GENERAL OVERVIEW OF CONTINUUM MECHANICS

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Contents

Summary

This chapter presents the essential concepts of continuum mechanics – starting from the basic notion of deformation kinematics and progressing to a sketch of constitutive behavior of simple materials. We intend neither to develop it with the mathematical sophistication used in the different topics related to this subject nor to develop every mathematical tool. On the contrary, we try to put in words the ideas behind this analysis as an initial approximation prior to further reading. This chapter is a general overview of the topic, an outline that covers the main and basic ideas. Our attempt is to develop a pedagogical approach accessible to a wide audience by introducing at every step the necessary mathematical machinery in conjunction with a physical interpretation. The purpose is neither to go deeply into the different areas involved nor to fully develop every concept and all mathematical tools associated with continuum mechanics. This

treatment assumes a working knowledge of vector and tensor notation, tensor calculus, and basic mechanics.

1. Preliminary Concepts: Definition and Theoretical Framework

Continuum Mechanics analyzes material bodies that undergo changes in motions and deformations when these bodies are subjected to loading. In particular, material bodies are solids, liquids and gases. By loading we consider in this preliminary definition any particular effect on the material body under study that changes its status and/or position. For example, this could be a change in temperature, an applied force, etc.

The atomic nature of the body is ignored and therefore it is assumed that the body can be infinitely divided obtaining in each partition the same body, i.e. the body is considered to be a continuous distribution of matter in space. Hence, the length scale involved in the continuum analysis is much longer than that associated with the analysis of atoms.

To deal with a material body mathematically, a body $\mathcal{B}$ in the continuum mechanics setting is taken as a set whose elements can be put into one-to-one correspondence with points of a region in three-dimensional Euclidean point space (more precisely, a body can be defined in term of what is known as a Borel set). The elements of the material body $\mathcal{B}$ are called material points or particles and the specification of the position of all particles of $\mathcal{B}$ with respect to a coordinate system at a particular time is called a configuration of the material body –a three dimensional picture of the body taken at a particular instant of time. Bodies are classified as either *deformable* or *rigid*. In the latter case, material points cannot move relative to each other under the action of loads, so that only translations and rotations of the body are possible. In contrast, material points of a deformable body can move relative to each other under the action of loading. Continuum mechanics is the study of deformable bodies.

The body moves in time so, in other words, the configuration of the body changes with time. The motion (and deformation) of the material body occurs in the four-dimensional space-time and the theoretical framework to which this analysis will be restricted here is classical mechanics, i.e., the Newtonian space-time framework. This means basically that at each instant of time, there corresponds one and only one configuration of the body. Since a unique configuration of the body is associated with each instant of time, the family of configurations gives the motion of the body.

Section 2 of this chapter is concerned with kinematics, which is a geometrical analysis of the body during its deformation and motion. No attention is given to the causes of that motion or the mechanical properties of the body. The motion of the body is just the continuous sequence of configurations in time. The change of configurations is viewed as the displacement of the body. A motion is called rigid when the distance between any two particles of the body does not change with time. The most general rigid motion or rigid-body displacement consists of simultaneous translation and rotation of the body. The Lagrangian/material and the Eulerian/spatial descriptions are analyzed.

Section 3 deals with balance laws including the first and second law of thermodynamics. For reference, Section 3.5 summarizes the number of equations and number of unknown fields in both Lagrangian and Eulerian description of a continuum problem.

The last section is a brief discourse on constitutive relations for solids and fluids mainly under isothermal processes. Nevertheless, in Section 4.1 basic principles that have to be obeyed by a constitutive relation are introduced . Key concepts regarding the behavior of a material model are explained. This gives rise to the thermo-mechanical behavior of a material. As a result the more general constitutive model is explained. The local constitutive theory which gives rise to the so called simple materials is presented. This section (4.1) is written in a self-contained manner, and the reader could skip these more advanced developments and focus on the next sections in which descriptions of well-known material models are given. Section 4.2 focuses on elastic solids while Section 4.3 focuses on fluids. Both sections consider isothermal processes. The formal formulation of an initial boundary value problem is presented.

2. Kinematics of Deformation

2.1. The Continuum Setting: Mathematical Description

In order to describe the motion and deformation of a material body the concept of frame of reference or observer is needed since it is an observer who captures the motion, i.e. the sequence of configurations.

A body $\mathcal{B}$ is referenced to a set of material points or particles which can be assigned coordinate labels with respect to an orthonormal basis in Euclidean space $\mathbb{E}$. The position of all the material points at a particular time defines the so called configuration of the body. Therefore, the configuration of a body is identified with a region in a three dimensional Euclidean space relative to an observer. It is convenient to choose a particular configuration as the reference and to identify that frame as the reference configuration of the material body (one can imagine that this is the body position at time equal to zero or the undeformed configuration). A current configuration is considered to be the body position at the current time (or the deformed configuration). It is clear that each configuration needs a coordinate system to describe the geometrical structure of the physical body at that particular instant of time. Nevertheless, while the reference configuration is defined by means of a reference coordinate system, the current configuration is defined by means of the current coordinate system. Therefore, this current coordinate system is used to specify all deformed configurations.

In a mathematical way we could say the following. Let $\mathcal{B}$ denote an abstract body and $P \in \mathcal{B}$ a typical material particle belonging to the body. Furthermore, let $K_{\mathrm{r}}(\mathcal{B})$ and $K_t(\mathcal{B})$ denote the reference and current placements of the body in Euclidean space, respectively. Here K_{r} and K_t are one-to-one mappings from $\mathcal{B}$ into the reference and current placements. Let $\mathbf{X}$, with Cartesian components $X_i, i \in (1,2,3)$ with respect to the basis $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$ denote the typical position vector of a material particle in the reference configuration $K_{\mathrm{r}}(\mathcal{B})$ of the body, and let $\mathbf{x}$, with components $x_i, i \in (1,2,3)$, with respect to the basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ denote the corresponding position vector of the same particle in the deformed configuration $K_t(\mathcal{B})$ at time t. It is usual to avoid cumbersome notation by respectively denoting the reference and current configurations by $\mathcal{B}_{\mathrm{r}}$ (or simply $\mathcal{B}$) and $\mathcal{B}_t$. Similarly, it is customary to suppose that the reference and current coordinate axes use only one basis. This is shown in Figure 1 where $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is the basis. The reference configuration coordinates $\mathbf{X}$ are called material coordinates and are identified with material points of the body. The coordinates $\mathbf{x}$ of the so called current configuration or also the deformed configuration are denoted as spatial coordinates, and assign to material points $\mathbf{X}$ places in space at a particular time. It is clear then that motion means that material particles change their locations in time. It is important to remark that while material coordinates are only used with the reference configuration, spatial coordinates are used for all other configurations.

Distinct material points must be mapped to distinct spatial points in order for the deformation to be physically plausible. In other words, different material points cannot occupy (at time *t*) the same physical location. It follows that there exists a mapping $\boldsymbol{\chi}$ which assigns to each point $\mathbf{X}$ just one point $\mathbf{x}$ at each instant t, i.e. there exists $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$. The mapping $\boldsymbol{\chi}$ depends on K_{r}, but for the sake of simplicity of notation, K_{r} is not usually used as a suffix.

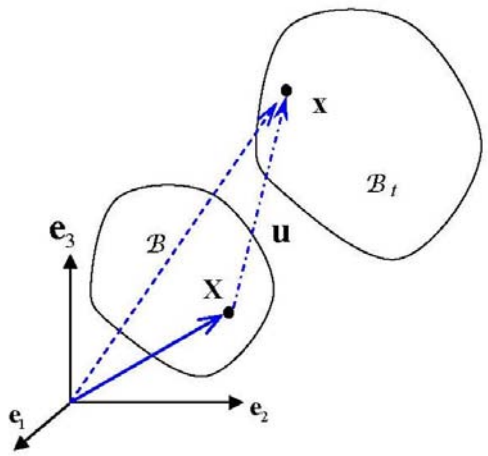

Figure 1. Displacement of a material point $\mathbf{u}$: $\mathbf{X} \rightarrow \mathbf{x}$. $\mathcal{B}$ denotes the body in the reference (or undeformed) state and $\mathcal{B}_t$ is the body at time *t*.

Mathematically speaking, the mapping (or motion) $\boldsymbol{\chi}$ must be *one-to-one* (the mapping $\boldsymbol{\chi}(\mathbf{X}, t)$ is one-to-one if $\boldsymbol{\chi}(\mathbf{X}_1, t) \neq \boldsymbol{\chi}(\mathbf{X}_2, t) \forall \mathbf{X}_1, \mathbf{X}_2 \in \mathcal{B}$ such that $\mathbf{X}_1 \neq \mathbf{X}_2$) and *onto*, i.e. it is assumed that this mapping is continuously differentiable and it has a continuously differentiable inverse (the physical interpretation is that the body does not trespass itself), $\mathbf{X} = \boldsymbol{\chi}^{-1}(\mathbf{x}, t)$. This will be true if and only if, for all material points $\mathbf{X} \in \mathcal{B}$ and $t \geq 0$,

$$J(\mathbf{X},t) \equiv \det(\nabla\boldsymbol{\chi}) = \det\left[\frac{\partial\chi_i}{\partial X_j}\right] = \left|\frac{\partial\chi_i}{\partial X_j}\right| = \begin{vmatrix} \partial\chi_1/\partial X_1 & \partial\chi_1/\partial X_2 & \partial\chi_1/\partial X_3 \\ \partial\chi_2/\partial X_1 & \partial\chi_2/\partial X_2 & \partial\chi_2/\partial X_3 \\ \partial\chi_3/\partial X_1 & \partial\chi_3/\partial X_2 & \partial\chi_3/\partial X_3 \end{vmatrix} \neq 0 . \quad (1)$$

It can be shown that $J(\mathbf{X},t)$ is a measure of the ratio of the deformed local volume at time t to the original volume so that its nonzero value in (1) makes physical sense. Since $\boldsymbol{\chi}(\mathbf{X},0) = \mathbf{X}$, one observes that $J(\mathbf{X},0) = \left|\frac{\partial\chi_i}{\partial X_j}\right|_{t=0} = 1$. Furthermore, continuity of $\boldsymbol{\chi}$ as a function of *t* implies that *J* is continuous in time. Consequently, $J > 0$ for $t \geq 0$. The latter analysis leads to the concept of a *proper* or *admissible* deformation as one in which $J > 0$ for $t \geq 0$.

The *displacement field* $\mathbf{u} := \mathbf{x} - \mathbf{X}$ is a smooth (the term 'smooth' means that the function $\mathbf{u}$, and its first and second derivatives are continuous), single-valued mapping from material points $\mathbf{X} \in \mathcal{B}$ to spatial points $\mathbf{x}$, so that $\mathbf{u} = \mathbf{x} - \mathbf{X} \in \mathcal{V}$, where $\mathcal{V}$ is the vector space associated with the Euclidean space $\mathbb{E}$.

It is also customary to use the notation $\mathbf{x}(\mathbf{X},t) = \boldsymbol{\chi}(\mathbf{X},t)$ and $\mathbf{X}(\mathbf{x},t) = \boldsymbol{\chi}^{-1}(\mathbf{x},t)$. In component form, one writes

$$x_i = x_i(X_1,\ X_2,\ X_3,t) \qquad X_i = X_i(x_1,\ x_2,\ x_3,\ t), \quad i = 1,\ 2,\ 3. \quad (2)$$

The mapping $\mathbf{X}(\mathbf{x},t) = \boldsymbol{\chi}^{-1}(\mathbf{x},t)$ is used to denote the material point that is located at $\mathbf{x}$ at time t.

2.1.1. Lagrangian and Eulerian Descriptions

Two coordinate systems have been introduced. Functions of the body in motion describing physical or kinematical properties can be expressed using either material coordinates $\mathbf{X}$ or spatial coordinates $\mathbf{x}$, and this gives rise to either material or spatial descriptions, respectively. In the material or *Lagrangian* description, attention is paid to the material points during the motion. In the spatial or *Eulerian* description, the focus is on events that take place at points in space.

Since the Lagrangian description follows a material point of the body, the rate of change of a quantity is associated with that particular material point. Consider a scalar function (for instance temperature) in Lagrangian description $G : \mathcal{B} \times [0,\infty) \to \mathbb{R}$, (so that *G* is a function of $\mathbf{X}$ and t). Then its *material time derivative* (this is also known as the total time derivative) is the time rate of change of G for a fixed material point. That is,

$$\frac{DG}{Dt}(\mathbf{X},t) \ := \frac{\partial G(\mathbf{X},t)}{\partial t}\bigg|_{\mathbf{X}} . \quad (3)$$

The Eulerian description considers quantities at fixed spatial points. The function G could be expressed as a function of the coordinates $\mathbf{x}$. Thus, let $g(\mathbf{x},t) := G(\boldsymbol{\chi}^{-1}(\mathbf{x},t),t)$. Then,

$$\frac{Dg}{Dt} = \left.\frac{\partial g}{\partial t}\right|_{\mathbf{x}} + \nabla g \cdot \left.\frac{\partial \mathbf{x}}{\partial t}\right|_{\mathbf{X}} \tag{4}$$

where $\nabla g \cdot \left.\frac{\partial \mathbf{x}}{\partial t}\right|_{\mathbf{X}} = \frac{\partial g}{\partial x_k}\mathbf{e}_k \cdot \left.\frac{\partial x_i}{\partial t}\mathbf{e}_i\right|_{\mathbf{X}} = \left.\frac{\partial g}{\partial x_k}\frac{\partial x_i}{\partial t}\right|_{\mathbf{X}} \mathbf{e}_k \cdot \mathbf{e}_i = \left.\frac{\partial g}{\partial x_i}\frac{\partial x_i}{\partial t}\right|_{\mathbf{X}}$. Here, the usual summation convention is assumed for a repeated index ($c_j\mathbf{e}_j = \sum_{j=1}^{3} c_j \mathbf{e}_j$). The term $(\partial\mathbf{x}/\partial t)|_{\mathbf{X}}$ is the velocity of the material point, that is, $\mathbf{V}(\mathbf{X},t) := \left.\frac{\partial \mathbf{x}}{\partial t}\right|_{\mathbf{X}} = \left.\frac{\partial x_k}{\partial t}\right|_{\mathbf{X}} \mathbf{e}_k = V_k\mathbf{e}_k$, where $V_k := \left.\frac{\partial x_k}{\partial t}\right|_{\mathbf{X}}$. Since Dg/Dt in (4) maps $(\mathbf{x},t) \in \mathcal{B}_t \times \mathbb{R} \to \mathbb{R}$, $\mathbf{V}(\mathbf{X},t)$ is replaced by its Eulerian (or spatial) description $\mathbf{v}(\mathbf{x},t) := \mathbf{V}(\boldsymbol{\chi}^{-1}(\mathbf{x},t),t)$ (this is simply the velocity of the material point $\mathbf{X}$ which is at $\mathbf{x}$ at time t), so that

$$\frac{Dg}{Dt} = \frac{\partial g}{\partial t}\Big|_{\mathbf{x}} + \nabla_{\mathbf{x}} g \cdot \mathbf{v}\,. \tag{5}$$

The time rate of change of the velocity of a material point $\mathbf{X}$ is its acceleration which, from (3) is given by,

$$\mathbf{A}(\mathbf{X},t) = \frac{D\mathbf{V}}{Dt} = \left.\frac{\partial \mathbf{V}}{\partial t}\right|_{\mathbf{X}} = \left.\frac{\partial V_i}{\partial t}\right|_{\mathbf{X}} \mathbf{e}_i\,. \tag{6}$$

If the Eulerian description of the velocity $\mathbf{v}(\mathbf{x},t)$ is prescribed, then an application of (5) yields,

$$\frac{D\mathbf{v}}{Dt} = \left.\frac{\partial \mathbf{v}}{\partial t}\right|_{\mathbf{x}} + \mathbf{v}\cdot\nabla_{\mathbf{x}}(\mathbf{v}) \tag{7}$$

or, in component form,

$$\frac{Dv_i}{Dt} = \left.\frac{\partial v_i}{\partial t}\right|_{\mathbf{x}} + \frac{\partial v_i}{\partial x_k} v_k \tag{8}$$

The first term on the right-hand side of (7) is called the *local acceleration* while $(\nabla_{\mathbf{x}}\mathbf{v})\cdot\mathbf{v}$ is the *convective acceleration*. The material time derivative of a tensor field is also expressed with a dot above the letter representing the tensor. In what follows, the distinction between material description and spatial description will be clear when the arguments $(\mathbf{x},t)$ or $(\mathbf{X},t)$ are included. However, when they do not appear, then the spatial description will be designated via small letters. On the other hand, a Lagrangian description will use capital letters or a bar above the letter. It follows, for instance, that one may use $\bar{\mathbf{v}}(\mathbf{X},t)=\mathbf{V}(\mathbf{X},t)$.

2.1.2. Material and Spatial Differential Operators

The initial or reference configuration is described by a system with coordinates $\mathbf{X}$ while the current configuration is described by a system having coordinates $\mathbf{x}$. The functions under study depend on these coordinates as has been explained previously. These functions can be either scalars, vectors or tensor valued functions. Since there are two coordinate systems, differential operations on these tensor fields have to be clearly distinguished in order to avoid confusion. Different authors follow different notations. For instance, while differential operators in the initial configuration are usually denoted with a capital letter, such as Grad, Curl, Div, in the current configuration these same differential operators are denoted as grad, curl, div. These ideas are developed in what follows.

Let $\phi:\mathbb{R}^3\to\mathbb{R}$ be a scalar function of the spatial point $\mathbf{x}\in\mathbb{R}^3$. To first order (in this Taylor series, higher order terms $(d\mathbf{x})^2,(d\mathbf{x})^3,...$ have been neglected), the total change in ϕ is given by

$$d\phi=\nabla_{\mathbf{x}}\phi\cdot d\mathbf{x}=\frac{\partial\phi}{\partial x_i}dx_i \tag{9}$$

where $d\mathbf{x}=dx_i\mathbf{e}_i$, and $\nabla_{\mathbf{x}}=\mathbf{e}_i\dfrac{\partial}{\partial x_i}$ is called the *spatial gradient*. On the other hand, if Φ is a function of the material point $\mathbf{X}$, then

$$d\Phi=\nabla_{\mathbf{X}}\Phi\cdot d\mathbf{X}=\frac{\partial\Phi}{\partial X_i}dX_i \tag{10}$$

where $d\mathbf{X}=dX_i\mathbf{e}_i$ and $\nabla_{\mathbf{X}}=\mathbf{e}_i\dfrac{\partial}{\partial X_i}$ is the *material gradient*. The notation that will be used here to distinguish between differential operators in the initial configuration and the differential operators in the current configuration will follow in accord with the example just given regarding the material gradient $\nabla_{\mathbf{X}}$ and the spatial gradient $\nabla_{\mathbf{x}}$. Nevertheless, differential operators could also be denoted as either Grad, Curl, Div, etc, for the coordinates $\mathbf{X}$ or grad, curl, div, etc for the coordinates $\mathbf{x}$.

2.2. Displacement, Deformation and Motion

The displacement field $\mathbf{u}=\mathbf{x}-\mathbf{X}$ was introduced in Section 2.1.1. One can consider either the material description of the displacement field denoted as

$\bar{\mathbf{u}}(\mathbf{X},t)=\mathbf{x}(\mathbf{X},t)-\mathbf{X}$, or its spatial description $\mathbf{u}(\mathbf{x},t)=\mathbf{x}-\mathbf{X}(\mathbf{x},t)$. If the focus is on the displacement of a material point (i.e., attention is given to the position of a material point $\mathbf{X}$ in time), then the description is related to the trajectory of that particle. It follows that the first and second time derivatives of the material point position (or its displacement) gives the particle velocity and particle acceleration, respectively. The examination of relative displacements between material points that is due to the deformation of the body is known as deformation analysis. Thus, time does not play any role in this analysis. If time is included, then, the analysis is known as motion analysis. An arbitrary motion or displacement of the body involves deformation as well as the most general rigid motion, i.e. translation and rotation of the body.

In continuum mechanics, it is usual to distinguish between deformation and motion analysis. In the former analysis, two configurations of the body - the initial or undeformed and a current or deformed one – are considered, regardless of the sequence of configurations between the initial configuration and the current one. On the other hand, motion analysis focuses on the rate with time at which the deformation takes place. Therefore, the sequence of configurations is crucial.

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Biographical Sketches

Jose Merodio gained his first degree in Mechanical Engineering at Universidad del País Vasco, Spain. He completed his PhD in Engineering Mechanics and a MS in Applied Mathematics at Michigan State University, USA. Currently, he is an Associate Professor in the Department of Continuum Mechanics and Structures at Universidad Politécnica de Madrid, Spain. His main research interests are in nonlinear elasticity theory, with particular application to the mechanics of soft biological tissues, bifurcation phenomena as well as mathematical analysis of constitutive models. He is also co-editor of the biomechanics volume for the EOLSS-UNESCO encyclopedia. He has delivered a solid intellectual contribution to the field of continuum mechanics, evidenced by a steady stream of archival publications as well as by regular invitations to present his work at conferences and to contribute to different books. He has organized several international conferences and international courses for researchers in this area of work at several International Centers. He is in the editorial board of the journal Mechanics Research Communications and has served as guest editor for different international journals.

Anthony D. Rosato received his PhD in Mechanical Engineering from Carnegie Mellon University in 1985. He has been at the New Jersey Institute of Technology since 1987 where he holds the rank of Professor of Mechanical Engineering. Dr. Rosato’s research interests are in the broad field of particle technology, with a focus on computational modeling and experimental studies of granular flows related to the solids handling and processing industries. From 1995-1999, he served as the founding director of NJIT's Particle Technology Center, and he has been the director of the Granular Science Laboratory in the Mechanical and Industrial Engineering Department since 1999. Dr. Rosato has held visiting appointments at Lawrence Livermore National Laboratory, Worcester Polytechnic Institute, the Lovelace Institutes (Albuquerque, NM), ESPCI in Paris and Stanford University. He is a Fellow of the ASME, Editor-in-chief of *Mechanics Research Communications*, a Fulbright Senior Research Fellow, and a member of the Academy of Mechanics, the ASCE Engineering Mechanics Institute, and Sigma Xi.