STRUCTURAL DYNAMICS

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Contents

Summary

This chapter addresses the response of structures subject to dynamic loads. The majority of the material presented in this chapter has been condensed and replicated from Tedesco et al. (1999); the reader is encouraged to refer to this for a more comprehensive discussion of the topics presented herein.

1. Introduction

In this chapter, dynamic means time varying. That is, the application and/or removal of the loads necessarily varies with time (Irvine, 1986). Moreover, the response (i.e., resulting deflections, internal stresses, etc.) of a structure resisting such loads is also time dependent or dynamic in nature.

In reality, no loads that are applied to a structure are truly static. Since all loads must be applied to a structure in some particular sequence, a time variation of the force is inherently involved. However, whether or not a load should be considered dynamic is a relative matter. The most significant parameter influencing the extent of the dynamic effect a load has upon a structure is the *natural period of vibration* of the structure, *T*. Briefly stated, the natural period of vibration is the time required for the structure to go through one complete cycle of *free vibration*. If the application time for the load is large compared to the natural period of the structure, then there will be no dynamic effect, and the load can be considered static. If, on the other hand, the application time for the load is in close proximity to the natural period of the structure, it will induce a dynamic response.

Situations in which dynamic loading must be considered are quite numerous. Examples include: the response of bridges to moving vehicles; the action of wind gusts, ocean waves, or blast pressures upon a structure; the effect of landing impact upon aircraft; the effect on a building structure whose foundation is subjected to earthquake excitation; and the response of structures subjected to alternating forces caused by oscillating machinery (Tauchert, 1974). Under these types of loading conditions, either the entire structure or certain components of the structure are set in motion (i.e., caused to vibrate). Therefore, it is necessary to apply the principles of dynamics rather than those of statics to evaluate the structural response. It will be demonstrated throughout this chapter that the maximum deflections, stresses, strains, and various other response quantities exhibited by a structure are generally more severe when loads of a given amplitude are applied dynamically rather than statically.

The response of a structure to dynamic loads may be categorized as either *deterministic* or *nondeterministic* (Clough and Penzien, 1975). If the magnitude, point of application, and time variation of the loading are completely known, the loading is said to be *prescribed*, and the analysis of the structural response to this prescribed loading is defined as a deterministic analysis. However, if the time variation and other characteristics of the loading are not completely known, but can be defined only in a statistical sense, the loading is referred to as *random*, and the corresponding analysis of the structural response is termed nondeterministic. This chapter emphasizes the deterministic response of structures to prescribed dynamic loading.

To expedite the dynamic analysis of structures, it is convenient to classify dynamic loads as either *periodic* or *nonperiodic*. Periodic loadings repeat themselves at equal time intervals. A single time interval is called the period, T_0. The simplest form of periodic loading can be represented by a sine function as shown in Figure 1a. This type of periodic loading is referred to as *simple harmonic*. Another form of periodic load is illustrated in Figure 1b. This loading is termed *periodic, nonharmonic*. Most periodic loads may be accurately represented by summing a sufficient number of harmonic terms in a *Fourier series*. Any loading that cannot be characterized as periodic is nonperiodic. Nonperiodic loads range from short-duration impulsive types, such as a wind gust or a blast pressure (Figure 1c), to fairly long duration loads, such as an earthquake ground motion (Figure 1d).

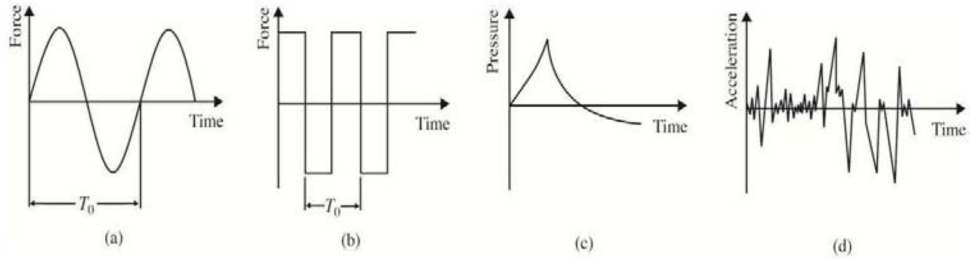

Figure 1. Types of dynamic loadings: simple harmonic; (b) periodic, nonharmonic; (c) nonperiodic, short duration; (d) nonperiodic, long duration

A structural dynamics problem differs from its static counterpart in two essential aspects (Craig, 1981). The first and most obvious difference is the time-varying nature of the excitation (applied loads) and the response (resulting deflections, stresses, etc.). That is, both are functions of time in a structural dynamics problem. This precludes the existence of a single solution. The analyst must investigate the solution over a specific interval of time to fully evaluate the structural response. Thus, a dynamic analysis is inherently more computationally intensive than a static analysis.

However, the most important feature differentiating a dynamic problem from the corresponding static problem is the occurrence of *inertia forces* when the loading is dynamically applied. Consider the vertical cantilever structure shown in Figure 2. If a force F is applied statically at the tip of the cantilever, as illustrated in Figure 2a, the resulting shear force, V, bending moment, M, and associated stresses and deflections in the structure can be computed from the basic static structural analysis principles, and are directly proportional to the force, F. If, however, a time-varying force $F(t)$ is applied to the tip of the cantilever, as illustrated in Figure 2b, the structure is set in motion, i.e., it vibrates and experiences accelerations. Inertia forces proportional to the mass then develop in the structure that must resist these forces. The significance of the contribution made by inertia forces to the shear force, $V(t)$, bending moment, $M(t)$, and related stresses and deflections in the structure determines whether a dynamic analysis is warranted.

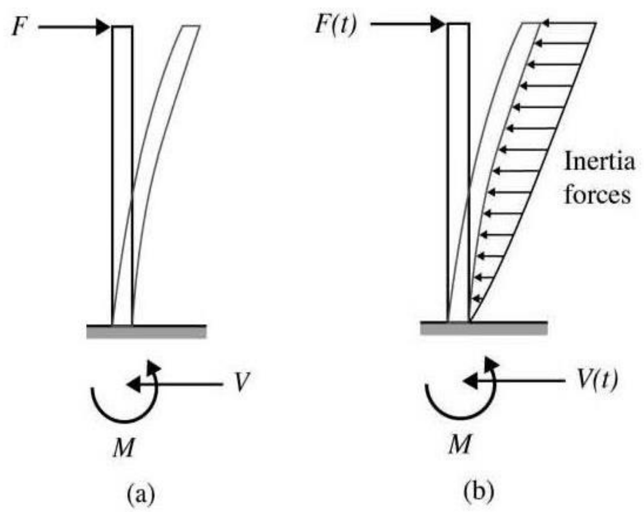

Figure 2. Cantilever structure subjected to (a) a static load; (b) a dynamic load

Typical of any problem in engineering mechanics, an appropriate methodology for conducting a dynamic structural analysis is essential to achieve a viable solution. One such methodology is summarized in Figure 3, which defines three basic phases of a dynamic analysis: (1) identification of the physical problem, (2) definition of the mechanical model, and (3) solution of the mechanical model.

Phase 1 entails recognition of the problem as it exists in nature. This includes accurately identifying and describing the physical structure, or structural component, and the source of the dynamic loading. Phase 2 requires an interpretation of the physical problem into a form conducive to available analysis techniques. This involves defining a mechanical model that accurately represents the dynamic behavior of the physical problem in terms of geometry, kinematics, loading, and boundary conditions. The idealization of the physical problem to a mechanical model conducive to available analysis techniques generally involves some simplifying assumptions, which influences the formulation of the differential equations governing the structural response. In Phase 3, the governing differential equations are solved to obtain the dynamic response. The solution is only as accurate as the representation provided by the mechanical model. Therefore, this step generally requires an assessment for accuracy. If the predefined accuracy criteria are met, the mechanical model has then been solved with a satisfactory level of confidence, and the analysis results can be interpreted in a meaningful manner. For complex structures, it may be necessary to refine the analysis by considering a more detailed mechanical model or to introduce design improvements for structural optimization, which leads to further analyses involving several iterations.

The complexity of the analysis depends largely on the physical problem under consideration and on the mechanical model that must be employed to obtain a sufficiently accurate response prediction. A linear analysis can be a routine task, although a fully three-dimensional solution may require a significant amount of human effort and computing resources. On the other hand, a nonlinear dynamic analysis can represent a major challenge to the ingenuity of the analyst and require very significant resources.

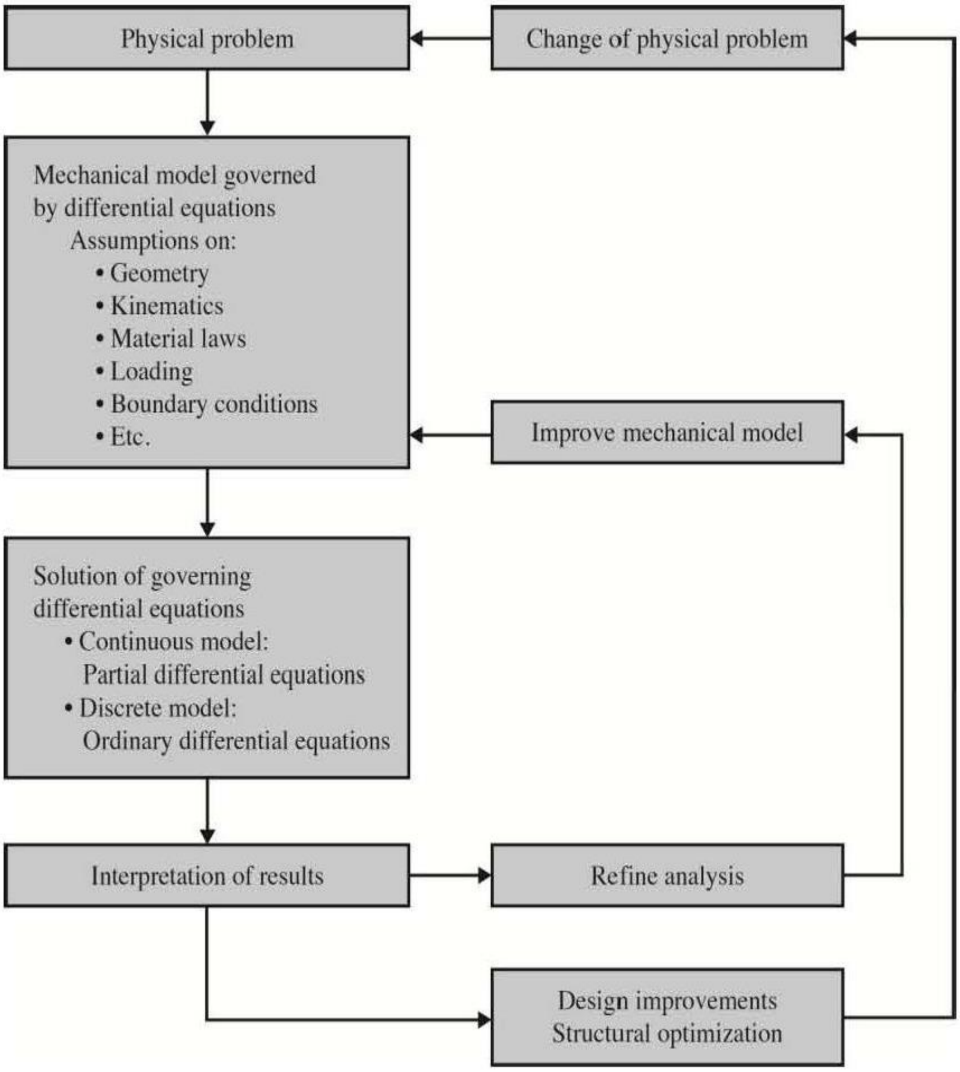

Figure 3. Methodology for conducting a dynamic analysis

Indeed, the most important step in the dynamic analysis procedure is defining a mechanical model that accurately represents the physical problem. Theoretically, all structures possess an infinite number of *degrees of freedom* (DOF). In other words, an infinite number of independent spatial coordinates are required to completely specify the position of all points on the structure at any instant of time (Craig, 1981). However, most practical analyses are conducted on mechanical models having a finite number of DOF. For each DOF exhibited by a structure, there exists a *natural frequency* (or natural period) of vibration. For each natural frequency, the structure vibrates in a particular mode of vibration.

For most large, complex structures, however, it is not necessary to determine all the system natural frequencies, since relatively few of these vibration modes contribute appreciably to the dynamic response. Therefore, the mechanical model should be defined in such a manner that only those vibration modes that significantly contribute to the dynamic response are accurately represented.

In general, the mechanical model can be categorized as either *continuous* or *discrete*. The type of mechanical model employed for an analysis affects the nature of the governing differential equations and their subsequent solution. For a continuous model, the mathematical formulation of the problem results in a system of partial differential equations. However, for a discrete system the mathematical formulation yields a set of ordinary differential equations, one for each DOF. Analytical solutions for partial differential equations and for large systems of ordinary differential equations are quite cumbersome, if not impossible in many cases. Therefore, in most practical applications numerical solution techniques must be employed. The focus of this chapter is the dynamic analysis of discrete systems.

2. Vibration of Single Degree of Freedom Systems

2.1. Basic Concepts

Vibrations are generally classified as either *free vibrations* or *forced vibrations*. Free vibration occurs in the absence of externally applied forces. The impetus for the free vibration is usually an initial displacement and/or velocity imparted to the mass. A system undergoing free vibration will oscillate at one or more of its natural frequencies. A single degree of freedom (SDOF) system has only one natural frequency.

Forced vibration occurs under the excitation of externally applied forces. If the excitation is transient (i.e., of short duration), the system response is at its natural frequency (once the disturbance terminates). However, if the excitation is oscillatory (periodically repetitive) and continues with time, the system vibrates at the excitation frequency. In situations where the excitation frequency coincides with the natural frequency of the system, a condition known as *resonance* occurs. At resonance, the amplitude of vibration becomes extremely large, and damage to the system is imminent if the vibration continues at the resonant frequency.

All structures exhibit some form of energy dissipation, or damping. Typically, the energy dissipation is due to frictional resistance or material hysteresis. In most practical engineering structures and mechanical systems, the damping is relatively small and, therefore, has very little influence on the natural frequency. However, even a small level of damping has a significant effect in limiting the amplitude of vibration, especially at resonance.

This section addresses the free vibration and harmonically excited forced vibration of undamped and viscously damped SDOF systems.

2.2. Formulation of the Equation of Motion

The dynamic response of a discrete SDOF system is described by a single, second-order ordinary differential equation. This mathematical expression, which defines the dynamic equilibrium of a system, is called the *equation of motion* of the structure. An important result from the solution of the equation of motion is the displacement-time history of a structure subjected to a prescribed time-varying load. However, before establishing methodologies for formulating the equations of motion for SDOF systems, it is important to define the basic components comprising the vibrating system. These include mass, stiffness, damping, and forcing. Damping is the energy loss mechanism, and forcing is the source of excitation.

The mechanical model for a simple SDOF vibrating system is depicted in Figure 4. It consists of a rigid body of mass m, constrained to move in only one translational direction, whose position is completely defined by the single displacement coordinate x. A spring of constant stiffness k, fixed at one end and attached at the other end to the mass, provides elastic resistance to displacement. The energy dissipation mechanism is represented by a damper or dashpot having a damping coefficient c. The external excitation to the system is provided by the time-varying force $F(t)$. Vibration in the absence of externally applied forces is also possible. Such vibration is referred to as *free vibration*.

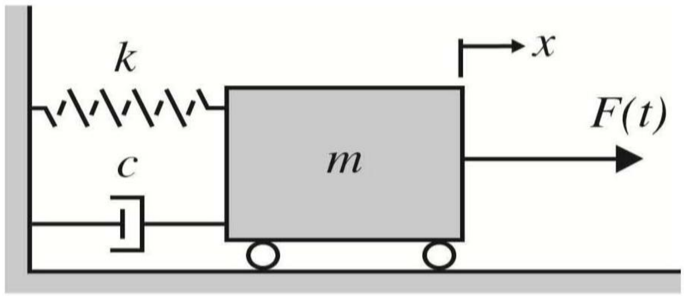

Figure 4. Mechanical model for a simple SDOF system

The displacement of the mass is measured from its static equilibrium position and is defined as a function of time by the spatial coordinate $x(t)$. The motion of the mass is resisted by the force F_s that develops in the spring and is defined by

$$F_s = kx \tag{1}$$

where k is the spring constant. The units of k are generally defined as force per distance (e.g., pounds per inch, lb/in, or Newtons per meter, N/m). The relationship between the deformation in the spring and the force in the spring is illustrated in Figure 5. For a completely elastic system, the spring serves as an energy storage device. The energy stored in the spring is called the *strain energy*, or potential energy, of the system. The strain energy V is calculated as the area under the force-displacement curve of the spring and is given by

$$V = \frac{1}{2}kx^2 \tag{2}$$

A *conservative system* will continue to vibrate indefinitely even after the external excitation has ceased. However, all practical structures exhibit energy dissipation, or damping, that prevents this from happening. Damping is a very complex phenomenon for which numerous analytical models exist to describe its effect. The most commonly employed analytical damping model is the linear viscous dashpot model (Beards, 1983). The damping force F_D is proportional to the velocity $\dot{x}$ of the mass and is given by

$$F_D = c\dot{x} \tag{3}$$

where *c* is the viscous damping coefficient having units of pound-seconds per inch (lb-sec/in) or Newton-seconds per meter (N-sec/m).

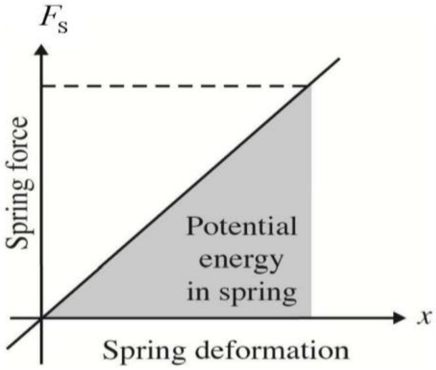

Figure 5. Force-deformation relationship for a linear spring

It was mentioned in Section 1 that the primary feature distinguishing a dynamic problem from the corresponding static problem was the presence of inertia forces in a vibrating system. The inertia force F_I is the product of the mass and the acceleration of the mass $\ddot{x}$ and is given by

$$F_I = -m\ddot{x} \tag{4}$$

The negative sign indicates that the inertial force opposes the acceleration of the mass.

D'Alembert's principle of dynamic equilibrium is a convenient method for establishing the equations of motion for simple SDOF and MDOF systems. It essentially involves invoking Newton's second law of motion to the system. Newton's second law states that the rate of change of momentum is proportional to the applied force and occurs along the line of action of the force. For a constant mass, the rate of change of momentum is equal to the product of the mass and its acceleration.

A free-body diagram of the SDOF system illustrated in Figure 4 is shown in Figure 6. The expression for dynamic equilibrium, using d'Alembert's principle, is given by

$$\sum (forces)_x - m\ddot{x} = 0 \tag{5}$$

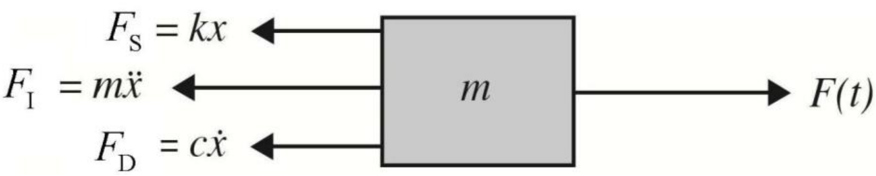

Figure 6. Free-body diagram for SDOF system

Thus, by introducing the appropriate inertial force, it can be reasoned that the applied force on the mass is in equilibrium with the inertia force. Therefore, the dynamic problem is reduced to an equivalent problem of statics. Applying Eq. (5) to the free-body diagram in Figure 6 results in the equation of motion for the system:

$$F(t) - kx - c\dot{x} - m\ddot{x} = 0 \tag{6}$$

or

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{7}$$

Dividing Eq. (7) through by m results in

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{F(t)}{m} \tag{8}$$

or

$$\ddot{x} + \frac{c}{m}\dot{x} + \omega^2 x = \frac{F(t)}{m} \tag{9}$$

where the term ω is called the *natural circular frequency* of the system, with units of radians per second, and is given by

$$\omega = \sqrt{\frac{k}{m}} \tag{10}$$

As illustrated by Eq. (10), the natural frequency is defined solely by the system's mass and stiffness characteristics. Natural frequency plays a vital role in vibration analysis and will be referred to extensively throughout the chapter.

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Biographical Sketches

Dr. Tedesco is currently the Elizabeth D. Rockwell Dean of the Cullen College of Engineering, University of Houston. He obtained his B.S. in Civil Engineering from the University of Notre Dame (1971), M.S. in Civil Engineering from Tufts University (1974) and Ph.D. in Civil Engineering from Lehigh University (1982).Professor Tedesco is a Registered Professional Engineer in the states of Florida and Alabama and has over 38 years of diversified experience ranging from practical structural design to highly mission oriented research projects. He has been principal investigator on more than thirty federally funded research projects involving the dynamic response of structures and materials to high intensity, short duration impulse loadings, including blast and impact. He is a six time U.S. Air Force research fellow, and has worked in residence at several AFRL's, including Tyndall AFB and Eglin AFB. Dr. Tedesco has been involved in research involving highly nonlinear numerical simulations of structures and materials subject to blast and impact loading, experimental research to assess the dynamic response characteristics of a variety of construction materials, and explosive tests of full-scale building structures. Dr. Tedesco is a member of the American Society of Civil Engineers (ASCE), the American Concrete Institute (ACI), and is a Charter Member of the Structural Engineering Institute (SEI).He is member of the ASCE Committee on Blast, Shock and Vibratory Effects and the ACI Committee on Short Duration Dynamics and Vibratory Load Effects. He is also the past chairman of the ASCE Technical Administrative Committee (TAC) on Dynamic Effects. Dr. Tedesco is past Managing Editor of the ASCE Journal of Structural Engineering and Associate Editor of the International Journal of Computers and Structures. Dr. Tedesco's teaching background includes courses on structural design, structural analysis and structural dynamics. He has written more than 110 technical publications including 70 refereed articles, 4 books, 2 book chapters and 38 technical reports. He is the principal author of the internationally renowned textbook titled Structural Dynamics: Theory and Applications (Prentice Hall, 1999).

Dr. Krauthammer is currently Theodore R. Crom Professor of Civil Engineering at the University of Florida, and Director of the Center for infrastructure Protection and Physical Security (CIPPS). Previously, he was Professor of Civil Engineering at the Pennsylvania State University, and the founding Director of the Protective Technology Center (PTC). He obtained his Ph.D. in Civil Engineering from the University of Illinois at Urbana-Champaign. His main research and technical activities are directed at structural behavior under severe dynamic loads, including considerations of both survivability and fragility aspects of facilities subjected to blast, shock, and impact effects. He has specialized in the nonlinear behavior of structures (including medium-structure interaction) under impulsive loads, and the development of numerical simulations and testing techniques for structural assessment. His work has included the development of design recommendations for enhancing structural performance, physical security and safety of buildings, facilities and systems. Dr. Krauthammer is a Fellow of the American Concrete Institute (ACI), a Life Member of the American Society of Civil Engineers (ASCE), a member of the American Institute of Steel Construction (AISC), and is involved with other national and international professional organizations. He serves on ten technical committees of ASCE, ACI, and AISC. Dr. Krauthammer was Chair of the ASCE/SEI Committee of Blast, Shock, and Impact Effects, and the

ASCE Task Committee on Structural Design for Physical Security. Also, he was the founding Chair of ACI Committee 370 on Short Duration Dynamics and Vibratory Load Effects, Chair of the ASCE Engineering Mechanics Committee on Experimental Analysis and Instrumentation and of the ACI Committee 444 Experimental Analysis of Concrete Structures, and of the Joint ACI-ASCE Committee 421 on Design of Reinforced Concrete Slabs. He was a member of the National Research Council Defensive Architecture Committee. Dr. Krauthammer is the author of the book “Modern Protective Structures” that was published in 2008, and has written more than 470 research publications. Dr. Krauthammer's teaching background includes courses on structural design and behavior, structural analysis, advanced dynamics, protective structures, and numerical methods. He is very active in organizing cooperative international scientific, technical, and engineering education activities. Dr Krauthammer has been invited to lecture in the USA and abroad, and he has been a consultant to industry and governments in the USA and abroad.