MECHANICS: STATICS AND DYNAMICS

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Keywords: mechanics, statics, dynamics, equilibrium, kinematics, kinetics, motion, impact

Contents

Summary

A comprehensive overview on the fundamentals of mechanics is presented in this chapter. Classical mechanics is a foundation of various mechanics topics such as strength of materials, fluid mechanics, machine design, mechanical vibrations, automatic control, finite elements, and so on. First, statics is illustrated with mathematical definitions of a force vector and subsequent force equilibrium requirements for particles. The concept of the moment of a force is introduced as static equilibrium requirements for rigid bodies. Then, dynamics is explained from kinematics arguments of motion to kinetics analysis of particles and rigid bodies. Various kinetic methods are explained through vector (Newtonian) methods, energy methods, and momentum methods. Finally, advanced dynamic topics such as 3-D kinematics and the Lagrangian approach are illustrated.

1. Introduction

The science of mechanics is centered on the study of the motion of a physical object subjected to various types of mechanical loading. From the causality point of view, a mechanical cause (applied load) to a physical object will result in mechanical responses (motion). Four entities are involved in this causality relationship:

- Physical objects – Three common states of physical objects are gas, fluid, and solid. Thus, mechanics studies are often named by their medium, i.e. gas dynamics, fluid mechanics, and solid mechanics. Furthermore, mathematical idealization is adopted to consider physical objects as particles, or as either rigid or non-rigid deformable bodies.
- Mechanical causes of motion – There are many mechanical causes of motion such as force, moment, work, impulse, and power, etc.
- Mechanical responses – Two types of spatial motion for a physical object are translation and rotation. A general motion consists of these two motion components, which are independent of each other. This lays an important theoretical basis for rigid-body kinematics.
- Cause and effect relationship – The governing physical laws are Newton's three laws of motion and Euler's equations. When Newton's second law of motion is integrated, it becomes either the principle of work and energy or the principle of impulse and momentum. These laws are the foundations of all mechanics studies.

Statics and dynamics concentrate on Newtonian or classical mechanics, which disregards the interactions of particles on a sub-atomic scale and the interactions involving relative speeds near the speed of light. Over a broad range of object sizes and velocities, classical mechanics is found to agree well with experimental observations. In his Principia, Sir Isaac Newton stated the laws upon which classical mechanics is based. When interpreted in modern language: (Greenwood 1988)

I. Every body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change its state by forces acting upon it (Law of inertia, N1L).
II. The time rate of change of linear momentum of a body is proportional to the force acting upon it and occurs in the direction in which the force acts (Law of motion, N2L).
III. To every action there is an equal and opposite reaction; that is, the mutual forces of two bodies acting upon each other are equal in magnitude, but opposite in direction (Law of action and reaction, N3L).

An understanding of Newton's laws of motion is easily achieved by applying them to the study of particle motion, where a particle is defined as a mass concentrated at a point. When the three basic laws of motion are applied to the motion of a particle, the law of motion (N2L) can be expressed by the equation

$$\boldsymbol{F} = m\boldsymbol{a} \tag{1}$$

where m is the mass of the particle, $\boldsymbol{a}$ is its acceleration, $\boldsymbol{F}$ is the applied force. In the SI system of units, the force is expressed in Newton (N), the acceleration in meter per second squared (m/sec^2), and the mass in kilogram (kg). In the U.S. customary system of units, the force is expressed in pound (lb), the acceleration in foot per second squared (ft/sec^2), and the mass in slug (slug). Note that one pound of force can cause a particle with one slug of mass to have one foot per second squared of acceleration.

2. Statics

From a Newtonian mechanics point of view, statics problems are a special case of dynamics problems in that the right-hand side of Eq. (1) becomes zero. It should be noted that zero acceleration implies two motion conditions, either zero displacement (stationary) or uniform velocity motion. Commonly, two idealized physical objects are considered for theoretical development in statics and dynamics. A particle is a point object consisting of a mass, whereas a rigid-body is an object with infinite stiffness ("rigid") with little local deformation. More detailed treatments of the following static topics can be found in reference 1.

2.1. Force Vectors

A physical quantity having a direction and a magnitude is called a *vector*, which requires recursive mathematical definition.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \tag{2}$$

where F is the magnitude of the vector and λ is the unit direction vector parallel to $\boldsymbol{F}$. Unlike scalar quantities, vectors are added up, according to the parallelogram law (Figure 1a). A point of application is also important in defining a force vector. A force vector acting on a particle has a well-defined point of application (the particle itself), whereas a force vector acting on a rigid body obeys the principle of transmissibility (Figure 1b), indicating that the mechanical effects will be the same as long as the point of application lies along the line of action of the force vector.

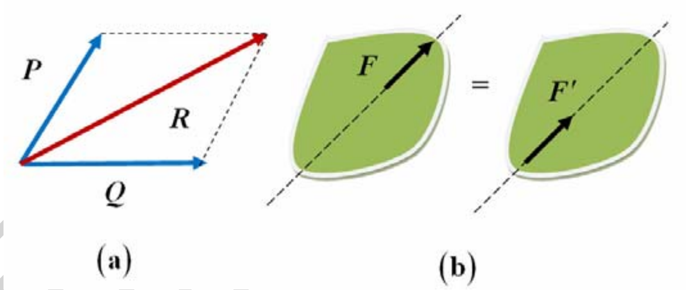

Figure 1. Major characteristics of a force vector, (a) the parallelogram law (the resultant of two force vectors is found by drawing a parallelogram with its diagonal becoming the resultant), and (b) the principle of transmissibility (two force vectors with the equal magnitude and direction are mechanically equivalent when their points of application lie along the line of action)

A force vector acting on a rigid body results in two mechanical responses, translational and rotational motions of the rigid body. Translational motion obeys Newton's second law of motion (Eq. (1)), while rotational motion follows a similar physical law called Euler's equation.

$$\boldsymbol{M} = I\boldsymbol{\alpha} \tag{3}$$

where I is the mass of moment of inertia of the rigid body, $\boldsymbol{\alpha}$ is its angular acceleration, $\boldsymbol{M}$ is the applied moment of a force vector $\boldsymbol{F}$. Thus, a moment of a force is the mechanical cause of rotational motion of a rigid body.

Force vectors are often mathematically represented in a rectangular coordinate system such as

$$\boldsymbol{F} = \boldsymbol{F_x} + \boldsymbol{F_y} + \boldsymbol{F_z} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k} \quad (4)$$

where $\boldsymbol{F}_x, \boldsymbol{F}_y$, and $\boldsymbol{F}_z$ are rectangular components in x, y, and z directions, respectively, whereas F_x, F_y, and F_z are magnitudes of each rectangular components. The unit vectors $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$ are used to represent directions along each rectangular coordinate axis.

Direction cosines are also used to represent a force vector. Mathematically, they are rectangular components of the given unit vector in such a way that

$$\boldsymbol{F} = F\boldsymbol{\lambda} = F\cos\theta_x\boldsymbol{i} + F\cos\theta_y\boldsymbol{j} + F\cos\theta_z\boldsymbol{k} \quad (5)$$

where $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are direction cosines, while θ_x, θ_y and θ_z are direction angles (Figure 2).

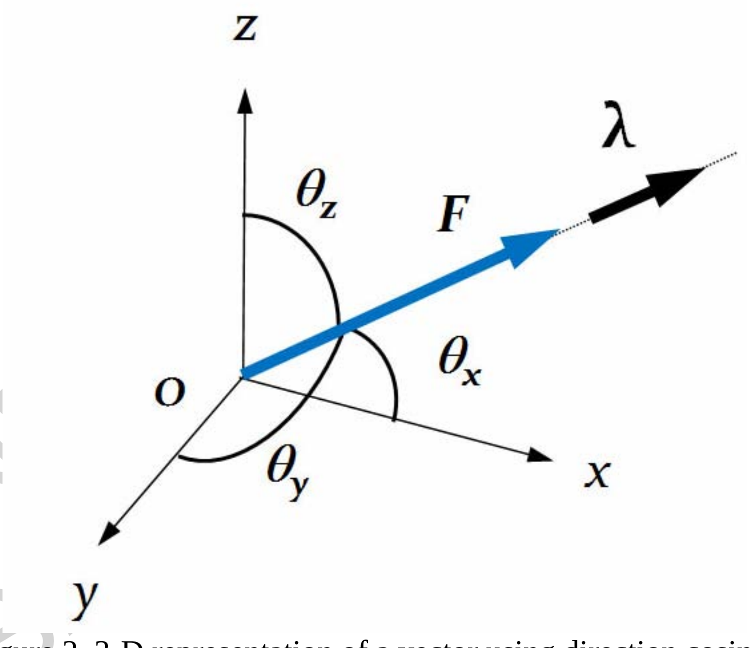

Figure 2. 3-D representation of a vector using direction cosines

2.2. Static Equilibrium for Particles

Any physical objects undergoing translational motion can be considered as particles. All the applied forces to such physical objects form a concurrent force system, meaning that the lines of action of all the forces intersect at the same point (Figure 3a). *A particle is in static equilibrium if and only if the resultant* $\boldsymbol{R}$ *or the sum of all the forces acting on the particle is zero.* In other words, the magnitudes of the components R_x, R_y, and R_z of the resultant are zero. Graphically, all the applied force vectors to the particle form a closed polygon if the particle is in static equilibrium (Figure 3b).

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \tag{6}$$

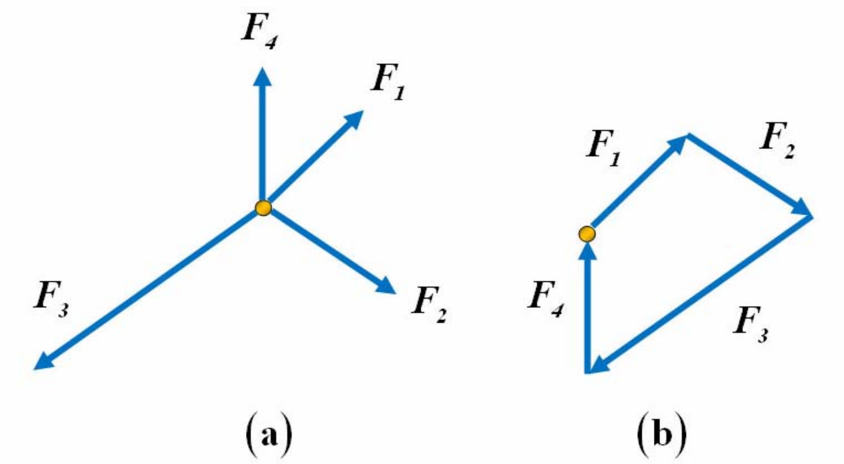

Figure 3. (a) Concurrent force system, (b) graphical representation of static force equilibrium

2.3 Moment of a Force Vector

A force vector on a rigid body can produce a general motion depending on its point of application. The translational motion effect is described by the Newton's law of motion (N2L) in Eq (1), and the rotational motion effect by the Euler's equation in Eq. (3). Such rotational motion effect is caused by a moment of a force. The moment of a force vector $\boldsymbol{F}$ about point *O* is mathematically defined by a vector product (Figure 4)

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = rF\sin\theta \boldsymbol{u}_a = Fd\boldsymbol{u}_a \tag{7}$$

where $\boldsymbol{u}_a$ is a unit direction vector perpendicular to both $\boldsymbol{r}$ and $\boldsymbol{F}$. The parameter d is the perpendicular distance between point *O* and the line of action of the force vector $\boldsymbol{F}$ and is called a *moment arm* (Figure 4).

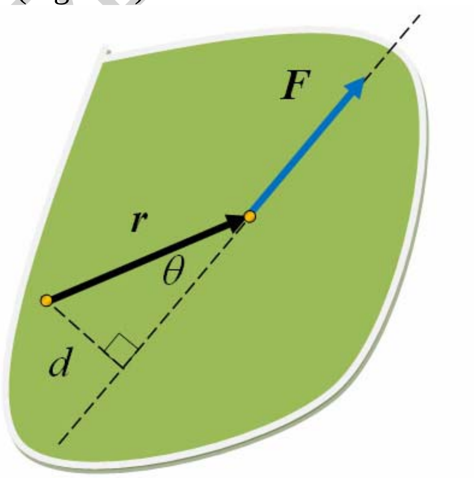

Figure 4. Moment of a force about *O*

Alternatively, the moment of a force vector $\boldsymbol{F}$ about point *O* is defined using the rectangular coordinate system as

$$\begin{aligned} \boldsymbol{M}_O &= \boldsymbol{r} \times \boldsymbol{F} \\ &= (x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}) \times (F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}) \\ &= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k} \end{aligned} \tag{8}$$

Physically, the magnitude of $\boldsymbol{M}_O$ measures the tendency of the force $\boldsymbol{F}$ to make the rigid body rotate about a fixed axis directed along $\boldsymbol{M}_O$. In the SI system of units, the moment of a force is expressed in Newton-meters (N-m), whereas in the U.S. customary system of units, in lb-ft or lb-in.

2.3.1. Varignon's Theorem

The distributive property of vector product can be used to determine the resultant moment of several forces acting at the same point.

$$\boldsymbol{M}_O = \boldsymbol{r} \times (\boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots) = \boldsymbol{r} \times \boldsymbol{F}_1 + \boldsymbol{r} \times \boldsymbol{F}_2 + \cdots \tag{9}$$

In words, the moment about a given point *O* of the resultant of several concurrent forces is equal to the sum of the moments of the various forces about the same point *O*. The property is known as *Varignon's theorem*. The theorem is typically applied when calculating the moment of a force in such a way that a force vector is decomposed into its rectangular components and then their individual moment arms are readily obtained from simple geometry arguments.

2.3.2 Moment of a Force Couple

When two force vectors $\boldsymbol{F}$ and $-\boldsymbol{F}$ have the same magnitude but opposite directions, they are said to form a (force) *couple* (Figure 5). Since the net force or their resultant is zero, the force couple only produces a moment perpendicular to the plane of the force couple such that $M = Fd$ where d is the perpendicular distance between the lines of action of $\boldsymbol{F}$ and $-\boldsymbol{F}$.

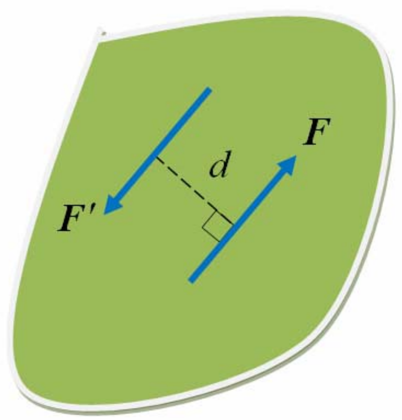

Figure 5. Force couple

Note that the moment of a force couple does not need a point of reference and thus becomes a free vector. Also note that two force couples having the same moment magnitude are equivalent, thereby resulting in the same rotational motion effect to a given rigid body.

2.3.3 Equivalent Force-Couple System

Any force $\boldsymbol{F}$ acting on a rigid body can be transferred from its own point of application to an arbitrary point if an additional couple moment $\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$ is added as long as it is equal to the moment about the new location of $\boldsymbol{F}$ at its original point of application (Figure 6). The additional couple moment shall cause the same rotational motion to the rigid body about the new location as the force $\boldsymbol{F}$ produced prior to its transfer to the new location. The additional couple moment is represented by a couple moment vector $\boldsymbol{M}_O$, which is perpendicular to the plane containing $\boldsymbol{r}$ and $\boldsymbol{F}$. Though the couple moment is a free vector, it is usually attached at the new location for convenience, together with $\boldsymbol{F}$, and the combination obtained is referred to as an *equivalent force-couple system*. Thus, the force $\boldsymbol{F}$ acting at its original point of application is mechanically equivalent to a force-couple system at the new location, indicating the same rigid body motion is imparted. This equivalent force-couple system is a theoretical basis of static equilibrium of a rigid body.

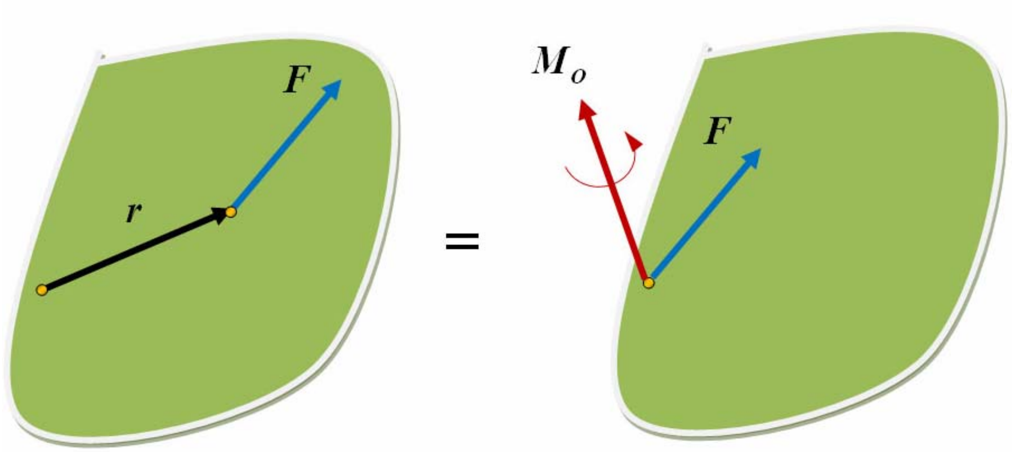

Figure 6. Equivalent force-couple system where $\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$

2.3.4 Static Equilibrium for Rigid Bodies

Consider a system of forces $\boldsymbol{F}_1, \boldsymbol{F}_2,$ and $\boldsymbol{F}_3, \ldots$ acting on a rigid body at the points $P_1, P_2, P_3, \ldots$ defined by the position vectors $\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \ldots$ etc. As noted in the previous section, any force vector can be transferred to a new location to form an equivalent force-couple system. Now each force vector $\boldsymbol{F}_i$ acting on the rigid body is transferred to a given point *O*, thereby resulting in an individual couple moment $\boldsymbol{M}_i = \boldsymbol{r}_i \times \boldsymbol{F}_i$ about *O*. Thus, the resultant force and the resultant moment of the force-couple systems for the system of forces after transfer to point *O* are obtained.

$$\boldsymbol{R} = \sum \boldsymbol{F}_i \quad \boldsymbol{M}_O = \sum \boldsymbol{r}_i \times \boldsymbol{F}_i \qquad (10)$$

The necessary and sufficient conditions for the static equilibrium of a rigid body are *that the resultant force-couple system becomes zero such that*

$$\boldsymbol{R} = \sum \boldsymbol{F}_i = \boldsymbol{0} \quad \boldsymbol{M}_O = \sum \boldsymbol{r}_i \times \boldsymbol{F}_i = \boldsymbol{0} \tag{11}$$

In a rectangular coordinate system, these become a total of six scalar equations:

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array} \tag{12}$$

2.3.5. Free-Body Diagram

In solving a problem concerning the equilibrium of a rigid body, it is essential to consider all of the forces acting on the body. Therefore, the first step in determining the solution of a problem should be to draw a free-body diagram of the rigid body under consideration. Four steps are typically involved in drawing a free-body diagram: 1) isolating a body of interest, 2) indicating all the known applied forces, 3) indicating unknown reactive forces and moments due to supports and constraints, and 4) putting appropriate dimensions as needed.

3. Dynamics

Dynamics deals with the analysis of physical bodies in motion. The most significant contribution to dynamics was made by Sir Isaac Newton (1642-1727) who formulated his fundamental laws of motion as described in Section I. Dynamics include kinematics and kinetics. The former is the study of motion to relate displacement, velocity, acceleration and time, without reference to the cause of the motion. The latter is the study of motion in relation to the forces acting on a body. There are two types of kinetic problems. One is forward dynamics problems to predict the motion with the given forces acting on a body. The other is inverse dynamic problems to determine the forces required to produce a given motion of a body. More detailed treatments of dynamics topics on planar kinematics and kinetics can be found in reference 2.

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