ELECTROMAGNETIC WAVES

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Contents

Summary

James Clerk Maxwell synthesized the unified theory in which all electric and magnetic phenomena could be described using four equations. This theory predicts that transverse electromagnetic waves are produced by accelerating charges, propagating through space with time at the speed of light. The electromagnetic spectrum includes electromagnetic waves of a continuous variety of wavelengths, from radio and microwaves to visible light to X-rays and γ-rays. As a transverse wave, light and other electromagnetic waves can be polarized, with a definite direction of the electric field vector. Electromagnetic waves carry energy and momentum from one region of space to another. They offer the possibility of transmitting information over long distances like for radio and television wireless transmission. A hot body emits, and absorbs, radiation consisting of electromagnetic waves. The spectrum of radiation emitted by black bodies at a given temperature was explained by Max Planck in terms of molecular oscillations. Important evidence for the Big Bang theory of the Universe was the discovery of the

cosmic microwave background radiation, which conforms to a blackbody radiation at a temperature of 2.7 K. The wave theory of light was strongly supported by the observations exhibiting interference and diffraction. Planck's formula and other radiation phenomena led, however, to a unified wave-particle description in terms of quantum states: the photons. Photons in radiation and discrete energy levels in atoms are the fundamental concepts for lasers. A laser is a device that can produce a very narrow intense beam of monochromatic coherent radiation.

1. Introduction: Physical Reality of the Field

The existence of electromagnetic waves that propagate through space with time, at the speed of light, was the most dramatic prediction of *Maxwell's equations*, published in 1864. These equations represent a complete and consistent set of fundamental physical laws for electricity and magnetism and show that these two disciplines are fundamentally connected or, better, unified. We shall discuss the orientation and relationship of the *electric and magnetic fields* that propagate in space, forming electromagnetic waves; the *energy* and *momentum* carried by these waves, and *polarization*, a phenomenon that does not appear in acoustic waves. In acoustic waves, the perturbation that propagates in space and time is the local pressure, a scalar magnitude, whereas in electromagnetic waves the perturbation is a vector field. Systems in which accelerating charges generate electromagnetic waves are the *broadcasting antennas*, for which the electromagnetic field propagates to very long distances. The properties of the radiation emitted by a *black body* cavity will be analyzed, as well as the *cosmic microwave background* that fills the entire Universe.

The realization that light is a form of electromagnetic radiation has led to a complete understanding of all the properties of light. By the early XIX century, it had become apparent that certain observations demand that light behaves like a wave. For example, when we look very closely, under controlled conditions, *light does bend around corners*. Although light appears to travel in straight lines, a fact that manifests itself in the sharpness of shadows, there are situations in which light does not cast *sharp* shadows. Christian Huygens, in 1687, had already proposed a wave theory of light, contrary to the view of Isaac Newton, who supported a picture of light composed of particles, based on its rectilinear motion. The wave theory of light can explain everything that the particle theory of light can, but not the opposite: *interference and diffraction* can be explained only by the wave theory. The fact that the electric and magnetic fields propagate in space-time like a wave implies that they transport, as we shall see, energy and momentum. As a consequence, the field has a physical reality, as much as matter, and it is not a mere mathematical construct.

In the XX century, we had to revise our view once more as new experimental evidence led to the picture of electromagnetic radiation in terms of *photons*, a concept that unifies the *quantum* of energy $\mathrm{E} = \mathrm{h}\nu$, a particle property, with the frequency ν of radiation, a wave property. The link between particle and wave behaviors is established through the Planck constant h, a fundamental and universal constant with the physical dimension of *action* = *energy* × *time*. On the basis of photons and atomic energy levels, we can understand the principles of the *laser*, a light source that produces a beam of highly coherent and very nearly monochromatic light. The quantum description of electromagnetic radiation in terms of photons unifies the particle properties of energy and momentum with the wave properties of frequency and wave number. The physical reality of photons is apparent in their direct detection by particle detectors.

The knowledge on the generation, propagation and detection of the electromagnetic waves has led, in the XX century, towards the modern methods of communication.

2. Maxwell's Equations

Maxwell's equations summarize the content of electricity and magnetism. They describe fully electric and magnetic fields in the presence of electric charges and currents, in the sense that the charge and current input is able to generate the electromagnetic field through space: $(Q, I) \rightarrow (\vec{E}, \vec{B})$.

The first pair of equations refers to the vector properties of the fields $\vec{E}$ and $\vec{B}$, independently of the generating sources.

1) The first is Gauss' law for magnetic fields

$$\oiint \vec{B}.\overrightarrow{dA} = 0 \qquad (1)$$

which says that the magnetic flux through a closed (Gauss) surface vanishes. In other words, any magnetic flux which goes into a volume has to come out, and vice versa. You cannot have localized magnetic sources and the magnetic field $\vec{B}$ is said to be *solenoidal*. Magnetic monopoles, which would be the magnetic analogues of electric charges, have never been discovered. The $\vec{B}$- *field lines*, which are tangent to the $\vec{B}$-vector at each point, are closed lines. As the number of field lines crossing a given surface is proportional to the magnetic flux, the equality of the number of field lines entering a closed surface to the number of those leaving it leads to Gauss' law. Equation (1) holds even for time - dependent magnetic fields. In Figure 1, the $\vec{B}$- field lines generated in a solenoid are given.

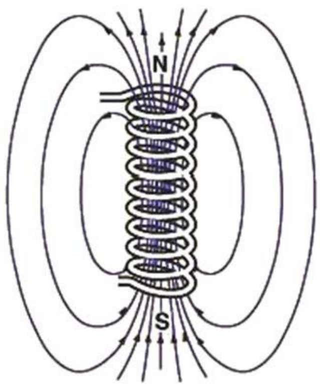

Figure 1. Magnetic field lines generated in a solenoid

2) Faraday's law describes the induced electric field generated by a changing magnetic

flux. It says that the e.m.f. (electromotive force, the source of energy), given by the integral of the tangential component of the induced electric field around an arbitrary closed loop, is determined by the opposite of the rate of change of the magnetic flux through *any* open surface bounded by the closed loop.

$$\oint \vec{E}.\, d\vec{s} = -\frac{d}{dt}\int\int_S \vec{B}.\, d\vec{A} \qquad (2)$$

The reader understands, with the help of the magnetic Gauss' law (1), why the right-hand-side of Eq. (2) is independent of the specific open surface S to be used. This is a very helpful result because it shows that we can choose for convenience the surface through which to calculate the magnetic flux. There is a *right-hand rule* that works as follows: If the fingers of the right hand curl in the direction of the loop, the thumb indicates the direction of the surface for calculating the flux. The minus sign is very important: it says that the induced electric field, were it to act on charges, would give rise to an induced current that opposes the change in the magnetic flux, formulated as *Lenz's law*. Induced electric fields described by Eq. (2) differ fundamentally from those generated by localized electric charges. The last ones are always associated with conservative forces, and the work done by them in moving a charge around a closed loop is always zero, contrary to Eq. (2). The induced electric field is not conservative, and it cannot be described by a potential that is only a function of space.

The second pair of Maxwell's equations contain the generation of the electromagnetic field by means of electric charges and currents.

3) Gauss' law for electric fields reads

$$\oiint \vec{E}.d\vec{A} = Q/\epsilon_0 \qquad (3)$$

It relates the electric flux through a closed surface to the charge Q enclosed. The coefficient $1/\epsilon_0$, where ϵ_0 is the vacuum dielectric constant, is an artifact of the International System of Units based on meter, Kilogram, second and Coulomb. The charge Q is the total charge contained within the closed surface. This law is actually a generalization of Coulomb's law. Whereas Coulomb's law is correct only for static charges, Gauss' law holds even if the charges are not stationary, i.e., even if the electric field varies with time. To derive Coulomb's law from Gauss' law we center a (closed) Gaussian sphere on a point charge q, as shown in Figure 2. By symmetry arguments, all directions around the point charge are equivalent. The only configuration of the field $\vec{E}$ that does not favor some particular direction is a radial field. The surface element $d\vec{A}$ of a Gaussian sphere is also radial, thus $\vec{E}.\overrightarrow{dA} = E\, dA$. Moreover, symmetry also implies that $\vec{E}$ will have the same magnitude everywhere on the surface, so that Eq. (3) implies

$$\oiint \vec{E}.\, d\vec{A} = \oiint E dA = E \oiint dA = E\left(4\pi r^2\right) = \frac{q}{\epsilon_0} \Rightarrow E = \frac{1}{4\pi\,\epsilon_0}\frac{q}{r^2} \qquad \text{[Coulomb]}$$

The field E is positive, radially outward, for a positive internal charge. The Coulomb law for the force between two charges q and q' follows if we put the second charge q' in the electric field generated by the first charge q and use $\vec{F} = q'\vec{E}$.

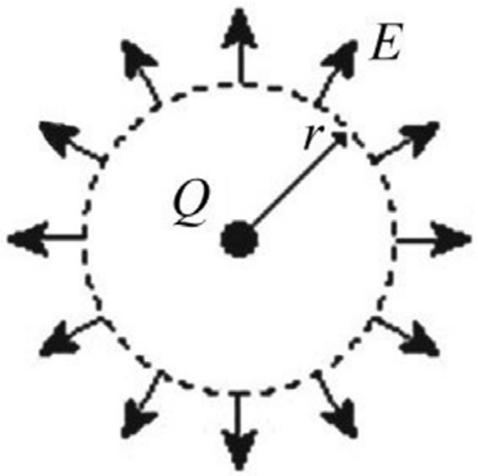

Figure 2. Gaussian sphere around a point charge

4) Ampère's law describes the relation between a magnetic field and the current that gives rise to that field. When there is time dependence in the current, this law has a logical flaw. If there is no charge buildup anywhere, then currents must be steady, consistent with charge conservation. As currents flow in a capacitor, the charge builds up and eventually the current that brings the charge must decrease. In Ampere's law, the quantity I (enclosed) that refers to the current enclosed by a *closed path* would be thus ambiguous, as seen in Figure 3.

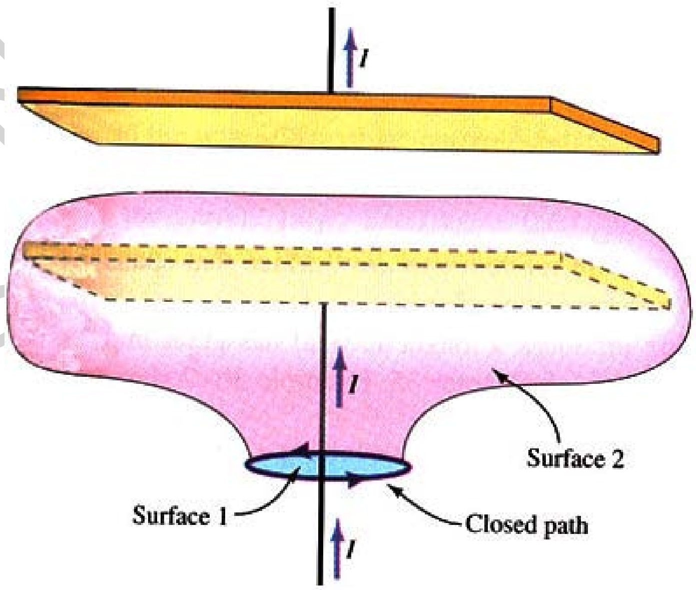

Figure 3. Ambiguity in the current enclosed by a closed path: Compare Surface 1 with the current I enclosed with Surface 2 with no current

Maxwell noted that, even if no current passes through surface 2 in Figure 3, *there is a changing electric flux through it*. Maxwell called the changing flux term the *displacement current* $I_d \equiv \in_0 d\Phi_E/dt$ (where the subcript E denotes the electric flux). Even though the current is not continuous when capacitors are present, the sum of the ordinary current and the displacement current is continuous. Maxwell's generalized form of Ampère's law is accordingly

$$\oint \vec{B}.\, d\vec{s} = \mu_0 \mathbf{I} + \mu_0 \in_0 \frac{d}{dt} \int\int_S \vec{E}.\, d\vec{A} \qquad (4)$$

The left-hand side is the expression for the integral of the magnetic field's tangential component along a closed loop. The right-hand side has two contributions: one is the current flowing any surface S bounded by the closed loop, the other is the rate of change of the electric field flux through such a surface, the "displacement current" contribution. The presence of μ_0, the vacuum permeability, is again a consequence of the choice of SI units.

In Maxwell's equations, loops and surfaces (Gaussian or not) are imaginary, i.e., mental constructs where the fields $\vec{E}$ or $\vec{B}$ are defined. The equations display a certain degree of symmetry between electric and magnetic fields: compare (1) with (3), and (2) with (4). This symmetry is not perfect, due to the non-existence of magnetic monopoles. For example, Faraday's law contains no term like the $\mu_0 I$ term in Ampère's law, because there is no free magnetic charge to form a magnetic current. In vacuum, where there are no electric charges, the symmetry is perfect.

In the presence of matter, electromagnetic phenomena can be described by a modified form of Maxwell's equations. For most types of materials, we can simply replace $\in_0$ by $\in$, because the additional replacement of μ_0 by μ does not affect matters much (except for ferromagnetic materials).

3. The Propagation of Electromagnetic Fields

A glance at Eqs. (2) and (4) shows that *when the electric and magnetic fields are time dependent, they influence each other*: they are said to be coupled. These coupled configurations of the field can transport energy and momentum over very long distances, much longer than the typical ranges of electrostatic and magnetostatic fields. The coupled fields produce traveling waves called *electromagnetic waves*. These waves are all around us: Radio and Television, Microwaves, visible Light, X-rays, etc.

We may understand how charges in motion can give rise to electromagnetic waves. Consider, as in Figure 4a, a straight wire that is aligned with the x-axis and carries a current I. A magnetic field forms in rings around the wire; if the current is constant, the magnetic field is also constant. If the current increases, the magnetic field increases, as does the magnetic flux through an area A_1 in the x-z plane. According to Faraday's law, Eq. (2), a changing magnetic flux induces an e.m.f. around the boundary of this area, and it is associated with the induced electric field of Figure.4b. Lenz's law determines the direction of this electric field.

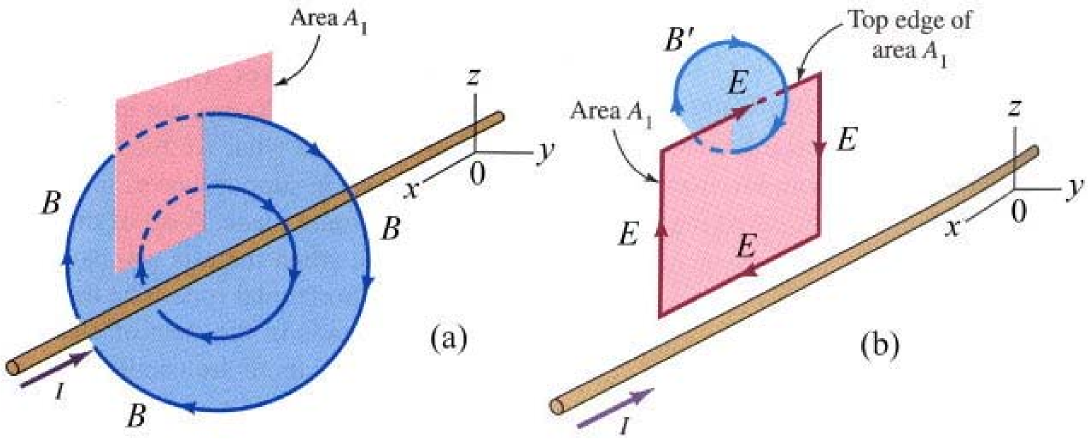

Figure 4. (a) Generation of electromagnetic waves; (b) Induced secondary electric and magnetic fields. See the text for the discussion.

Consider now the top edge of area A_1. Along that edge, the electric field has been induced in the $x-$direction. This induced electric field changes, because it was due to a changing magnetic field. But, according to Maxwell's law, Eq. (4), *a changing electric field produces a magnetic field by giving rise to a displacement current* and we do not need any flowing charges from now on. At this point we have discovered how the propagation works: The displacement current produces a secondary magnetic field $\vec{B}'$, at higher values of z as in Figure 4b. In the xz plane, $\vec{B}'$ is perpendicular to that plane and it is changing with time. As a consequence, this secondary magnetic field produces an induced e.m.f. aligned in the x-direction at still higher values of z. The process repeats itself to still higher and higher z values, leading to the propagation of the electromagnetic field in space regions without charges or currents.

4. Sinusoidal Electromagnetic Waves

In the configuration of Section 3, with the imaginary loop in the xz-plane, the fields propagate in the z-direction. Propagation only in the z-direction is the physical situation if the single current- carrying wire is replaced by a sheet of current oriented in the xy-plane, as indicated in Figure 5. The electric field is aligned along the x-axis, parallel to the current, and the magnetic field is aligned along the y-axis, perpendicular to both the electric field and to the direction of propagation. This is a general feature of electromagnetic waves.

The current that is the original source of the fields must change with time or, equivalently, charges that produce *propagating* electric and magnetic fields must be accelerating.

The propagating fields E_x and B_y depend therefore on (z,t). From Maxwell's equation (4), one can derive.

$$-\frac{\partial B_y}{\partial z} = \mu_0 \in_0 \frac{\partial E_x}{\partial t} \tag{5}$$

From Faraday's Law (2), one can get

$$-\frac{\partial B_y}{\partial t} = \frac{\partial E_x}{\partial z} \tag{6}$$

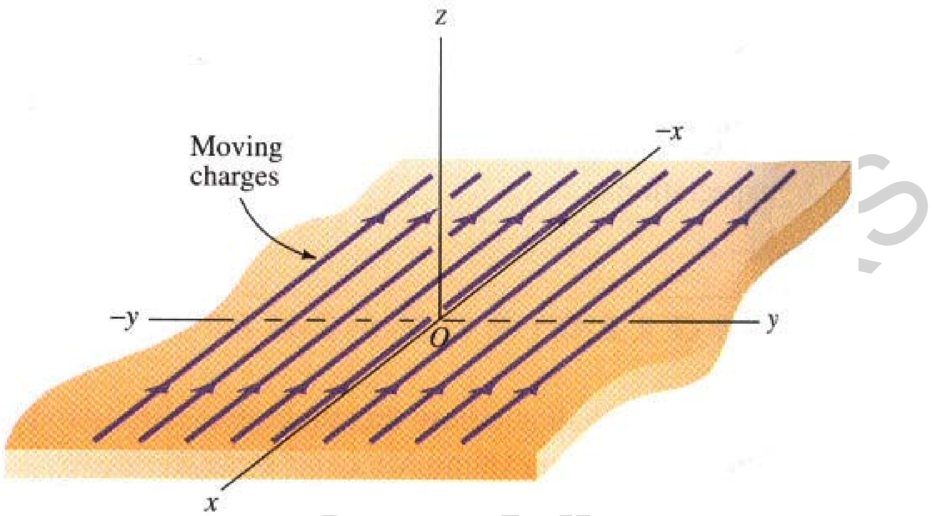

Figure 5. Generation of plane waves

These Eqs. (5) and (6) couple the two fields. Taking the derivative of (5) with respect to time and the derivative of (6) with respect to z, the result is the *Wave Equation*

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \in_0 \frac{\partial^2 E_x}{\partial t^2} \tag{7}$$

which describes how the perturbation, the field, propagates in a correlated way with respect to z and to t. The Wave Equation provides the connection between the variation of the field with time, at a given point, and the variation of the field with space, at a given time.

A solution of this wave equation is a harmonic plane wave propagating in the z-direction

$$E_x = E_0 \cos(kz - \omega t + \phi) \tag{8}$$

where E_0 is the amplitude, k the wave number and ω the angular frequency. Equation (8) represents a wave of wavelength $\lambda = 2\pi/k$ and frequency $\nu = \omega/2\pi$. The speed of the wave's propagation is $c = \lambda\nu = \omega/k$. Its value is found from the wave equation (7).

We have

$$c = \frac{1}{\sqrt{\mu_0 \in_0}} \tag{9}$$

giving the speed of light $c = 3.00 \times 10^8 \text{m/s}$.

The wave equation for the magnetic field is similar. From Eqs. (5) and (6), the correct spatial and time dependence of B_y is again harmonic

$$B_y = B_0 \cos(kz - \omega t + \phi) \tag{10}$$

with the same wave number k and angular frequency ω. As seen in Eqs. (8) and (10), the fields E_x and B_y are in phase: when the electric field is a maximum, the magnetic field is also a maximum; when one is zero, the other is zero, and so forth. The solutions found for these plane waves have the property that the electric field and magnetic field are perpendicular to each other $\vec{E}.\vec{B} = 0$ and both are *transverse* to the direction $\hat{z}$ of propagation of the wave. In Figure 6 one has an instantaneous view of the transverse electric and magnetic fields that propagate along the z-axis.

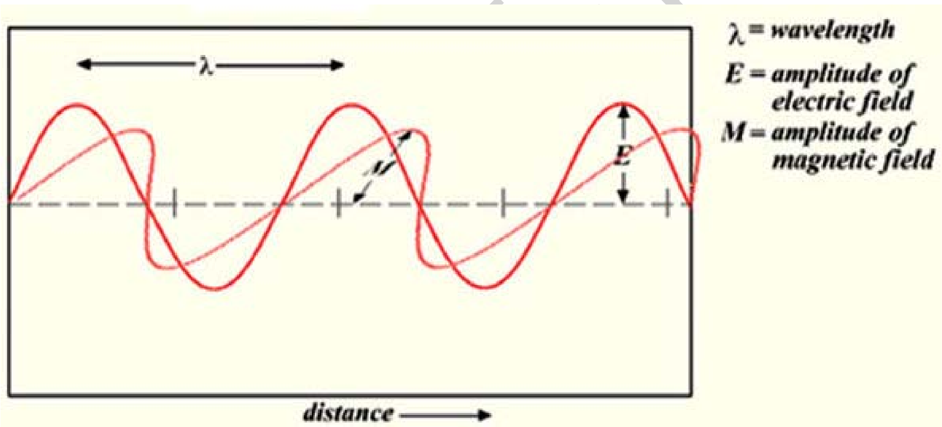

Figure 6. Instantaneous view of the transverse electromagnetic field propagating in a definite direction

Again, from Eq. (6) one finds the relation between the electric and magnetic amplitudes in an electromagnetic wave

$$E_0 = cB_0 \tag{11}$$

which is independent of the original currents. From Eq. (5), one finds

$$B_0 = \mu_0 \in_0 cE_0 \tag{12}$$

which is compatible with (11) only if c is given by (9).

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Biographical Sketch

Jose Bernabeu - Spanish physicist of high energy elementary particles. Born in Mutxamel (Alicante) in 1945. Licenciado in Physics by the University of Valencia in 1967, he obtained the Extraordinary Prize and the National Prize in Physics. Under the advice of Professor Pedro Pascual, he obtained the Ph. D. in Physics with Extraordinary Prize in 1970.

Scientific Activity

In 1971 he initiated a postdoctoral stay at the European Laboratory for Particle Physics (CERN), with site in Geneva, Switzerland, where he had the different positions of fellow, associate researcher and fixed-term staff member until 1978, in a period in which Spain was still not a Member State of CERN (it is since 1983). He was the first Spanish staff in the Theoretical Division of CERN. In 1976 he obtained the Chair of Theoretical Physics in the University of Barcelona and then that of Valencia in 1977. Most of his scientific activity has been shared between Valencia and CERN, where he maintains a regular collaboration.

Besides Valencia, he had teaching activities in the Universities of Barcelona, Louvain-la-Neuve, Bergen, Lyon, La Plata, Oviedo and Paris-Orsay. He was the Director of the Department of Theoretical Physics of the University of Valencia from 1978 to 1988, when he was appointed the Chairman of the National Programme of High Energy Physics in the National Plan of Science and Technology, from 1988 to 1992.

His research work has been mostly devoted to elementary particle physics, in the field of unified electroweak interactions within and beyond the standard theory. His results on the non-decoupling effects associated to the spontaneous breaking of the gauge symmetry, leading to quantum virtual contributions of the top quark mass in certain observables at the Z peak, have been influential in the scientific community. Another area in which he has international recognition is neutrino physics, with contributions which go from models of neutrino mass and mixing and lepton flavor violation to phenomenological analyses of the physics reach with new observables and facilities, results of relevance for particle physics, astroparticle physics and cosmology. A third research activity to be pointed out is in the field of symmetries and, in particular, CP violation: on phenomenology, he was among the first authors to signal the crucial role to be played by the b-quark sector to understand the origin of CP violation first observed in the neutral Kaon system; on theory, he studied the conditions for the existence of a CP operator in quantum field theory and, in particular, the CP Restrictions on Quark Mass Matrices in the standard theory. These results are of importance for the Baryogenesis of the Universe. His article on parity violation by neutral currents in muonic atoms is the source for the discussion of atomic parity violation in several standard textbooks.

He has regularly been Principal Investigator of Grants obtained by the Theory Group of Valencia. He has been invited, in scientific stays from 2 months to 1 year, in more than a dozen of European and American centers of excellence. He has given more than 100 research seminars and he is a regular Lecturer in important scientific events.

He has published around 200 research articles in the journals of highest impact index and he has received around 3000 citations. He has written a book on the unified electroweak interaction. He is a regular referee for journals, funding agencies and evaluation systems. He has also written more than 30 articles for outreach and has given a considerable number of talks addressed to a non-specialized audience in order to bring the frontier of physics to the society.

Prizes and Distinctions

Sir of the Order "Alfonso X the Wise" and Academician of the National Academy of Exact, Physical and Natural Sciences of Argentina. National Prize of Canada-Blanch Foundation. In 2001 he obtained the Distinction to the Cultural Merit of Generalitat Valenciana. He is and has been Member of Committees related to the scientific life in Spain and Europe: National Director of the InterUniversity Group of Theoretical Physics from 1984 to 1987, Member of the Scientific Policy Committe of Generalitat Valenciana from 1985 to 1988, Member of LHC Committee at CERN from 1994 to 1997, Member of the Appeal Committee of the University of Valencia from 1997 to 2004 and of its Consulting Board since 2005. Member of the Scientific Committee of the Science Museum of Valencia since 2000. He was the Chairman of the HEPP Board of the EPS from 2003 to 2005. From 2004 to 2007 he was the Chairman of the International Advisory Committee of the Canfranc Underground Laboratory. Since 2005 he is a Member of the International Advisory Committee for the Modane Underground Laboratory. At present he is the Project Leader for a Large Scale Facility of Research on Medical Physics, to be organized as a Consortium between the Government of Spain and the Government of Generalitat Valenciana.