of entangled photons, *Letters to Nature, Nature* **418**, 304-306 [One of examples that the photon pure states are not necessarily destroyed by a temporary absorption by a macroscopic environment (plasmons). C.f. also the ideas of Leggett A.J. (2002)]

Avron J.E., and Simon B. (1981) Almost periodic Hill's equation and the rings of Saturn, *Phys. Rev. Lett.* **46**, 1166-1168 [The description of the Saturn rings proposed by the authors was not accepted by the astronomers; yet the paper is a surprising example of how the Schrodinger's equation can become a model for purely macroscopic phenomena]

Ballentine L.E. (1970) The statistical interpretation of quantum mechanics, *Rev. Mod. Phys.* **42**, 358-381 [The author sticks to the "ensemble interpretation" of QM, denying that the state vector ("wave function") can represent the single particle state and negating the reality of the state collapse (reduction)]

Ballentine L.E., Pearle Ph., Walker E.H., Sachs M., Koga T., and Gerver J. (1971, April) Quantum-mechanics debate, *Phys. Today* **24**, 36-44; (1971, October) Still more quantum mechanics, ibid 11-15

Ballentine L.E. (1991) Comment on "Quantum Zeno effect", *Phys Rev A* **43**, 5165-5167 [Following the ensemble interpretation, L.E. Ballentine questions the need of using the reduction postulate in the explanation of the 'quantum Zeno effect'. In his parallel polemic paper, "Limitations of the projection postulate" (1990) *Found. Phys*. **20**, 1329-1343, he concludes: "...the projection postulate may accurately be described as either redundant or wrong". See, however, different opinions e.g. in (von Neumann 1932, Dirac 1935, Wheeler and Zurek 1983, Itano et al. 1990, Bennet et al. 1993, Penrose 1994, Greenberger and Zeilinger 1995, Penrose 2004)]

Bell J.S. (1966) On the problem of hidden variables in quantum theory, *Rev. Mod. Phys.* **38**, 447-452 [The historical paper in which the simplest form of John S. Bell inequality was derived. An equally relevant case known as CHSH inequality (Clauser J.F., Horne M.A., Shimony A. and Holt R.A. (1969) *Phys. Rev. Lett.* **23**, 880-884) is frequently evoked to show the non-locality of the quantum EPR correlations; see, e.g. (Leggett A.J. 2002) The still more general forms of Bell's inequalities due to Mermin N.D., are collected e.g. in (Cereseda J.L. (2001) *Phys. Lett. A* **286**, 376-382)]

Bell J.S., and Hallet M. (1982) Logic, quantum logic and empiricism, *Phil. Sci.* **49**, 355-79 [The fundamental paper questioning Putnam's arguments on the validity of quantum logical axioms and the necessity of the Hilbert space formalism in quantum theory]

Bell J.S. (1987) *Speakable and unspeakable in quantum mechanics*, Cambridge: Cambridge University Press [The summa of John S. Bell ideas on the interpretation of quantum theory. The reader will enjoy his humorous comments on some generally accepted concepts and opinions]

Beltrametti E.B., and Casinelli G. (1981) The logic of quantum mechanics, in *Encyclopedia of Mathematics*, Rota G.C. (Ed) London, UK: Addison Wesley

Bennett C.H., Bessette F., Brassard G., Salvail L., and Smolin J. (1992) Experimental quantum cryptography, *J. Cryptology* **5**, 3-28

Bennett C.H., Brassard G., and Ekert A.K. (1992) Quantum cryptography, *Sci. Am.* **267**, 50-57

Bennett C.H., Brassard G., and Crépeau C. (1993) Teleporting an unknown quantum state via dual classical and Einstein-Podolski-Rosen channels, *Phys. Rev. Lett.*, **70**, 1895-1899 [A classical paper explaining in simple terms the mechanism of teleportation of an arbitrary, unknown state between two spatially separated detectors in the EPR experiment. The effect was experimentally verified by works of Insbruck group, C.f. (Zeilinger 2000)]

Bergquist J.C., Hulet R.G., Itano W., and Wineland D.J. (1986) Observation of quantum jumps in a single atom, *Phys. Rev. Lett.* **57**, 1699-1702 [One of the well known papers on the sudden luminosity jumps observed in the radiation of the 3-level system, affected by the resonant 2-frequency electromagnetic waves. A suggestive interpretation is that the jumps correspond to the spontaneous reductions of the 3-level states]

Bialynicki-Birula I., and Mycielski J. (1976) Nonlinear-wave mechanics, *Ann. Phys.* **100**, 62-93 [The scheme of quantum mechanics based on the Schrodinger's wave equation with the logarithmic non-linearity. The theory makes possible a natural formulation of the collapse hypothesis for packets composed of several space separated parts]

Biberman L., Sushkin N., and Fabricant V. (1949) Difraktsiya poocheredno letyashchikh elektronov, *Docl. Acad. Sci. USSR* **66**, 185-186 [i.e., "The diffraction of separately flying electrons"; the paper reports the first experiment in which the wave behavior is observed for an electron beam of so low intensity that it can hardly be attributed to a collective effect of many electron interactions]

Birkhoff G., and von Neumann J. (1936) The logic of quantum mechanics, *Ann. Math.* **37**, 823-843 [An emblematic paper suggesting that the *complementarity* considered by the Copenhagen School is indeed the manifestation of a new type of the *logic* obeyed by quantum phenomena]

Blokhintsev D.I. (1968) *The philosophy of quantum mechanics*, Dordrecht, Holland: Reidel [Dimitri Ivanovich Blokhintsev, an outstanding follower of the "ensemble interpretation" of QM; a doctrine opposed to the Copenhagen School, derived from Einstein's ideas, popular also in the former Soviet Union for ideological reasons]

Bohm D., and Aharonov Y. (1957) Discussion of experimental proof for the paradox of Einstein, Rosen, and Podolsky, *Phys. Rev.* **108**, 1070-1076 [The spin variant of the EPR thought-experiment illustrates most transparently the non-local aspects of the reduction (collapse) axiom for the antisymmetric singlet state]

Bohm D., and Bub J. (1966) A proposed solution of the measurement problem in quantum mechanics by a hidden variable theory, *Rev. Mod. Phys.* **38**, 453-469; A refutation of the proof by Jauch and Piron that hidden variables can be excluded in quantum mechanics, ibid 470-475 [One of the most interesting intents of constructing the hidden variables for the spin measurements. Even if the generalization to the higher dimension is questionable (Kochen S. and Specker E.P., 1967) and even if it could not explain the EPR statistics for the singlet state, the paper is worth of attention for rising some wider questions as, e.g., whether the *decoherence* is indeed sufficient to describe the mechanism of the measurement. The authors doubt: '...When this packet arrives at the film (...) it is "broken up" into many very small packets, which cease to interfere coherently. What is still unexplained, however, is that only one of these packets contains an electron...' (p.459, col.1, lines 13-8 from the bottom). C.f. also (Haroche S. 1998, Leggett A.J. 2002)]

Bohr N. (1913) On the constitution of atoms and molecules, *Phil. Mag.* **26**, 1-25, 476-502, 857-875 [The observation that the atomic spectral frequencies are differences between some fixed spectral terms lead Bohr to his historical discovery of the distinguished, stable orbits of the atomic electrons]

Bohr N. (1928) The quantum postulate and the recent development of atomic theory, *Nature* **121** (Suppl.) 580-590

Bohr N. (1935) Can quantum-mechanical description of physical reality be considered complete?, *Phys. Rev*. **48**, 696-702 [Niels Bohr's famous answer to the equally famous objections in (Einstein A., Podolsky B. and Rosen N., 1935) about the non-locality of quantum measurements]

Bohr N. (1948) On the notions of causality and complementarity, *Dialectica* **2**, 312-319 (reprinted in: *Science* **111** (1950) 51-54)

Born M. (1926a) Zur quantenmechanik der stossvorgänge (The quantum mechanics of the impact process) *Z. Physik* **37**, 863-867. Translated in Wheeler and Zurek (1983) [In his first, intuitive intent to formulate the statistical interpretation of the wave packet Max Born (Nobel Prize in Physics, 1954) postulated the probabilities proportional to the transition amplitudes. An improved, though still imprecise version (probabilities = amplitude squares) was added in the last moment, in his *note added in proof*. The correct formula was finally published in (Born 1926b)]

Born M. (1926b) Quantenmechanik der stossvorgänge, *Z. Physik* **38**, 803-827

Born M. (1969) *Atomic physics*, New York: Dover Pub. Inc.

Born M (1971) *The Born-Einstein letters*, Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955, with commentaries by Max Born, New York: Walker and Company

Bouwmeester D., Ekert A., and Zeilinger A. (2001) *The physics of quantum information*, New York: Springer [The book by known leaders of quantum information, of notable interest to the advanced students (C.f. also Nielsen M.A. and Chuang I.L, 2000)]

Brumer P., and Shapiro M. (1995) Laser control of chemical-reactions, *Sci. Am.* **272**, 56-58

Brune M, Schmidt-Kaler F., Maali A., Dreyer J., Hagley E., Raymond J.M., and Haroche S. (1996) Quantum Rabi oscillations: a direct test of field quantization in a cavity, *Phys. Rev. Lett.* **76**, 1800-18003 [The patient research of the experimental/theoretical group in the *Ecole Normale Superieure* (Paris) shows the reality of the superposed energy states in a single atom. The superposed "pink state" is created by generating the Rabi rotations between the "red" and "blue" energy eigenstates of an atom passing through an optical cavity. Perhaps, it can answer a part of the doubts expressed in (Primas 1983)?]

Cervantes-Cota J.L., Galindo S., Klapp J., and Rodríguez-Meza M.A. (2005) *Las Mejores Historias del Joven Einstein* (in Spanish), México: Ediciones del Milenio [A sequence of historical studies on the origin of quantum and relativistic ideas at the beginning of XX c. The reader will find lot of unknown facts, amazing anecdotes and polemic disputes relevant for the shape of the present day physical theories]

Chu S. (1992) Laser trapping of neutral particles, *Sci. Am.* **266**, Num 2, 48-54

Cohen-Tannoudji C., Diu B., and Lalöe F. (1977) *Quantum mechanics*, Paris, France: Hermann and Wiley

Cohen-Tannoudji C., and Phillips W.D. (1990, October) New mechanisms for laser cooling, *Phys. Today*, 33-40 [Two of the 1997 Nobel Prize winners about one of the most ingenious methods of controlling the atomic motion]

Cohen-Tannoudji C., Dupond-Roc J., and Grynberg G. (1992) *Atom-photon interactions*, New York: Wiley [A careful review of the radiation theory, Floquet spectra and the principal methods of quantum optics. Transparently written, of high relevance for the advanced students]

Cohen-Tannoudji C. (1998) Manipulating atoms with photons, *Rev. Mod. Phys.* **70**, 707-719

Compton A.H. (1921) The magnetic electron, *J. of the Franklin Institute* **192**, 145-155

Compton A.H. (1961) Scattering of x-rays as particles, *Am. J. Phys.* **29**, 817-818 [One of crucial experiments confirming the granular nature of the light, described retrospectively by the author himself. When Arthur H. Compton (Nobel Prize in Physics, 1927) reported his result in 1923, his colleagues were reluctant to believe. Compton had even some difficulties with publishing his paper. Today, one of obligatory chapters in the quantum mechanics textbooks]

Corson E.M. (1953) *Introduction to tensors, spinors and relativistic wave equations*, London: Blackie [A classical though forgotten monograph, still worth consulting]

Davisson C., and Germer L.H. (1927) Diffraction of electrons by a crystal of nickel, *Phys. Rev.* **30**, 705-740 [The interference effects for an electron beam reflected by the crystalline lattice confirm the strange idea of Luis de Broglie]

de Broglie L. (1924) *Recherches sur le théorie des quanta*, Paris: Masson [The historical hypothesis of Luis de Broglie (Nobel Prize in Physics, 1929) that the beam of any micro-particles must show the wave behavior. In his times, it seemed almost a science-fiction story]

de Broglie L. (1960) *Non-linear wave mechanics: A causal interpretation*, Amsterdam: Elsevier de la Peña L. (1991) *Introducción a la mecánica cuántica*, México D.F.: Fondo de Cultura Económica

Deutsch D., Barenco A., and Ekert A. (1995) Universality in quantum computation, *Proc. R. Soc. Lond. A* **449**, 669-677

Deutsch D. (1985) Quantum theory, the Church-Turing principle and the universal quantum computer, *Proc. R. Soc. London A* **400**, 97-117 [The fundamental ideas by one of the pioneers of quantum computing. Recommended as the first (and second) contact with the subject]

Dirac P.A.M. (1928a) The quantum theory of the electron part I, *Proc. R. Soc. London* **117**, 610-624; part II, *Proc. R. Soc. London* **118**, 351-361

Dirac P.A.M. (1928b) On the quantum theory of the electron, *Phys. Z.* **29**, 561-563 [The 1928 papers of Paul A.M. Dirac (Nobel Prize in Physics, 1933) presenting the theory of the relativistic electron were for a long time under a persistent criticism of Niels Bohr and other authors. The situation changed after the discovery of positron. See also the historical studies of Moyer D.F. (1981a-c)]

Dirac P.A.M. (1935) *The principles of quantum mechanics* (2nd Ed.), Oxford: The Clarendon Press [The bestseller textbook of P.A.M. Dirac, explaining the concepts, notation and elementary algebraic methods of quantum mechanics. Based on the informational interpretation of quantum states (the *pure state* is a maximal set of non contradictory information about the quantum system), it yields the most transparent interpretation of quantum theory. In spite of the difference in epochs, the heuristics and formalism of Dirac are highly recommended for the present day students of physics, mathematics and physical chemistry]

Dirac P.A.M. (1938) Classical theory of radiating electrons, *Proc. R. Soc. London A* **167** (A929), 148-169

Dodonov V.V. (2002) Nonclassical states in quantum optics: a 'squeezed' review of the first 75 years, *J. Opt. B-Quantum and Semiclassical Optics* **4**, R1-R33

Einstein A. (1905) Über einen die erzeugung und verwandlung des lichtes betreffenden heuristichen gesichtspunkt, *Annalen der Physik* **17**, 132-148 [The fundamental paper of Albert Einstein (Nobel Prize in Physics, 1921) on the quantum nature of the light revealed by the photoelectric effects. Curiously, opposed for a long time by Planck and Bohr in spite of their crucial contributions to quantum theory]

Einstein A., Rosen N., and Podolsky B. (1935) Can quantum-mechanical description of physical reality be considered complete?, *Phys. Rev.* **47**, 777-780 [The thought experiment on the entangled two particle state (shortly, the EPR experiment), with predictable non-local aspects leads Albert Einstein, Boris Podolsky and Natan Rosen to question the interpretation of quantum mechanics. The effect is up to now the subject of fundamental discussions, and simultaneously, the theoretical basis of the control operations essential for quantum computing. The non-local correlations for the spin and polarization variants of the EPR experiment are recently checked with an increasing precision (see, e.g., Tittel W., et al. 1998)]

Einstein A., and Infeld L. (1966) *The evolution of physics*, Mass. USA: Murray Printing Co. [A fascinating book outlining the successes and defeats of physical ideas during several centuries. The error of the idea of "caloric", the non-existence of ether and other problems should be also remembered by the present day vanguard theories (as e.g. the "dark energy")]

Ekstrom P., and Wineland D. (1980) The isolated electron, *Sci. Am.* **243**, Num 2, 90-101

Elitzur A.C., and Vaidman L. (1993) Quantum mechanical interaction-free measurements, *Found. Phys.* **23**, 987-997 [One of the most provocative papers in the last decade of XXc. Due to the editor's indecision it circulated unpublished (as the preprint TAUP 1865-91) until the reinterpretation of Hardy L. (1992) *Phys. Lett. A* **167**, 11-16 opened a world wide discussion. The problem as to, whether the reduction of the quantum state can affect the past of quantum systems is imminent. An attempt to solve this problem was presented by Cramer in the frame of the "Transactional Quantum Mechanics". However, the recent discussion in (Elitzur A.C. and Dolev Z. (2007) Multiple interaction-free measurement as a challenge to the transactional interpretation of quantum mechanics, in *Frontiers of Time: Retrocausation - Experiments and Theory*, AIP Conf. Proc. **863**, pp. 27-43) implies that even the "transactional interpretation" fails. A visible challenge for the old and new interpretations of QM might indicate the need of a new theory]

Ferguson A.J., Cain P.A., Williams D.A., and Briggs A.D. (2002) Ammonia-based quantum computer, *Phys. Rev. A* **65**, 034303

Feynman R.P. (1948) Space-time approach to non-relativistic quantum mechanics, *Rev. Mod. Phys.* **20**, 367-387 [The idea of *path integrals* presented for the first time by Richard Feynman at the level of the non relativistic quantum mechanics. Today, one of the principal formulations of all quantum theories. It permits to avoid the difficulties affecting the canonical quantization, introducing some unified background into the different quantum field theories]

Feynman R.P., Leighton R.B., and Sands M. (1963) *The Feynman Lectures on Physics Vol. 1*, New York: Addison-Wesley.

Feynman R.P. (1982) Simulating physics with computers, *Int. J. Theor. Phys.* **21**, 467-488

Finkelstein D. (1963) The logic of quantum physics, *Trans. NY Acad. Sci.* **25**, 621-637

Fivel D.I. (1991) Geometry underlying no-hidden-variable theorems, *Phys. Rev. Lett.* **67**, 285-289 [The John S. Bell's theorem is shown equivalent to the triangle inequality in an abstract metrical space defined by the probability measures on a Boolean algebra]

Gasiorowicz S. (1974) *Quantum physics*, Singapore: John Wiley and Sons

Gerlach W., and Stern O. (1921) The experimental proof of magnetic moment of silver atoms, *Z. Physik* **8**, 110-11 (Sp. Iss. Yrs 1921/2 Dec-Feb 1921) [The historical experiments in which the spin spectral effects were observed still before the existence of the spin was accepted]

Gerlach W., and Stern O. (1922) The experimental evidence of direction quantization in the magnetic field, *Z. Physik* **9**, 349-352

Gilson J.G. (1968) On stochastic theories of quantum mechanics, *Proc. Cambridge Phil. Soc.* **64**, 1061-1070

Gisin N., and Rigo M. (1995) Relevant and irrelevant nonlinear Schrödinger equations, *J. Phys. A: Math. and Gen.* **28**, 7375-7390 [A fundamental theorem showing that the non-linear quantum mechanics for one particle states and the conventional QM for the entangled many particle states, together with the reduction axiom, would imply the possibility of reading the superluminal messages in the EPR-type experiments]

Gisin N., Ribordy G.G., Tittel W., and Zbinden H. (2002) Quantum cryptography, *Rev. Mod. Phys.* **74**, 145-195

Glauber R.J. (1963) Coherent and incoherent states for the radiation field, *Phys. Rev.* **131**, 2766-2788

Glauber R.J. (1965) Optical coherence and photon statistics, in DeWitt C., Blandin A., and Cohen-Tannoudji C. (Eds), *Quantum optics and electronics*, New York: Gordon and Breach [A nicely written article, offers a transparent guide to the ideas of Roy J. Glauber (Nobel Prize in Physics 2005)]

Golub R., Mampe W., Pendlebury J.M., and Ageron P. (1979) Ultracold neutrons, *Sci. Am.* **240**, 106-119

Greenberger D.M. (1983) The neutron interferometers as a device for illustrating the strange behavior of quantum systems, *Rev. Mod. Phys.* **55**, 875-905

Greenberger D.M., and Yasin A. (1989) Haunted measurements in quantum-theory, *Found. Phys.* **19**, 679-704

Greenberger D.M., Horne M.A., Shimony A., and Zeilinger A. (1990) Bell's theorem without inequalities, *Am. J. Phys.* **58** (12), 1131-1143

Greenberger D.M., and Zeilinger A., Eds. (1995) Fundamental Problems in Quantum Theory, *Ann. N. Y. Acad. Sci.*, Vol. 755, New York, N.Y. [One of the most interesting Conference Proceedings, dedicated to the entangled states, non-locality, hidden variables and other fundamental problems of quantum theory]

Gribbin J. (1996) *Schrodinger's kittens and the search for reality: Solving the quantum mysteries*, New York: Orion Publishing Co. [An excellent, humorous book about the models of the Schrodinger's cat "in little"]

Haag R. (1996) *Local quantum physics, fields, particles, algebras,* 2nd Ed., Berlin: Springer Verlag (See Chapter VII) [A profound mathematical presentation of the idea of locality, local observables and measurements by Rudolf Haag, one of the principal leaders of the C*-algebraic approach to quantum field theories. In the introduction the reader will find an ample conceptual study including the "event enhanced quantum mechanics"]

Hackermüller L., Uttenthaler S., Hornberger K., Reiger E., Brezger B., Zeilinger A., and Arndt M. (2003) The wave nature of biomolecules and fluorofullerenes, *Phys. Rev. Lett.* **91**, 090408

Haroche S., and Raimond J.M. (1996) Quantum computing: Dream or nightmare?, *Phys. Today* **48** (8), 51-52

Haroche S. (1998) Entanglement, decoherence and the quantum/classical boundary, *Phys. Today* **51** (7) 36-42 [Serge Haroche, one of the best known leaders in the foundations and phenomenology of QM. Together with the team of *Ecole Normale Superieure* (Paris) he is developing a systematic study to check the reality of the quantum mechanical "thought experiments" decisive for the meaning of quantum theory. One of the most surprising results of the ENS-group is the laboratory observation of a single photon in an optical cavity. The atom crossing the cavity "borrows" the photon which causes the Rabi rotation of the atomic states. When the atom leaves the cavity, the photon is honestly turned back without any energy loss: however the transaction is documented in form of a changed phase of the atomic state... Haroche reports also the robust behavior (stability) of the eigenstates of a measured observable and the volatility of the superposed states; though (as he states) the mechanism of the choice still involves a mystery]

Hau L.V. (2004) Frozen light, *Sci. Am. Special* **13**, Number 1, p 44

Havel T.F., Cory D.G., Lloyd S., Boulant N., Fortunato E.M., Pravia M.A., Teklemariam G., Weinstein Y.S., Bhattacharyya A., and Hou J. (2002) Quantum information processing by nuclear magnetic resonance spectroscopy, *Am. J. Phys.* **70**, 345-362

Heisenberg W. (1927) Über den anschlauchichen Inhalt der quantentheoretischen Kinematik und Mechanik, *Zeitschrift für Physik* **43**, 172-198 (English translation, Wheeler and Zurek 1983, p 62-84)

Heisenberg W. (1930) *The physical principles of quantum theory*, Chicago: University of Chicago Press [The formulation of QM by Werner Heisenberg was parallel to the wave mechanics of de Broglie and Schrödinger but obeyed a different idea of an "empirical realism" (indeed, a neopositivist philosophy in QM). According to this approach, the physical quantity "in itself" makes no sense unless associated with a well defined measuring prescription. However, the experimental prescriptions defining the physical observables might be incompatible, which means that some properties or concepts ("faces of reality") are *complementary* i.e. not simultaneously meaningful. "It makes as little sense to speak of the frequency of the light wave at a definite instant as of the energy of an atom at a definite moment" (translated in Wheeler J. and Zurek W. 1983, p.67). Following the idea, Heisenberg proposed an ample collection of the "uncertainty principles" including position-momentum, phase-action, time-energy, etc. His description of observables and the formulation of the dynamic theory obeyed the rule of the *canonical quantization* exploring the analogy between the Poisson brackets and commutators in a linear algebra]

Heisenberg W. (1955) The development of the interpretation of the quantum theory, in W. Pauli (Ed), *Niels Bohr and the development of physics*, London: Pergamon Press, pp 12-29

Heisenberg W. (1958) Physics and philosophy, New York: Harper and Brothers

Hepp K. (1998) Toward the demolition of a computational quantum brain, in *Quantum Future*, P. Blanchard, and A. Jadczyk (Eds), *Lecture Notes in Physics*, **517**, 92-104 [Evaluating the orders of magnitude, Klaus Hepp arrives at a skeptical conclusion about the collapse of the wave packet as a mechanism of our thinking. Our brain is macroscopic and so are our thoughts, he says. See also a sharp criticism of the "quantum computing" and its inadequacy to describe the working of our brain (Koch Ch and Hepp K (2006), Quantum mechanics in the brain, *Nature* **440**, 611-612)]

Holland P.R. (1993) *The quantum theory of motion*, Cambridge, UK: Cambridge University Press [An ample monograph dedicated to the possibility of the classical background of quantum mechanics. A particular attention is dedicated to the hidden variable scheme of David Bohm (1952), *Phys. Rev*. **85**, 166-193 (reprinted also in Wheeler J. and Zurek W. 1983, Ch.III.3, pp. 369-396) presented by Holland in a wider perspective, including the mathematical and philosophical aspects, application to many particle systems, non-locality and other fundamental problems]

Huttner B., Muller A., Gautier J.D., Zbinden H., and Gisin N. (1996) Unambiguous quantum measurement of non-orthogonal states, *Phys. Rev. A* **54**, 3783-3789 [A surprising result about the techniques of distinguishing unmistakably the non-orthogonal pure states of a quantum ensemble, at the cost of loosing a part of the ensemble individuals. The effect illustrates the influence of an absorbing medium onto the quantum states of micro-systems even if they apparently escaped interaction. The authors discuss its possible advantages for quantum cryptography]

Itano W.M., Heinzen D.G., Bollinger J.J., and Wineland D.G. (1990) Quantum Zeno effect, *Phys. Rev. A*, **41**, 2295-2300 [A widely commented article about the three level optical system under the influence of two competing, resonant electromagnetic fields. One of them causes a slow Rabi rotation between a pair of levels, while the other one, in form of short pulses and rest intervals, is supposed to act as a sequence of measurements inducing the collapse phenomena. If frequent enough, they should block the Rabi rotations; an effect indeed observed, taken by the authors as a proof of the physical reality of the state collapse (however, see the strong objections of other authors; e.g., Ballentine L.E. 1991)]

Jammer M. (1966) *The conceptual development of quantum mechanics*, New York: McGraw-Hill

Jammer M. (1974) *The philosophy of quantum mechanics; the interpretations of quantum mechanics in historical perspective*, New York: Wiley [Both books of Max Jammer are one of the most complete sources of historical and conceptual information. Who will read them carefully, without avoiding the labyrinth of footnotes, can find a lot of forgotten truth about the origin and development of physical ideas without excluding amazing anecdotes (e.g., Max Planck did not believe in the existence of photons, Wolfgang Pauli was an antagonist of the electron spin, etc.)]

Joffe A. (1913) Sitz. Ber. Bayer. Akad. Wiss. (München) *Math. Phys. Kl.* **43**, 19 [One of crucial experiments confirming the existence of photons, absent in most textbooks and monographs. See also (Meyer E. and Gerlach W. 1914)]

Kim J., Benson O, Kan H., and Yamamoto Y. (1999) A single-photon turnstile device, *Nature* **397** (2) 500-503 [Thanks to new methods of cavity quantum electrodynamics, an explicit proof of the existence of a single photon interacting with an atomic system could be obtained; see also (Haroche 1998)]

Klein F. (1893) Vergleichende Betrachtungen über neuere geometrische Forschungen, Mathematische Annalen **43**, 63-100 [The famous *Verlangen program* of Felix Klein considers groups as fundamental, geometries as secondary. Klein classifies the different geometries according to their invariance groups. The point of view which might seem inverting the natural order of concepts, turns useful to explain the origin of some physical geometries where the group was indeed first. Thus, e.g., the simplectic geometry of the classical phase space appears as the invariant of the classical motion group, while the Hilbert space geometry in QM is the product of the (*unitary*) state transformations generated by the Schrodinger (or Dirac) evolution equations. Quite similarly, in the special relativity, the Lorentz group was first, the geometry of the Minkowski space-time appeared later]

Klein O. (1926) The quantum theory and five-dimensional relativity theory, *Z. Phys.* **37**, 895-906

Kochen S., and Specker E.P. (1967) The problem of hidden variables in quantum mechanics, *J. Math. Mech.* **17**, 59-87

Kofman A.G., and Kurizki G. (2000) Acceleration of quantum decay processes by frequent observations, *Letters to Nature*, *Nature* **405**, 546-549

Kwiat P., Weinfurter H., and Zeilinger A. (1996, November) Quantum Seeing in the Dark, *Sci. Am.* **275**, 52-58 [The consequences of the *interaction free experiment* of Avshalom C. Elitzur and Lev Vaidman are analyzed. It makes possible to detect a physical object by a photon which passed far away without ever illuminating it]

Lalöe F. (2001) Do we really understand quantum mechanics? Strange correlations, paradoxes, and theorems, *Am. J. Phys.* **69**, 655-701 (erratum in *Am. J. Phys.* **70** (2002) 556) [The Author reconsiders the interpretation, paradoxes and unsolved conceptual problems of quantum mechanics, collecting carefully all arguments on favor or against the widely shared convictions. In spite of all disquieting questions, he finds some important advantages of the orthodox Copenhagen interpretation. Interesting to read, recommended to all students of the exact sciences]

Leggett A.J. (2002) Testing the limits of quantum mechanics: motivation, state of play, prospects, *J. Phys.: Condens. Matter* **14**, R415-R451 [The possible steps of QM towards a new quantum theory are considered. The principal challenge on this way is the measurement paradox (known also as the paradox of the Schrodinger's cat). Against the belief of some other authors, Anthony J. Leggett (Nobel Prize in Physics, 2003) considers the state reduction as a real process and the question about its mechanism as a key to further development of the theory. He dedicates a systematic study to the mesoscopic systems (such as the currents crossing the Josephson junction) which can produce the superpositions of macroscopically different states]

Leonard M.A. (1998) Computing with DNA, *Sci. Am.* **279**, 54-61

London F., and Bauer E. (1939) *La Théorie de l'observation en mécanique quantique*, Paris: Hermann et Cie. [The metaphysical hypothesis that the *consciousness* of an intelligent observer is what indeed reduces the wave packet of a quantum system. (English version see, Wheeler J. and Zurek W. 1983, Ch.II.1) The idea is supported in (Wigner E.P. 1962); in a more realistic sense in (Penrose R. 1996), objected in (Bell J.S. 1987, Ch.15, p.117), then in (Hepp K. 1998)]

Madore J. (1995) *An introduction to noncommutative differential geometry and its applications*, Cambridge, UK: Cambridge University Press [The readers willing to focus their attention on the simplest physical ideas, might also like Madore J (1992) Fuzzy Physics, *Ann. Phys*. **219**, 187-198. For more mathematically minded: Majid S (1995) *Foundations of Quantum Group Theory*, Cambridge, UK, Cambridge University Press. Highly recommended as well one of the pioneer papers: Woronowicz S.L. (1987) Compact matrix pseudo groups, *Commun. Math. Phys*. **111** 613-665]

Maeda H., and Gallagher T.F. (2004) Nondispersing wave packets, *Phys. Rev. Lett.* **92** 133004 [An experimental research showing the existence and stability of the circulating "planetary" states in the Rydberg atoms. (For the mathematically minded readers, worth reading are also: Delande D, Zakrzewski J, *Phys. Rev. A* **58**, 466 (1998); Bialynicki-Birula I, *Phys. Rev. Lett.* **93**, 020402 (2004) and the literature cited there)]

Magnus W., and Winkler S. (1973) *Hill's Equation*, New York: Dover

Margenau H. (1936) Quantum-mechanical description, *Phys. Rev.* **49**, 240-242

Marshall T.W. and Santos E. (1989) Stochastic optics: A local realistic analysis of optical tests of Bell inequalities, *Phys. Rev. A* **39**, 6271-6283 [The paper represents the still existing opposition against the reality of the light quanta. The authors defend the original ideas of Max Planck and Niels Bohr that the electromagnetic wave is continuous. If it is absorbed in portions, this is only because of the mechanism of absorption (relaxation phenomena), not due to the quantum structure of the light itself]

Mehra J. (1974) *The quantum principle: Its interpretation and epistemology*, Boston, USA: D. Reidel Publishing Company

Mermin N.D. (1998) What is quantum mechanics trying to tell us? *Am. J. Phys.* **66**, 753-767 [The points of view of Mermin are not easy to classify. In his widely discussed papers he insists that most of the quantum mechanical concepts are secondary if not illusionary. The only phenomenon described by quantum theory are correlations (coincidence probabilities) between quantum events. "Correlations and only correlations!" he says. This would be close to the ensemble interpretation, but simultaneously Mermin considers the probabilities well defined for each single micro-system. His point of view is questioned by several other authors, e.g. (Griffiths R.B. (2003) *Found. Phys*. **33**, 1423-1459), but yet, worth attention for the fans of quantum information]

Mermin N.D. (2003) From cbits to qbits: Teaching computer scientists quantum mechanics, *Am. J. Phys*. **71**, 23-30

Meyer E., and Gerlach W. (1914) Über den Photoelektrischen Effekt an ultra microskopischen Metallteilchen (The Photoelectric effect on ultra microscopic metallic particles), *Ann. der Phys* **45**, 177-236 [A crucial (and forgotten) experiment showing that the energy quanta absorbed in photo-effects cannot be explained by a gradual accumulation of the continuous field energy (see also Joffe 1913)]

Mielnik B. (1974) Generalized quantum mechanics, *Commun. Math. Phys.* **31**, 221-256 [An outline of a statistical theory in which the *pure states* do not need to correspond to the unit vectors, the transition probabilities are not the moduli squared of the scalar products and the "quantum logic" is not necessarily orthocomplemented. The existence of such structures might mean that the formalism of Hilbert spaces is not the only option for quantum theories. Supported in (Bell J.S. and Hallett M. 1982), discussed also in (Penrose R. 1997)]

Mielnik B., and Tengstrand G (1980) Nelson-Brown motion: Some question marks, *Int. J. Theor. Phys.* **19**, 239-250 [It is argued that the model of Brownian motion used by E. Nelson to explain the Schrodinger's wave equation has some interpretational gaps; if analyzed carefully, it becomes just a statistical variant of the pilot wave theories]

Millikan R.A. (1948) Franklin's discovery of the electron, *Am. J. Phys.* **16**, 319

Misra B., and Sudarshan E.C.G. (1977) The Zeno's paradox in quantum theory**,** *J. Math. Phys.* **18**, 756-763 [A famous result showing that the frequently repeated measurement can inhibit the evolution of quantum systems. A mathematically equivalent theorem was proved earlier in (Khalfin L.A. (1958) JETP **6**, 1053), though the work of Misra and Sudarshan turned better known and more inspiring in the fundamental research]

Moyer D.F. (1981a) Origins of Dirac's electron, 1925-1928, *Am. J. Phys.* **49**, 944-949

Moyer D.F. (1981b) Evaluations of Dirac's electron, 1928-1932, *Am. J. Phys.* **49**, 1055-1062

Moyer D.F. (1981c) Vindications of Dirac's electron, 1932-1934, *Am. J. Phys.* **49**, 1120-1125 [In his three articles of 1981 D.F. Moyer describes the historical polemic in which the arguments of P.A.M. Dirac (Nobel Prize in Physics, 1933), at the beginning, seemed in disadvantage (the negative energy levels of the free electron, the hypothetical "electron sea" with an infinite number of electrons occupying the continuum of negative levels, etc., all this under the persistent attacks of Bohr and other physicists) But then, one of his most risky conjectures, about the "positive electron", turned real. This started a fast development of quantum field theories, though against the skepticism of Dirac himself]

Nagourney W., Sandberg J., and Dehmelt H. (1986) Shelved optical electron amplifier: Observation of quantum jumps, *Phys. Rev. Lett.* **56**, 2797-2799 [One of widely discussed 1986 papers showing the instability and sudden transitions (quantum jumps) in a single atom. Tentatively, illustrating the reduction of the wave packet]

Nelson E. (1967) Dynamical theories of Brownian motion, Princeton University Press, Princeton, USA [A polemic hypothesis of Edward Nelson (known as the *stochastic quantum mechanics*). that the wave-like behavior of micro-particles described by the Schrodinger's equation is indeed the consequence of a fundamentally statistical but classical mechanics based on Newton's equations of motion]

Newburgh R., Peidle J., and Rueckner W. (2006) Einstein, Perrin, and the reality of atoms: 1905 revisited, *Am. J. Phys*., **74**, 478-481 [An interesting historical and fundamental essay analyzing the true sources of our knowledge]

Nielsen M.A., Knill E., and Laflamme R. (1998) Complete quantum teleportation using nuclear magnetic resonance, *Letters to Nature, Nature* **396**, 52-55

Nielsen M.A., and Chuang I.L. (2000) *Quantum computation and quantum information*, Cambridge: Cambridge University Press

Nogues G., Rauschenbeutel A., Osnaghi S., Brune M., Raimond J.M., and Haroche S. (1999) Seeing a single photon without destroying it, *Nature* **400**, 239-242 [One of the most ingenious experiments at the end of XX c: the existence of a photon is documented without absorbing its energy]

Pais A. (1991) *Niels Bohr's times, in physics, philosophy, and polity*, Oxford, UK: Clarendon Press

Park J.L. (1968) Nature of quantum states, *Am. J. Phys.* **36**, 211-225 [The Ph.D. thesis of James L. Park contains one of the strongest arguments against the reduction of the wave packet, on favor of the ensemble interpretation: due to the non-unique mixture decomposition into the pure states, the single particle after the measurement is in no state at all (so, neither it is in an eigenstate!)]

Paul W. (1993) in *Nobel lectures in physics 1981-1990*, Singapore: World Scientific, p 610

Pauli W. (1925a) Concerning the influence of the equilibrium ability of the electron mass on the Zeeman effect, *Z. Physik* **31**, 373-385 [In an amazing historical *quid pro quo*, Wolfgang Pauli was one of the most outstanding adversaries of the electron spin. Once forced to recognize its existence, he contributed to the formal description. Paradoxically, the spin 1/2 of the electrons is called today the *spin of Pauli*]

Pauli W. (1925b) On the connection of the arrangement of electron groups in atoms with the complex structure of spectra, *Z. Physik* **31**, 765-783

Pauli W. (1927) Über gasentartung und paramagnetismus, *Z. Physik* **43**, 81-102

Pauli W. (1964) Nobel Lecture, December 13, in *Nobel Lectures, Physics 1942-1962*, Amsterdam: Elsevier Publishing Co.

Penning F.M. (1936) The spark discharge in low pressure between coaxial cylinders in an axial magnet field, *Physica* **3**, 873-894

Penrose R. (1989) *The Emperor's New Mind*, Oxford, UK: Oxford University Press

Penrose R. (1994) *Shadows of the Mind*, Oxford, UK: Oxford University Press

Penrose R. (1996) Mechanism, microtubules, and the mind, *Journal of Consciousness Studies* **1**, 241-249

Penrose R. (1997) *The large, the small and the human mind,* Cambridge, UK: Cambridge University Press

Penrose R. (2004) *The road to reality*, London: Jonathan Cape. [Roger Penrose, one of the most talented authors and philosophers of physics, approaching the questions on the frontier of quantum and relativistic theories. In his four monographs he describes the origin, puzzles and the possible future of physical theories. Analyzing the links between the quantum and classical information, he is convinced about the reality of quantum states and state reduction for the single particles. He thinks that the reduction (collapse) events might be precipitated by the gravitational interactions, and he also believes that they may be essential in our brain activity (in a way, supporting the "psychological reduction mechanism" of London F. and Bauer E. 1939, Wigner E.P. 1962) In his search for the realistic mechanism of the state collapse, Penrose is aware of the numerous *quid pro quo* and unavoidable paradoxes. Citing Bob Wald, he writes "If you truly *believe* in quantum mechanics, then you cannot take it *seriously*" (Penrose 1997, p.72). His interest is also dedicated to the possibility of the *non-linear graviton*, for which he proposes a plausible analytical description, though the problem of the statistical interpretation remains open, see e.g. his most recent book (Penrose 2004). In the same monograph he comments an excess of arbitrary parameters and *ad hoc* models in the present day physics, making the vanguard theories hard to falsify, and hence, vulnerable to degenerate into never ending stories with little chances to be either rejected or verified. A fascinating literature for the young and old readers]

Phillips W.D., and Metcalf H.J. (1987) Cooling and trapping atoms, *Sci. Am.* **256**, 36-42 [As a surprising consequence of the Doppler effect, the electromagnetic waves can efficiently slow down the running atoms. William D. Phillips together with Steven Shu and Claude Cohen-Tannoudji received the Nobel Prize in Physics in 1997]

Piron C. (1964) Axiomatique quantique, *Helv. Phys. Acta* **31**, 439-468 [One of the most careful axiomatic studies deducing the Hilbert space formalism of quantum theories from the orthocomplementarity and weak modularity of quantum logic (for mathematically minded readers)]

Primas H. (1983) *Chemistry, quantum mechanics and reductionism*, Berlin: Springer-Verlag [One of exceptional monographs presenting the fundamental problems, interpretation and evolution of quantum ideas on the abstract and applied level. Truly priceless as a source of systematically presented problems and abundant, transparent bibliography. Apart of the encyclopedic achievements, the author discusses some uneasy questions about the applicability limits of quantum theory, e.g.: "Why so many stationary states not exist?", or "Why can approximations be better than the exact solutions?" (Ch.1.4, pp 12-13). He also offers unexpected interpretations of some traditional concepts, helping to understand the essence of quantum theory, e.g. "Pure quantum states are objective but not real" (Ch.3.4, p.103). It can be read almost as a novel.]

Rabi I.I. (1937) Space quantization in gyrating magnetic field, *Phys. Rev.* **51**, 652-654

Rabi I.I., Ramsey N.F., and Schwinger J. (1954) Use of rotating coordinates in magnetic resonance problems, *Rev. Mod. Phys.* **26**, 167-171 [More than 50 years after the Rabi's 1937 paper, the results of Rabi and collaborators proved crucial for the selective manipulations of the bound states of the spin and/or atomic systems. C.f. the present day techniques of creating the superposed energy states in optical cavities (Brune M. et al. 1996))]

Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond JM, and Haroche S. (2000) Step-by-step engineered multiparticle entanglement, *Science* **288**, 2024-2028

Redhead M. (1987) *Incompleteness nonlocality and realism: A prolegomenon to the philosophy of quantum mechanics*, New York: Oxford University Press [A transparent, compactly written monograph, explains the physical background of the mathematically advanced ideas (for mathematically minded readers)]

Reed M., and Simon B. (1978) *Methods of Modern Mathematical Physics IV*, New York: Academic Press [An extensive "river-encyclopedia" painting the mathematical aspects of modern physics. Not very easy, but extremely useful]

Riesz F., and Nagy B. (1990) *Functional analysis*, New York: Dover Pub Inc.

Romer A. (1948) Zeeman's discovery of the electron, *Am. J. Phys.* **16**, 216-218

Rosenfeld L (1953) Strife about complementarity, *Science Progress* **41**, 393-410 (Revised version of "L'évidence de la complementarité" in *Louis de Broglie, physician et penseur*, Paris: Albin Michel (1953), pp 43-65)

Rovelli C. (2004) *Quantum gravity*, Cambridge monographs on mathematical physics, London: Cambridge University Press

Sauter Th., Neuhauser W., Blatt R., and Toschel P.E. (1986) Observation of quantum jumps, *Phys. Rev. Lett.* **57**, 1696-1698

Schirmer S.G., Greentre A.D., Ramakrishna V., and Rabitz H. (2002) Constructive control of quantum systems using factorization of unitary operators, *J. Phys. A: Math. Gen.* **35**, 8315-8339 [The authors present efficient methods of the step by step manipulation of many level quantum systems, including the total population inversion. Mathematically elegant, some parts based on exact solutions]

Schrieffer J.R. (1964) *Theory of superconductivity*, Reading Mass.: W.A. Benjamin Inc.

Schrödinger E. (1978) *Collected papers on wave mechanics*, translated from the second 1928 German edition, New York: Chelsea Publishing Co.

Schrödinger E. (1935) Die gegenwrtige situation in der quantenmechanik, *Naturwiss* **23** 807-812 (English translation, see Wheeler and Zurek 1983, Ch. I.11) [One of the most relevant works of Erwin Schrodinger in which the informatics character of the wave function and its uncertainties are recognized. "The *psi*-function as Expectation catalog" (Sec.7). While one information is *sharp*, the other must be *blurred* (uncertainty principle). Schrödinger wondered, whether the blurring can affect also the macroscopic systems and he illustrated his doubts in a little, humorous tale about a living organism (a cat) coupled with an unstable atom, so that the animal must enter into a superposed state of "being alive" and "being dead". While most of the world difficult papers end up rather quickly in the dead archives, the problem of the Schrödinger's cat, presented in a simple hearted language (Schrödinger E 1935, translated in Wheeler J. and Zurek W. 1983, p.157, col.1), is still alive, subject of polemic discussions, which might turn decisive for the future of quantum theory (see also Leggett A.J. 2002)]

Scully M.O., and Zubairy M.S. (2002) *Quantum optics*, Cambridge, UK: Cambridge University Press

Shimony A. (1963) Role of the observer in quantum theory, *Am. J. Phys.* **31**, 755-773

Sommerfeld A. (1916) The quantum theory of spectral lines, *Ann. der Phys.* **51**, 1-94; ibid III, 125-167 [The Nils Bohr's quantization rules in old QM are generalized for the relativistic electrons, yielding the Bohr-Sommerfeld atomic levels]

Tittel W., Brendel J., Zbinden H., and Gisin N. (1998) Violation of Bell inequalities by photons more than 10 Km apart, *Phys. Rev. Lett.* **81**, 3563-3566

Thomson G.P. and Reid. A (1927) Diffraction of cathode rays by a thin film, *Nature* **119**, 890-890

Thomson G.P. (1927) Diffraction of cathode rays by thin films of platinum, *Nature* **120**, 802-802 [The historical experiments of Thomson confirmed the hypothesis of Luis de Broglie, marking the beginning of the modern quantum theory]

Uhlenbeck G.E., and Goudsmit S. (1925) Ersetzung der Hypothese von unmechanischen Zwang durch eine Forderung bezuglich des inneren Verhaltens jedes einzelnen Elektrons, *Naturwissenschaften* **13**, 953-954

Uhlenbeck G.E., and Goudsmit S. (1926) Spinning Electrons and the Structure of Spectra, *Nature* **117**, 264-265 [G.E. Uhlenbeck and S. Goudsmit, the authors of the hypothesis about the electron spin, faced negative reactions of the scientific community around 1925. Intimidated by the criticism of Wolfgand Pauli, Werner Heisenberg and others, they were at the point to give up their idea, but finally, supported by Ehrenfest, decided to publish it in their 1925, 1926 papers]

von Neumann J. (1932) *Mathematische Grundlagen der Quanten-mechanik*, Berlin: Springer-Verlag (English translation: 1955, Princeton: Princeton University Press) [The classical mathematical monograph introducing the formalism of Hilbert spaces in Quantum Mechanics. The reader may also find a study of the uncertainly principles with the detailed presentation of the Heisenberg's microscope, and the discussion of the collapse axiom with its intrinsic paradoxes]

Watson E.C. (1947) Jubilee of the electron, *Am. J. Phys.* **15**, 458-464

Weinacht T.C., Ahn J., and Bucksbaum P.H. (1998) Measurement of the amplitude and phase of a sculpted Rydberg wave packet, *Phys. Rev. Lett.* **80**, 5508-5511 (erratum *ibid* **81**, 3050-3050)

Weinberg S. (1989) Testing quantum-mechanics, *Ann. Phys. N.Y.* **194**, 336-386 [In order to test experimentally the linearity of quantum mechanics Stephen Weinberg assumed a non-linear model of the Schrodinger's wave equation (see also Bialynicki-Birula I. and Mycielski J. 1976) and evaluated the admissible non-linearity by calculating corrections to the energy spectra. Some general problems, like e.g. non-existence of the orthodox measurement theory remained open. The possibilities of formulating the non-linear QM for one particle states were soon limited by the theorem of Nicolas Gisin (c.f. Gisin N. and Rigo M. 1995) showing the basic difficulty of extending the theory to many particle systems]

Weyl H. (1950) *The theory of groups and quantum mechanics*, New York: Dover Publications Inc.

Wheeler J., and Zurek W. (1983) *Quantum theory and measurement*, Princeton: Princeton University Press [One of the most useful anthologies on the foundations, measurements and interpretation of quantum theory. Apart of the original ideas of Wheeler and Zurek (on the "delayed choice experiment") it contains the English translations of some classical papers, like e.g. (Schrodinger 1935, Heisenberg 1927), reprinted versions of Bohm, Zeh and others]

Wigner E.P. (1962) Remarks on the mind-body question, in I.J. Good (Ed), *The scientist speculates*, New York: Capricorn Books, Ch. 7.98 [A brilliant but perturbing paper in which Eugene P. Wigner (Nobel Prize in Physics, 1963) returns to the hypothesis (London and Bauer 1939) that the reduction of quantum states is caused by the consciousness of an intelligent observer. He then considers several observers watching the same quantum effect, trying to distinguish between the subjective but collective and solipsist "constructions of reality". He concludes at the end: "The present writer is well aware (...) that he is not the first one to discuss the questions (...) and that the surmises of his predecessors were either found to be wrong or improvable (...) He feels, however, that many of the earlier speculations on the subject, even if they could not be justified, have stimulated and helped our thinking (...) in the question which is, perhaps, the most fundamental question at all". (Reprinted also in Wheeler J. and Zurek W. 1983)]

Wigner E.P. (1963) The problem of measurement, *Am. J. Phys.* **31**, 6-15

Wineland D.J., and Itano W.M. (1979) Laser cooling of atoms, *Phys. Rev. A* **20**, 1521-1540

Witten E (1995) String theory dynamics in various dimensions, *Nucl. Phys. B* **443**, 85-126

Zeilinger A (2000, April) Quantum teleportation, *Sci. Am.*, 32-41 [Anton Zeilinger, one of the best known leaders of the experiments on quantum teleportation, an effect challenging our intuitions about quantum states, confirmed in the Insbruck labs. The Insbruck-Viena group, founded in Nov.2003, counts with a team of the world known specialists, headed by R. Blatt and R. Grimm (experimental physics), H. Briegel and P. Zoller (theoretical physics as scientific directors), and a laboratory in Vienna headed by A. Zeilinger (experimental physics). To be noticed is also their collaboration with Juan Ignacio Cirac (Principe de Asturias Prize on *Investigación Científica y Técnica* 2006), C.f. e.g. I. Cirac and P. Zoller (1995) *Phys. Rev. Lett.* **47**, 4091]

Biographical Sketches

Bogdan Mielnik is a specialist in fundamental problems and algebraic techniques of quantum theories. His research field includes the "quantum logic" and the possibility of new quantum geometries.

Born in 1936, Warsaw, Poland, in his early years he was mostly interested in humanities but later on his interests shifted to the exact sciences. After his studies and Master degree in 1958 in Physics Faculty in Warsaw, he was dividing his time between the Institute of Theoretical Physics in Warsaw and Mexican Center for Advanced Studies (Cinvestav), taking inspirations from both places. His papers on atypical quantum geometries were discussed by many authors including John Bell, Rudolf Haag and Roger Penrose, their fragments reproduced in several monographs, including "Encyclopedia of Mathematics¨ of Gian Carlo Rota. Recently, he is active in Mexican and Spanish groups working on the supersymmetric quantum mechanics and quantum control; interested specially in manipulation of quantum states and its consequences for fundamental problems. He has a permanent position of Profesor Titular in the Physics Department of Cinvestav (Mexico); retains also the Professor's title in Warsaw University.

Oscar Rosas-Ortiz is *Profesor Titular* in the Physics Department of the Center for Research and Advanced Studies (Cinvestav), Mexico, and a member of the Mexican Academy of Sciences. Born in 1969, Puebla, Mexico, he studied physics at Universidad Autónoma de Puebla, Mexico, and received his doctorate in Physics at Cinvestav, where he is the recipient of the 1997 *Arturo Rosenblueth Prize* for best exact sciences PhD thesis of the year. In 1998-1999, he was a Postdoctoral Research Fellow at the University of Valladolid, Spain. Dr. Rosas-Ortiz is organizer of the Advanced Summer Schools in physics and other scientific events in Mexico City.

He works in several areas of mathematical physics including, among others, the supersymmetric formalism of quantum mechanics (SUSYQM), the dynamical manipulation of quantum particles (Quantum Control), the analysis of resonances at low energy processes (Gamow vectors) and the study of differential equations and special functions arising in physics. Actively engaged in the application of Darboux transformations to construct new exactly solvable potentials, his results include a wide family of radial potentials with exactly the same spectrum as the hydrogen-like one as well as non-Hermitian radial operators with real spectra. His recent overview article in J. Phys. A: Math. Gen. (jointly with B. Mielnik) reports the progress and trends of SUSYQM. His construction (jointly with D.J. Fernández, V. Hussin and L.M. Nieto) of the coherent states associated with a distorted version of the Heisenberg-Weyl algebra has been recently extended in order to “linearize” the Darboux-deformed algebras appearing in SUSYQM; the subject which show signs of a dynamic expansion.