COMPLEX SYSTEMS AND NON-LINEAR DYNAMICS

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Contents

Summary

We discuss the origins and recent advances of the vast field of non-linear dynamics and complex systems. The main aspects of complex systems are illustrated with relevant examples and the connection with non-linear dynamics and chaos is presented. Since this is a truly interdisciplinary field of research, many applications in mathematics, physics, chemistry, biology and physiology are briefly covered.

1. Introduction

The topic of complex systems and non linear dynamics is a very broad one and therefore it is not possible to pay attention in detail to the very vast range of subjects involved. Therefore, we will select some particular topics to develop in more detail, but at the same time we will refer to other subjects as well, in order to give a broader view of the field.

The study of complex systems and non linear dynamics is essentially an interdisciplinary field of research, involving mathematics, physics, chemistry, biology, ecology, medicine, economics and social sciences alike. By essential we mean a truly interdisciplinary endeavor both in its scope as well as in its origins. According to its multidisciplinary character, it is a very important subject in connection to the future development of a sustainable world and the life support systems that encompass.

In order to provide guidance to this vast field we would like to establish a connection between non-linear dynamics, chaos and complex systems. Until very recently, the paradigm in science was to consider that simple systems (with a few variables) behave in a simple (ordered) way, and that complex systems (with many variables) behave in a complex (disordered) way. However, we have learned recently that even a simple system, but non linear, are able to behave in a chaotic way. This implies that if we observe in a system a disordered and complex behavior can be a complex system, but it is also quite possible that the system is simple but presenting a chaotic dynamics. On the other hand, we can have a situation in which a complex system behaves in a simple fashion through the phenomena of synchronization and self organization.

In Section 2, we discuss complex systems and then in Sec. 3, non-linear dynamics and chaos. In Section 4, we discuss the topics involved and at the end we will give some concluding remarks.

2. Complex Systems

The origin of modern science is based on the idea of reductionism: the reduction of a system to simpler parts that are easier to analyze in detail. This way of facing the challenge to understand nature has been very successful and is a logical manner to begin this endeavor. Using this paradigm, we have been able to understand the structure of matter based on more elementary structures, like molecules and atoms. These atoms, we know now, are made of smaller particles: protons, neutrons, electrons, and many other elementary particles. Even more, many of these particles are made of more elementary constituents: the quarks. The detail knowledge of the atomic structure, based on quantum mechanics, allowed us to understand in detail the properties of solids, liquids, gases and plasmas. Based on this theoretical framework, we have been able to modify the very structure of matter to produce controlled nuclear energy, to develop semiconductor devices and many other applications that have forged the modern world in which we live today.

However, there is an apparent paradox. Everything is formed of the same atoms: a piece of metal, a drop of wine, a living cell, or a human being. The question is then: how can we distinguish between the simple structure of a piece of metal and the complexity that we encounter on a living cell, if both are formed by exactly the same building blocks (atoms). This question cannot be answered using only the reductionism program. On the contrary, to understand the complexity that emerges in a complex system, like the cell in our example, we need to have in mind a new conceptual framework. This framework has various elements which are essential to understand the emergent properties. Firstly, it is necessary that the system is non linear, that is, the system cannot be understood merely as the sum of its simpler parts. Or, if you prefer, the whole is not a mere collection of its unassembled parts. Secondly, it is necessary that the system is out of thermodynamic equilibrium, that is, an open system that interacts with its environment. Both conditions can lead to emergent phenomena like pattern formation, synchronization, or life itself. On the contrary, in those systems in thermodynamic equilibrium, the situation is simpler and is, therefore, well understood nowadays. In this equilibrium case we cannot obtain emergent properties.

In order to clarify these ideas, let us consider, as an example, an egg in two different conditions: in the first case, we put the egg in a closed box that isolates completely the egg from the external world and that inhibits the heat flux towards its interior. In this situation, after some time, the structural complexity inherent in the egg stars to degrade and we witness a transition to a more disordered system. That is, the second law of thermodynamics acts in such a way that the system transits from an ordered state to a disordered one. Now, if we put the egg in an open box that allows the flux of heat inside and we apply an external source of heat, after some time, we observe an amazing transformation: the emergence of a much more complex system, in this case a living chicken that can walk, feel and see. The question is where all this complexity came from if the only difference is that we heat the system with a constant flux of energy. Well, the full answer is still out of reach, but what is clear is that we have a highly non-linear system and that, being an open system, is out of thermodynamic equilibrium.

These two conditions for the system, its non-linear character and being out of equilibrium, even though they are necessary for having emergent properties, are not sufficient. Other properties that characterize complex systems are needed as well. These systems usually have a large number of variables or elements that interact among themselves. They can be simple elements that interact nonlinearly, either locally or in a global way. Besides the kind of interaction among the elements of the system, it is important the architecture, the geometry and topology of the net of interactions. The issue here is that the complex system acquires, somehow, emergent properties through the interaction of its parts. The question is: Is it possible to understand the origin of the emergent properties from the interaction (coupling) of the elements of the system? To answer this question we need to understand more clearly the dynamics of nonlinear systems with many degrees of freedom. What is known now is that it is possible to generate global emergent properties from local interactions of non linear elements.

Let us illustrate these aspects of complex systems with some examples. Think of a school of fishes swimming in the sea. From the distance we observe an amorphous structure that moves slowly and constantly changing in shape, sometimes is long, some other times is round, compacting itself acquiring all kind of forms. We wonder if this is a new kind of marine animal. When we get closer to this bizarre creature we notice, to our surprise, that it is not a creature but hundreds or thousands of fishes. Each one of these fishes is an individual that forms the elements (agents) of our complex system, in this case a school of fishes. These individuals interact locally among themselves, that is, each fish interacts only with its neighbors. However, this local interaction is enough to span the whole system. The school of fishes can even acquire highly ordered forms and structures, like a huge vortex in which the fishes turn around a hole, forming a living donut. Another example, easier to observe, is a flock of birds forming flying patterns in the sky. Of course we can mention many other instances of the same phenomenon in which individuals form swarms with very complex degree of self organization. The individuals can be cells forming complex structures (organs) that perform specific functions: cardiac cells that synchronize in the beating of the heart; neurons that web the complex network that characterize the human brain. On other scale, we can think of individuals as people that, through their relations, form the texture of human society with all its complexity. The main point to emphasize here is that there is no leader or main individual that acts as an organizer or that orchestrates these complex structures. This complexity emerges through self organization, that is, all the individuals are basically the same and perform the same kind of behavior as the rest; it is only through its local interactions the self organized patterns emerge.

To understand the emergence of this kind of patterns and ordered structures from the coupling of individual components, several models have been designed in which it is supposed that the agents interact only locally and through very simple rules of behavior. The main idea of these studies is to discover if it is possible to obtain complex structures solely from simple rules. A model of this kind consists of a set of elements (individuals or agents) that can move freely and interact only with their neighbors through a local interaction which is finite or limited, that is, each agent can only interact with other agents that are within a finite region of space. In the case of a two-dimensional model, the region of interaction can be a circle of finite radius centered on each agent. The model has only three simple rules: 1) separation, that is, an agent tends to move away from another agent if the distance between them is very small, avoiding in this way a collision; 2) aligning, that is, an agent turns or rotates in order to align itself to the average direction of the agents in its neighborhood; 3) cohesion, that is, an agent will move towards the average position of the agents in its neighborhood. With these three simple rules, which can be implemented very easily on a computer, this model can produce a dynamics that is very similar to the one observed in the real world. That is, the flock in the computer is qualitatively the same as a real school of fishes or a flock of birds.

Other types of models are based on cellular automata. A cellular automaton is a dynamical system that is discrete both in space and time. It typically consists of a grid in one or more dimensions and every element on the grid can be in a finite set of discrete states. The state of the system is updated in discrete steps of time and the dynamics obeys certain local rules. One particular cellular automaton that illustrates in a clear way the emergence of complex structures from simple rules is the so called game of life. In this model the agents can be only in two states: alive or dead. The dynamics take place on a two-dimensional square grid and therefore each agent has eight neighbors. Depending on the state of its neighbors, that is, if they are alive or dead, the agent will be alive or dead in the next discrete time step. This dynamics, with a few simple rules, can give rise to a whole set of complex structures that propagate in this two-dimensional world as if they were living creatures, thus the name of the model.

The important message that we have to bear in mind is that we are able to understand, in principle, the emergent properties of self organization from rules of interaction that are both simple and local. Then the challenge consists in determining those simple rules for a given system.

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