STRUCTURAL RELIABILITY

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Contents

Summary

The safety of a structure is defined in terms of the structure's ability to perform its intended functions, and the inability to satisfy any such intended performance requirement is termed as its failure. From a demand/capacity perspective, a structure is safe when the total demand on the structure (in terms of load effects) for a typical performance scenario is less than its capacity. The reverse situation is considered to be the failure of the structure. For example, a design code prescribes how to calculate the moment capacity of a beam section under bending, and also how to calculate the total moment (demand) that acts on this section under various design scenarios. The total applied moment is checked against the moment capacity of the beam section to check if the design is safe. The reliability of a structure is generally understood as the probability that it will not fail to satisfy its intended functions. A more mathematical description of structural reliability is provided later in this chapter, however, that definition also conforms to this basic generic understanding of the reliability of a structure. Reliability of structures is a concept that is used both in structural design and analysis. While the reliability-based design focuses on achieving structural designs with a certain level of reliability, the structural reliability analysis is used to find the level of reliability that exists for a selected performance requirement.

1. General Introduction

1.1. Uncertainty and Reliability-Based Design

The deterministic design philosophy considers all the demand (load effects) and capacity (structural strength) parameters to be single-valued quantities. Therefore, it describes every design to be either "safe" (when capacity is more than demand) or "unsafe" (when capacity is less than the demand). However, in reality, these parameters describing the loading and the load-carrying capacities are not deterministic quantities, but are random variables. Thus, real life designs are never *absolutely* safe or unsafe. The reliability-based design concept acknowledges the uncertainties in the whole design process and incorporates the concept that there is some probability of failure in every design. Instead of ignoring uncertainties existing in real life, as in the case of deterministic design procedures, the reliability-based design method focuses on scientific quantification of these uncertainties and limiting the probability of failure to a desired level.

Uncertainties in the design procedure is usually not acknowledged by one trained in the deterministic design principles. However, a few common examples may clearly show how uncertainty is an integral part of the design procedure. For example, the imposed (live) load on a classroom floor can never be precisely estimated as a single-valued quantity. Even, there is uncertainty in estimating the maximum value of this load. Similarly, the characteristic compressive strength of concrete in a reinforced concrete beam is also not a specific value as specified by the designer (say 30 MPa). Three cylinder tests of the same concrete will most possibly show three different values for this parameter (say, 30.1 MPa, 30.4 MPa and 29.9 MPa). In reliability-based design and analysis, these variations (or, the uncertainties in estimating every parameter) are incorporated in a statistical sense.

1.2. Objectives

The basic objectives of this work are:

- To provide the statistics and probability backgrounds required for structural reliability analysis
- To introduce various methods of estimating reliability level/probability of failure for a selected design scenario ("structural safety analysis")
- To discuss various concepts involved in the reliability-based design process and code calibration
- To introduce to concepts of component and system reliability

2. Probability Basics and Random Variables

2.1. Basic Definitions

Let us consider the example of tension test of low carbon steel specimens for determining their yield strength. For n number of specimens, let the results be: $x_1, x_2, x_3, \ldots x_n$. Since the result for the *i*-th specimen, x_i, can be any positive number theoretically (even including zero), the sample space for this experiment consists of the continuous set of all positive numbers. This is an example of a *continuous* sample space. An example of a *discrete* sample space is the sample space for the number of students attending a course on Reliability of Structures. A sample event E_1 for the aforementioned tension test specimens can be defined by the outcomes in the range of 255-260 MPa, and another event E_2 for the range 270-280 MPa.

2.2. Probability Axioms and Probability Functions

The three axioms of probability are:

$$0 \le \mathrm{P}(E) \le 1 \tag{1}$$

where $\mathrm{P}(E)$ is the probability of occurrence of event *E*.

$$\mathrm{P}(S) = 1 \tag{2}$$

where S represents an event corresponding to the entire sample space.

$$\mathrm{P}\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} \mathrm{P}(E_i) \tag{3}$$

The left side of Eq. (3) represents the probability of the union of events $E_1, E_2, \ldots E_n$ (that is when any of these events occur).

Probability functions are based on sample events. They are functions of random variables. A *random variable* translates or maps an event onto real number intervals. For example, the parameter yield strength obtained from the tension test of low-carbon steel can be considered a random variable. It assigns various events of material yielding to an interval of numbers, such as 255-260 MPa or 270-280 MPa. The outcome of each experiment in terms of the yield strength is randomly varying from experiment to experiment, and its specific value cannot be predicted before an experiment is conducted. Hence, this parameter (yield strength) is considered to be a random variable. A random variable can be either *continuous* or *discrete* depending on the type of real number interval it maps an event onto. For example, the yield strength in the previous example is a continuous random variable. On the other hand, if a random variable can only be assigned a few discrete numbers (say, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 & 4.0), it is a discrete random variable. In this chapter, a random variable is denoted by a capital/uppercase letter while the value it takes is denoted by the corresponding lowercase letter.

The *probability mass function* (PMF) for a discrete random variable is defined as the probability that a random variable X is equal to a specific value x:

$$p_X(x) = \mathrm{P}(X = x) \tag{4}$$

p denotes the PMF and the subscript X denotes the random variable. x in the parenthesis denote the value the random variable takes. This format is followed for other probability functions as well.

The *cumulative distribution function* (CDF) for both discrete and continuous random variables is defined as the probability that a random variable X is less than or equal to a specific value x:

$$F_X(x) = \mathrm{P}(X \le x) \tag{5}$$

where F denotes the CDF.

The *probability density function* (PDF) for a continuous variable is defined as the first derivative of the CDF:

$$f_X(x) = \frac{\mathrm{d}}{\mathrm{d}x} F_X(x) \tag{6a}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\,\mathrm{d}u \tag{6b}$$

f represents the PDF, and u is a dummy variable.

Note that the CDF needs to be differentiable in order to define the PDF. Hence, the PDF can be defined only for continuous random variables. It should also be noted, that the CDF expresses a probability, while the PDF does not. Therefore the axioms of probability apply to any CDF. The CDF is a nondecreasing function of x. For F_X to be a proper distribution function, it also needs to satisfy the following:

$$F_X(-\infty) = 0 \tag{7a}$$

$$F_X(+\infty) = 1 \tag{7b}$$

$$\int_{-\infty}^{+\infty} f_X(u)\,\mathrm{d}u = 1 \tag{7c}$$

The PDF, although it does not represent any probability, has a value greater than zero.

The basic parameters used to describe a random variable X includes its *mean value* (μ_X) or *expected value* ($\mathrm{E}\left[X\right]$), and *standard deviation* (σ_X). The mean value (same as the expected value) is defined as

$$\mu_X = \int_{-\infty}^{+\infty} x f_X(x)\,\mathrm{d}x \tag{8}$$

The *variance* (σ_X^2) of the variable X is defined as

$$\sigma_X{}^2 = \int_{-\infty}^{+\infty} (x-\mu_X)^2 f_X(x)\,\mathrm{d}x \tag{9}$$

Also,

$$\sigma_X{}^2 = \mathrm{E}\left[X^2\right] - \mu_X{}^2 \tag{10}$$

The standard deviation is obtained from the variance:

$$\sigma_X = \sqrt{\sigma_X{}^2} \tag{11}$$

In addition, this σ_X can be normalized in terms of the coefficient of variation (V_X):

$$V_X = \frac{\sigma_X}{\mu_X} \tag{12}$$

2.3. Conditional Probability

The conditional probability that the event E_1 takes place, given the fact that another event E_2 has already taken place, is expressed as

$$\mathrm{P}\left(E_1 \mid E_2\right) = \frac{\mathrm{P}\left(E_1 \cap E_2\right)}{\mathrm{P}\left(E_2\right)} \tag{13}$$

where the vertical line (|) represents that the occurrence of event E_1 is studied conditional to the occurrence of E_2. The numerator on the right side of the Eq. (13) is the probability of occurrence of the intersection of the two events E_1 and E_2. If these two events are 'statistically' independent, then the probability of occurrence of one event does not remain conditional on the other's occurrence, and the hypothetical conditional probability reduces to the probability of occurrence of this event itself:

$$\mathrm{P}\left(E_1 \mid E_2\right) = \mathrm{P}\left(E_1\right) \tag{14}$$

The concept of conditional probability are used in cases such as this, where E_1 expresses an event that the plastic moment capacity of a steel section is greater than 45 kNm, and E_2 expresses an event that the material has an yield stress more than 265 MPa. Note that when the conditional probability of the occurrence of event E_1, given that E_2 has occurred, can be defined, one can also define the conditional probability P(E_2 | E_1). This signifies that the definition of P(E_1 | E_2) does not imply that E_2 occurs before (temporally) E_1.

2.4. Common Random Variable Distributions

A. Uniform Random Variables

If a random variable is equally likely to have any possible value in a given (continuous) range, then it is known as a *uniform random variable*. For example, if the probability that person A reaches office at a certain time instant between 9 to 9:30 am is equal to the probability that A reaches office at any other time instant between 9 to 9:30 am, then the arrival time for A is a uniform random variable between 9 to 9:30. The PDF of a uniform random variable *X* can be expressed as

$$f_X\left(x\right) = \frac{1}{b-a} \text{ for } a \le x \le b\text{; and 0 otherwise} \tag{15}$$

where a and b define the range of values for X. The mean and variance for this random variable can be easily obtained as

$$\mu_X = \frac{a+b}{2} \tag{16}$$

$$\sigma_X{}^2 = \frac{(b-a)^2}{12} \tag{17}$$

B. Normal Random Variables and the Standard Normal

Also known as the *Gaussian*, this distribution is expressed as having a PDF of the following form

$$f_X(x) = \frac{\exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right]}{\sigma_X\sqrt{2\pi}} \tag{18}$$

Note that the PDF is defined in terms of the mean (μ_X) and the standard deviation (σ_X) of the distribution. Figure 1 shows both the PDF and CDF of this distribution. The *standard normal variable* Z is a normal random variable that has $\mu_Z = 0$ and $\sigma_Z = 1$ (Figure 2). This variable can be obtained from any normal random variable X, using the simple transformation

$$Z = \frac{X-\mu_X}{\sigma_X} \tag{19}$$

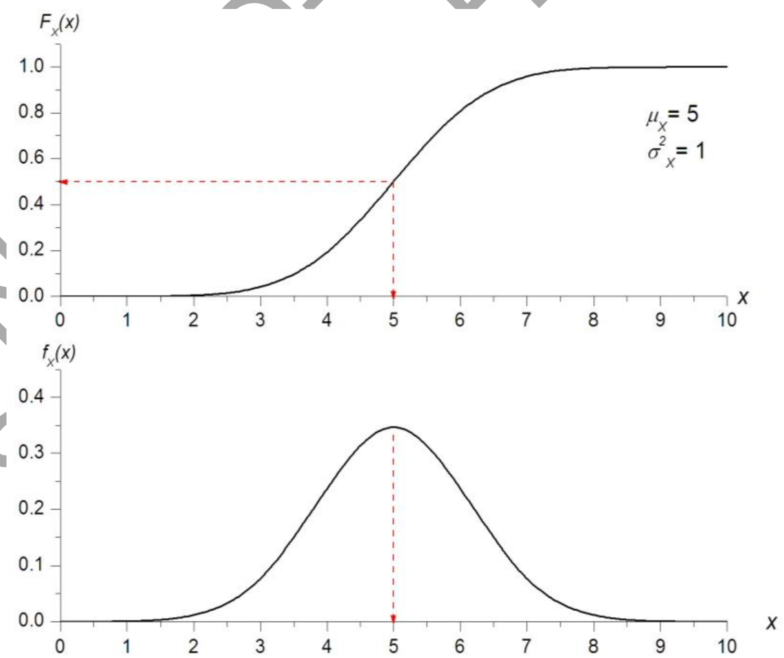

Figure 1. CDF and PDF of a typical normal random variable *X*.

For this unique random variable, the PDF, denoted by ϕ, is

$$\varphi = f_Z(z) = \frac{\exp\left[-\frac{1}{2}z^2\right]}{\sqrt{2\pi}} \tag{20}$$

This PDF is symmetric about its mean ($z=0$). The CDF of the standard normal variable Z is commonly denoted with Φ. Note that there is no subscript Z in this notation of the CDF, because the standard normal variable Z is unique. Values of this function $\Phi(z)$ for various z values are usually available in standard tables. Numerical/mathematical programs, libraries, and spreadsheet programs usually have a function associated with the CDF of the standard normal variable (for example, *normsdist* in LibreOffice Calc). These functions or tables can also be used to obtain the CDF of a normal random variable X, because

$$F_X(x) = \Phi\left(\frac{x-\mu_X}{\sigma_X}\right) \tag{21}$$

For example, *cdfnor* in Scilab and *normdist* in LibreOffice Calc are functions which return the CDF of a normal distribution with specific mean and standard deviation. Note that due to the symmetry in ϕ, we have

$$\Phi(-z) = 1-\Phi(z) \tag{22}$$

The inverse of this standard normal CDF, that is Φ^{-1}, is also a commonly used function in reliability analysis, as we will see later. Numerical/mathematical programs, libraries, and spreadsheet programs usually have a function associated with this inverse function as well (*normsinv* in LibreOffice Calc, for example).

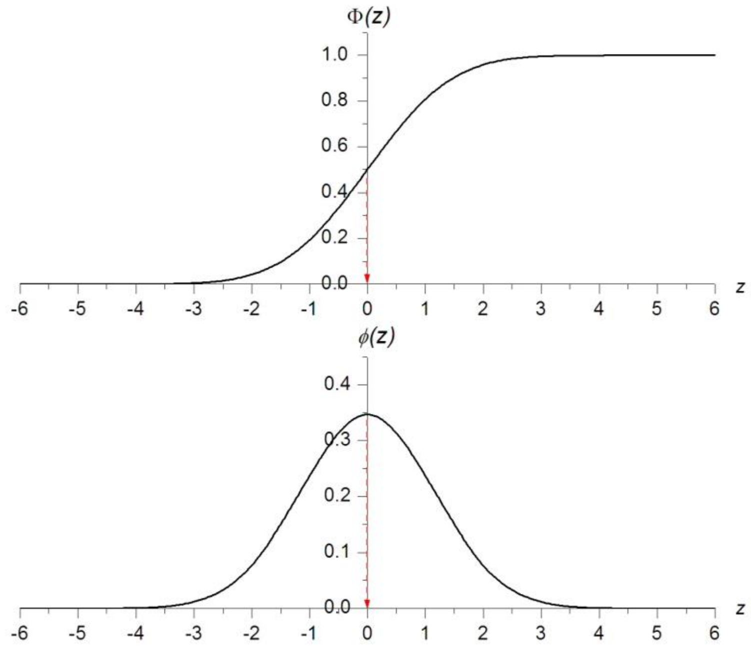

Figure 2. CDF and PDF of the standard normal random variable *Z*.

C. Lognormal Random Variable

A random variable X is called a *lognormal random variable* if $Y = \ln(X)$ is a normal random variable. Since $\ln(X)$ is a monotonically nondecreasing function of X, one can write

$$F_X(x) = \mathrm{P}(X \le x) = \mathrm{P}(\ln X \le \ln x) = \mathrm{P}(Y \le y) = F_Y(y) = \Phi\left(\frac{y-\mu_Y}{\sigma_Y}\right) \quad (23)$$

The mean and the variance for $Y = \ln(X)$ can be obtained from the known distribution parameters of X

$$\sigma^2_{\ln X} = \ln\left(V_X{}^2 + 1\right) \quad (24)$$

$$\mu_{\ln X} = \ln(\mu_X) - \sigma^2_{\ln X} / 2 \quad (25)$$

Figure 3 shows CDF and PDF of a general lognormal distribution. Note that this random variable can only have positive numbers.

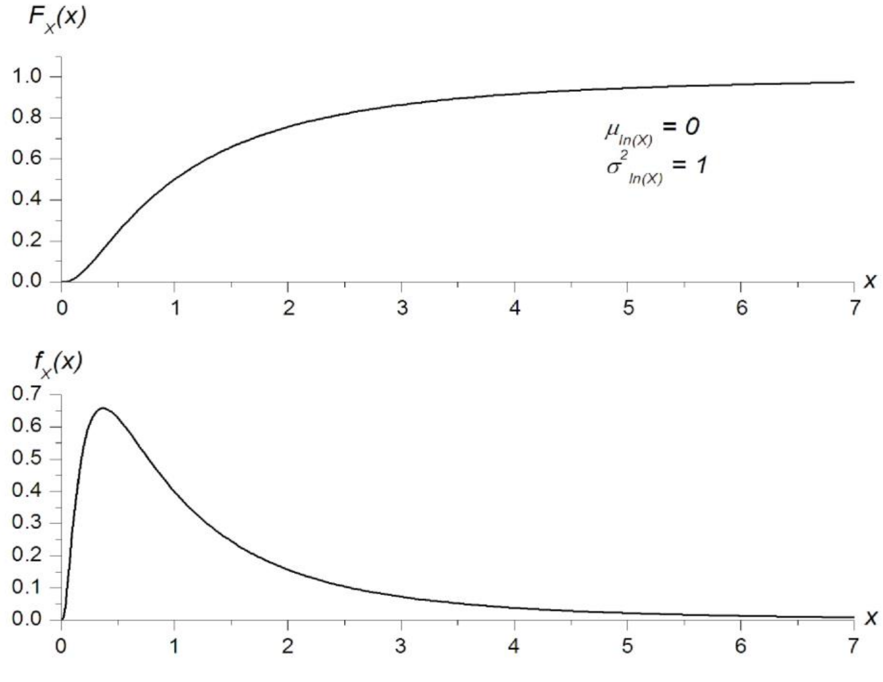

Figure 3. CDF and PDF of a typical lognormal random variable *X*.

The uniform, normal and lognormal distributions are the most commonly used distributions of random variables in reliability analysis. The other common distributions for continuous random variables are *gamma*, *extreme types I*, *II* and *III*. The commonly occurring discrete distributions are *binomial* and *Poisson* distributions.

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Biographical Sketch

Siddhartha Ghosh received his PhD from the University of Michigan, USA (2003), working in the area of performance-based seismic design of structures. Prior to that, he received his BCE from Jadavpur University, India (1997) and MTech from IIT Kanpur, India (1999). After his doctoral studies, he joined the Indian Institute of Technology Bombay and currently is an Associate Professor in the Department of Civil Engineering at IIT Bombay. He was also a guest professor at ETH Zürich, Switzerland for the academic year 2014-15. Professor Ghosh's research works focus in the area of structural earthquake engineering, with an emphasis to the application of probability/reliability concepts in seismic analysis and design of structures. His other research interests are in developing innovative building solutions using steel structures and in the mechanics of stone block masonry arches and domes. Professor Ghosh is a member of the Bureau of Indian Standards (BIS) committee CED7 on structural steel. He has been the principal investigator of several research projects funded by the Department of Science and Technology (India), Board of Research in Nuclear Sciences (India), ArcelorMittal R&D (Belgium), Atomic Energy Regulatory Board (India), etc. His research works have been published in the major journals in his area of research (Earthquake Engineering and Structural Dynamics, Engineering Structures, Journal of Earthquake Engineering, Nuclear Engineering and Design, Journal of Pressure Vessels and Piping, etc.). Prof. Ghosh received the Young Scientist award from the Department of Science and Technology, India in 2007. He is a member of the Earthquake Engineering Research Institute (EERI) and the Indian Society of Earthquake Technology (ISET).