MECHANICS OF MATERIALS

Jeff Brown
Hope College, Department of Engineering, 27 Graves Pl., Holland, Michigan, USA

Keywords: Solid mechanics, stress, strain, yield strength

Contents

Summary

Mechanics of Materials is a fundamental topic in the field of mechanical and civil/structural engineering. The primary goal of Mechanics of Materials is to understand how external loads applied to a structure cause the structure deform and develop internal stresses.

This knowledge and understanding can be used to determine the safe allowable loading for an existing structure or to determine the dimensions and material properties required for a new structure to safely resist the expected loading.
A solid understanding of Mechanics of Materials topics is considered a pre-requisite for further study in the areas of structural engineering and mechanical engineering design. This article provides a foundational treatment of topics typically found in a college/university level course in Mechanics of Materials.

1. Introduction

The overall goal of structural design is to ensure that each element of a structure can safely resist any loads that are applied. The study of Mechanics of Materials is concerned with developing an understanding of how a structural element will respond under specified loading conditions. Some specific objectives of this type of analysis include:

1. Determine the internal stresses that result from external forces.
2. Determine the deformations in a structural element that result from these stresses.
3. Evaluate the suitability/acceptability of a structural element for safely resisting a specific loading condition. In addition to basic strength parameters, this can also include stability (i.e. buckling).

These concepts can be applied to a wide variety of structural elements ranging from beams and columns in buildings to drive shafts and gears in automobiles. In some cases, the objective of the analysis is to determine whether or not a specific structural component will fail under specified loading conditions.

Another example might be to determine the required geometry (typically the cross-sectional area) for a structural element made of a specific material (e.g. steel, aluminum, concrete) to safely resist a given load. Finally, the objective may be to determine which material is suitable or required for a structural element of specified size to safely accommodate a given load.

Regardless of the overall objective of the analysis, the relationship between loading conditions, element geometry, and material properties is central to the study of mechanics of materials.

It should be noted very early, however, that the result of any mechanics of materials analysis is only as good or relevant as the quantities used to develop the structural model. This is particularly true for loading conditions and material strength properties.

Determining the loads that a structure might experience during its service life can be challenging. Determining how these loads interact with a structure to develop internal forces in each structural member may require a very complicated structural analysis. Finally, determining the strength characteristics of a specific material may also present challenges.

This is especially true when confronting the issue of environmental durability. A mechanics of materials analysis may indicate that a structural element is safe when it is first installed, but long-term effects, such as corrosion, fatigue, or creep, may lead to unsafe conditions over time.

Such considerations are beyond the scope of this chapter, but some relevant EOLSS references include: Structural Analysis (sample chapter), Linear Analysis of Structural Systems, Corrosion (sample chapter).

2. Stress and Strain

Stress is defined as the force per unit area that develops *at a single point* in a structural element due to external loading. Strain is a measure of the deformation experienced by a structural element (also at a single point). A fundamental distinction is made between *normal stress* (σ), which acts perpendicular to a plane of interest, and *shear stress* (τ), which acts parallel to a plane of interest.

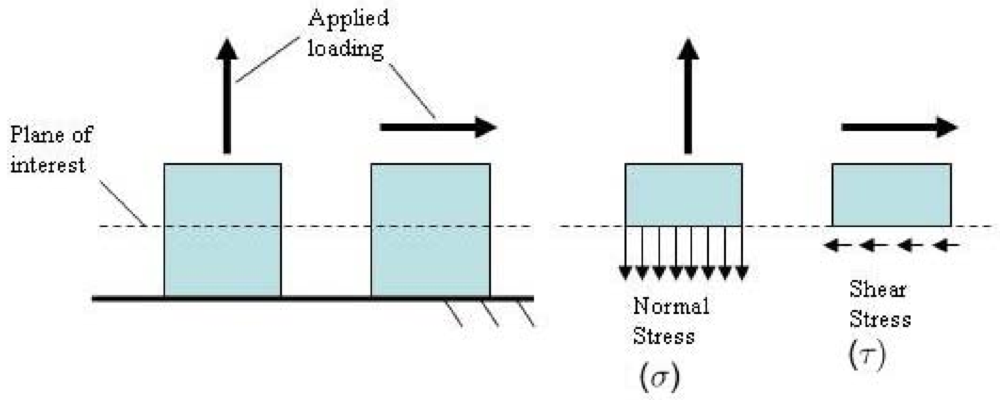

Figure 1. General concept of normal stress and shear stress for one-dimensional loading

For a general 3-D body subjected to generic loading, the state of stress at each point within the body can be summarized using a 3-D stress block. Each face of the 3-D block will experience one normal stress component and two shear stress components. The three unique normal stress components, σ_x, σ_y, and σ_z, are denoted with subscripts corresponding to the x, y, or z axes that project normally from each face. For the block to remain in translational equilibrium, the normal stress component acting on the positive x-face must be equal in magnitude but opposite in direction to the stress component acting on the negative x-face. The same principal applies to the y and z faces. Each component of shear stress, τ, is denoted with two subscripts. The first subscript corresponds to the face on which the shear stress is acting, and the second subscript corresponds to the directional sense of the stress component. Positive sign conventions for these stress components are shown in Figure 2. Again, for the block to remain in translational equilibrium, the shear stress components on opposite faces must be equal in magnitude and opposite in direction. To ensure rotational equilibrium for the stress block, the following relationships must also be true: $\tau_{xy} = \tau_{yx}$; $\tau_{xz} = \tau_{zx}$; and $\tau_{yz} = \tau_{zy}$. As a result, the six-sided stress block, with three stress components acting on each face (a total of 18 stress components), can be accurately described with only six values: σ_x, σ_y, σ_z, τ_{xy}, τ_{xz} and τ_{yz}.

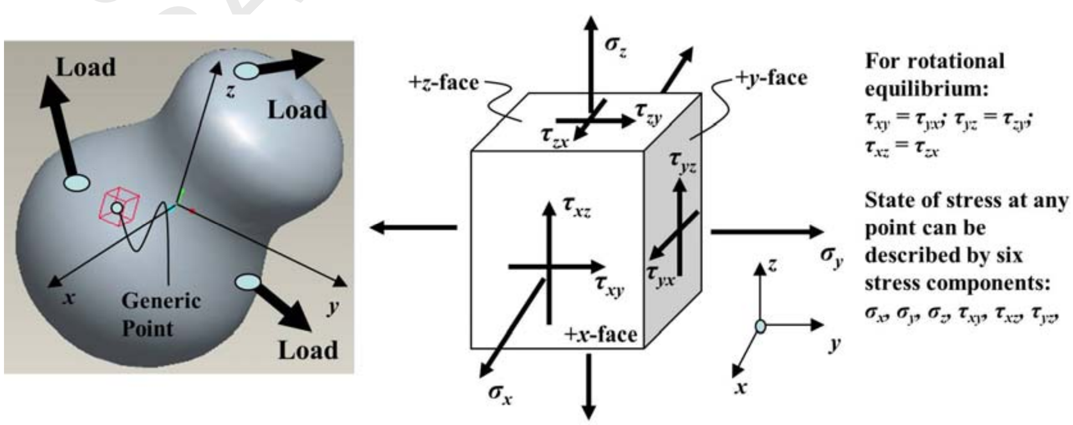

Figure 2. 3-D state of stress for general body

2.1. Axial

Axial Stress. Axially loaded structural elements often occur in the form of anchor bolts, hanging rods, cables, struts, or truss members. The internal axial force experienced by a structural element is typically determined using statics or more advanced structural analysis techniques. These internal axial forces develop normal stresses as shown in Figure 3. The general relationship between the internal axial force, P, and the normal stress, σ, that develops over the cross-sectional area of the member, A, is given by:

$$P = \int_A \sigma \cdot dA \qquad (1)$$

If the plane of interest is a sufficient distance from any external applied loads, the stress distribution is considered uniform and the *average normal stress* can be calculated as follows:

$$\sigma_{\text{avg}} = \frac{P}{A} \qquad (2)$$

The most common sign convention for axial stress is that tensile stress is considered positive and compressive stress is considered negative. This is the sign convention that will be used throughout the remainder of this document. It should be noted, however, that in reinforced concrete design the sign convention is often reversed so that compressive stress is considered positive and tensile stress is considered negative.

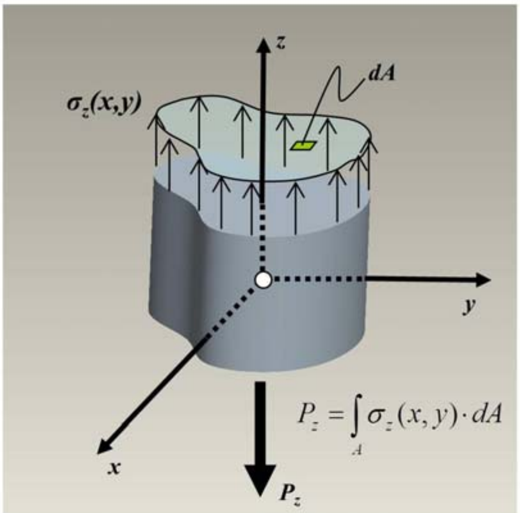

Figure 3. Normal stress in axially loaded members.

For locations near a point where a concentrated axial load is applied, the normal stress distribution is non-uniform. Non-uniform normal stress distributions can also result from axial loads if the resultant axial force does not pass through the centroid of the element’s cross-section (i.e. an internal bending moment is present). A rule of thumb is that the normal stress distribution approaches uniformity a distance equal to the largest dimension of the cross-sectional area away from the applied concentrated load. For the case of truss elements, where only axial forces develop, the average normal stress can be computed by dividing the calculated internal member force (obtained from a structural analysis) by the cross-sectional area of the member.

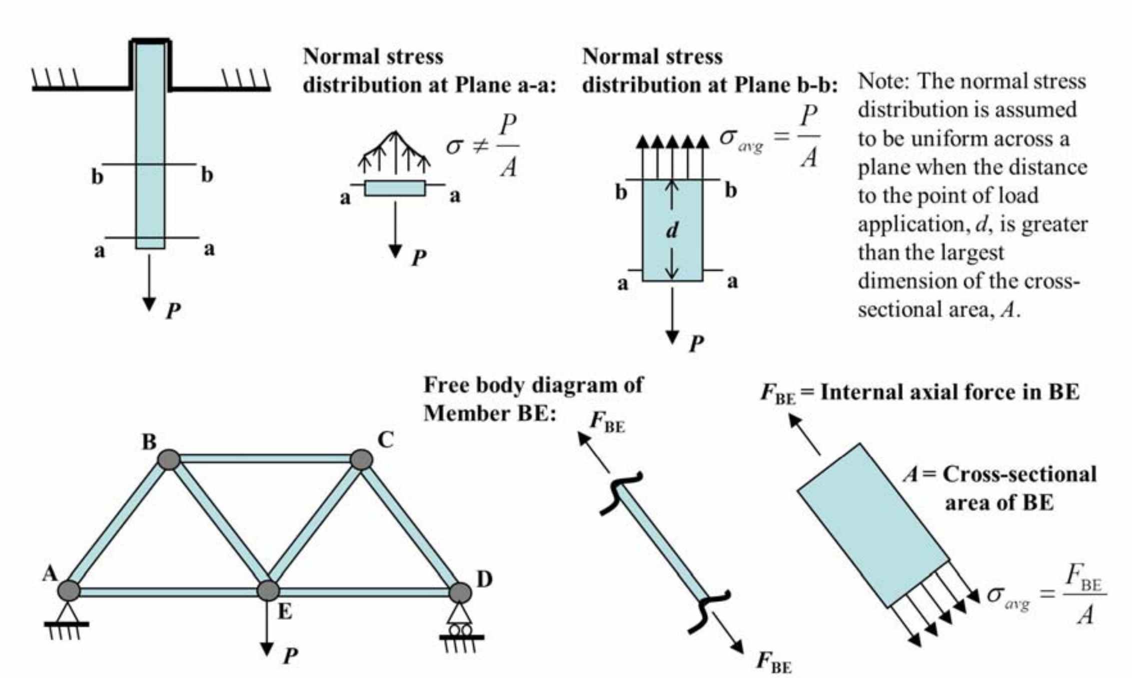

Figure 4. Normal stress distribution near applied loads and an example of normal stress in a truss element.

Axial Strain. Axial strain, ε, is a measure of the axial deformation experienced by a structural element due to internal axial loads. A common definition for *engineering* strain is the change in length occurring over a finite gage length divided by the original undeformed gage length. Similar to stress, strain is a quantity that describes the deformation experienced at a specific *point*. The following formula can be used to determine the *average* engineering strain over a specified gage length:

$$\varepsilon = \frac{\Delta L}{L_o} = \frac{L_f - L_o}{L_o} \tag{3}$$

where ΔL is the change in length, L_f is the final length, and L_o is the original length. Another phenomenon that occurs in axially loaded members is deformation (strain) that is transverse to the direction of loading. If a structural element is subjected to tensile loading, the cross-sectional area of the element is expected to decrease, while a structural element subjected to compressive loading will experience an increase in cross-sectional area. The relationship between the axial strain, ε, and transverse strain, ε_t, experienced by a structural element subjected to axial loading is described by Poisson's ratio, ν. Poisson's ratio is an intrinsic material property and is defined as follows:

$$\nu = -\frac{\varepsilon_{\mathrm{t}}}{\varepsilon} \tag{4}$$

The relationship between axial stress, σ, and axial strain, ε, also depends on the material under consideration, and determining this relationship is one of the most common objectives of materials testing. For the case of common metals, such as steel and aluminum, there is a linear relationship between stress and strain as long as the stress in the material does not exceed the *yield stress*, σ_y. The linear relationship between stress and strain is commonly expressed as *Hooke's Law* and is given by:

$$\sigma = E \cdot \varepsilon \quad \left(\text{for } \sigma \le \sigma_y\right) \tag{5}$$

where E is the *modulus of elasticity* or *Young's Modulus* of the material under consideration.
For the general case of 3-D loading, axial stress that develops in one direction due to external loading will tend to cause strain in all three orthogonal directions (one axial strain corresponding to the direction of the stress and two transverse strains that result from Poisson's ratio). The general relationship between strain and stress for 3-D loading is given as:

$$\begin{aligned}
\varepsilon_x &= \frac{\sigma_x}{E} - \frac{\nu \cdot \sigma_y}{E} - \frac{\nu \cdot \sigma_z}{E} \\
\varepsilon_y &= -\frac{\nu \cdot \sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu \cdot \sigma_z}{E} \\
\varepsilon_z &= -\frac{\nu \cdot \sigma_x}{E} - \frac{\nu \cdot \sigma_y}{E} + \frac{\sigma_z}{E}
\end{aligned} \tag{6}$$

Deformations in Axially Loaded Structural Elements. The relationships described above for axial stress and strain can be combined to determine the total deformation experienced by a structural element subjected to a generic axial loading. Consider the structural element shown in Figure 5 with variable cross-section, $A(x)$, known end reactions, P_{A} and P_{B}, and variable applied axial load, $P'(x)$. The internal axial force, $P(x)$, can be determined at any location between A and B using statics. A differential element, dx, will experience a portion of the total deformation, $d\delta_{\mathrm{AB}}$. The total deformation between A and B is given by:

$$\delta_{\mathrm{AB}} = \int_0^L d\delta_{\mathrm{AB}} \tag{7}$$

Recalling the fundamental definition of strain, $d\delta_{\mathrm{AB}}$ and dx are related by:

$$\varepsilon(x) = \frac{d\delta_{\mathrm{AB}}}{dx} \text{ or } d\delta_{\mathrm{AB}} = \varepsilon(x) \cdot dx \tag{8}$$

The strain experienced by dx, $\varepsilon(x)$, is related to stress, $\sigma(x)$, by Hooke's Law (Eq. (5)):

$$\sigma(x) = \varepsilon(x) \cdot E \text{ or } \varepsilon(x) = \frac{\sigma(x)}{E} \qquad (9)$$

Finally, the stress can be related to the internal axial force, $P(x)$, and the cross-sectional area, $A(x)$:

$$\sigma(x) = \frac{P(x)}{A(x)} \qquad (10)$$

Substituting these expressions back into Eq. (7) results in the following integral expression for the total deformation between A and B:

$$\delta_{\text{AB}} = \int_0^L \frac{P(x)}{A(x) \cdot E} dx \qquad (11)$$

For the case of a prismatic member with constant cross-section, A, total length, L, and uniform internal axial force, P, Eq. (11) can be simplified to the following expression for the change in length:

$$\delta_{\text{AB}} = \frac{P \cdot L}{A \cdot E} \qquad (12)$$

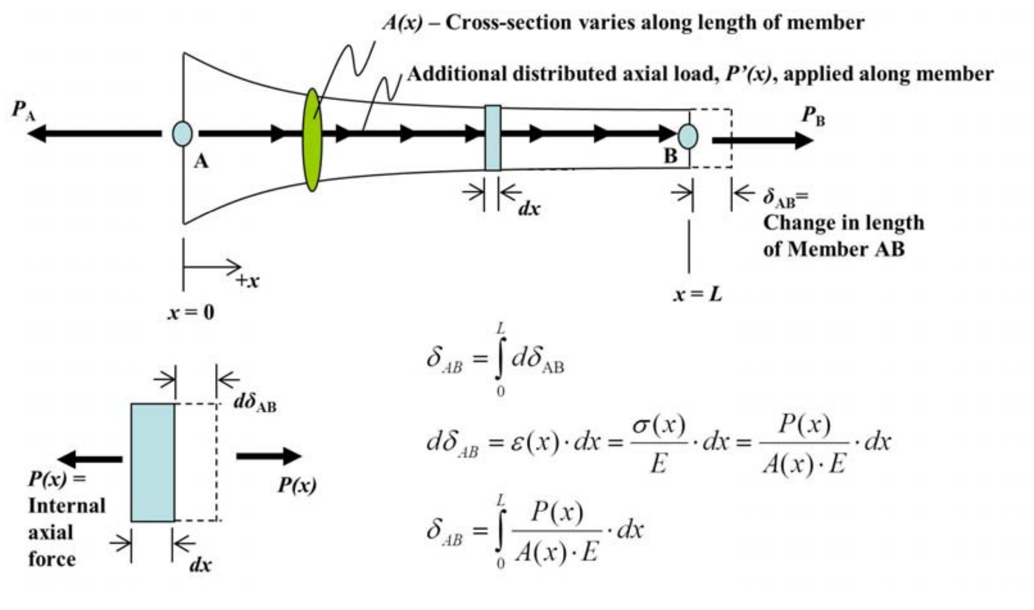

Figure 5. Determining axial deformations for structural element with varying cross-section and generic axial loading.

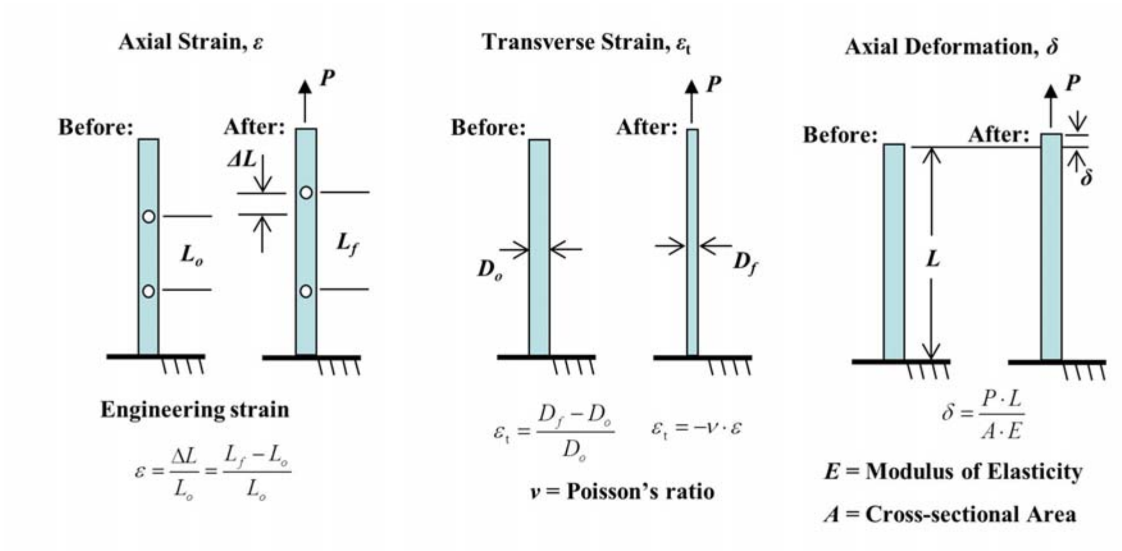

Figure 6. Summary of strain and deformation relationships for axially loaded members.

-
-
-

TO ACCESS ALL THE **37 PAGES** OF THIS CHAPTER,
Visit: http://www.eolss.net/Eolss-sampleAllChapter.aspx

Bibliography

Hibbeler, R.C. *Mechanics of Materials*. 7th ed. Upper Saddle River: Pearson Prentice Hall, 2008. [This textbook will provide further discussion on the topics presented in this article. Numerous example problems with solutions can also be found.]

Beer, Ferdinand Pierre. *Mechanics of Materials*. New York: McGraw-Hill, 2011. [This textbook will provide further discussion on the topics presented in this article. Numerous example problems with solutions can also be found.]

West, Harry H. *Fundamentals of Structural Analysis*. New York: Wiley, 1993. [This textbook includes additional background on the method of virtual work as well as more advanced topics in structural analysis that were not addressed in this article.]

Biographical Sketch

Jeff Brown is Assistant Professor in the college of engineering at Hope College, Michigan. Jeff received his B.S. degree in 1996 and his M.S. degree in 1998 in civil engineering both from the University of Central Florida. He then served on the Peace Corps in 1998 after which he continued his graduate studies and completed his Ph.D. from the University of Florida in 2005. He received the National Science Foundation Graduate Fellowship to pursue his doctoral studies at the University of Florida. He authored an award-winning paper in COMPOSITES 2004. At Hope College, he is responsible for teaching courses in solid mechanics and structural analysis. His research interests include performance and durability of reinforced concrete structures.