HISTORY OF CONTINUUM MECHANICS

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Summary

The history of Continuum Mechanics is traced from the early work of the Hellenic period up to the present century. This history is based upon early work in statics, deformable solids, dynamics, fluid mechanics and aerodynamics. The unifying theory of continuum mechanics came in the 1900s combined with the advances in thermodynamics and rheology. Truesdell was the major force to develop this unifying theory. This history has tried to summarize the major contributes to the development of continuum mechanics but so many contributed to the field that many have been over looked and only the individuals who made major contributions are listed. Many of the advances in new analytical methods that have their birth in the study of continuum mechanics have not been referenced.

1. History of the General Theories and Fundamental Equations

First, the reader is referred to "A History of Mechanics" by Rene' Dugas, 1955 [1]. This text gives an excellent historical review of Mechanics from the Hellenic period up to the 19th century. The early developments of mechanics focused on rigid bodies and the movement of particles. Mass points were defined to explain inertia. Newton's fundamental laws all dealt with the movement of a particle when subjected to forces and the gravitational attraction between masses. This established the theory that gravitational mass was equal to inertial mass. During the 18th and 19th centuries, the early concepts of continuum mechanics were developed by mathematical scientists. At the beginning of the 20th century, physics again returned to the study of molecules or corpuscles.

Today, in modern physics, matter is universally considered to be composed of molecules or particles that in turn are composed of atoms. Early in the 20^{th} century, the theories of relativity, wave mechanics and quantum mechanics were introduced. So physics began with the study of single particles or rigid bodies and then returned to models of particles and the interaction between them. However, we are surrounded by matter in the form of continuous media; deformable solids, fluids and gases. We will need a science that describes the responses of these materials to the external forces imposed upon them until, if ever, the science of particles can be developed such that it predicts the response of the aggregate. A. E. H. Love [2] wrote, “In a theory ideally worked out, the progress which we should be able to trace would be, in other particulars, one from less to more, but we may say that, in regard to the assumed physical principles, progress consists in passing from more to less.” Timoshenko [3] expressed similar thoughts; “Atomic structure will not be considered here. It will be assumed that the matter of an elastic body is homogeneous and continuously distributed over its volume so that the smallest element cut from the body possesses the same specific physical properties as the body.” The science of continuous media developed in two parts during the 15th through the 18th centuries, one from the deformation of solids and the other the flow of fluids.

The foundations of continuum mechanics up to and including the formulation of the linear constitutive equations occurred during the period from 1687 to 1788. Truesdell and Toupin[5] give a complete review of classical field theory and an exhaustive set of historical footnotes and references. The name of Clifford Truesdell will appear many times in the history of continuum mechanics. Rutherford Aris [6] wrote; “In the last decade there has been a renascence of interest in rational mechanics in the mathematical world. It has been fairly widespread and attracted the attention of many mathematicians whose abilities are of the best order. If one name is to be singled out, it is probably not unfair to the others to select that of Truesdell, whose deep scholarship and extensive writing have been of great influence.” Truesdell [7] also has written a history of this 100 year period.

In 1687, Newton (1642-1727) published “Principia”, containing Newton’s laws of motion and the law of gravitational attraction. Newton’s first three laws may be stated as follows:

1. Every body or particle continues in a state of rest or in uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it—that is, unless the external forces are not in equilibrium.
2. The change of motion of a body or particle is proportional to the net external force acting on the body or particle in the direction of the net external force.
3. If one body or particle exerts a force on a second body or particle, then the second exerts a force on the first that is equal in magnitude to, opposite in direction to, and collinear with the given force.

   The final law is the law of universal gravitational attraction:

4. Any two particles are attracted toward each other with a force whose magnitude is proportional to the product of their gravitational masses and inversely proportional to the square of the distance between them.

Leonardo da Vinci (1452-1519) noted “Mechanics is the paradise of Mathematical science because here we come to the fruits of mathematics.” This was proven to be the case. As the science of mechanics developed, so did applied mathematics, calculus, field theory, partial differential equations, vector analysis and tensor analysis. Applied mathematics developed to fill the need to understand and predict the response of materials to external stimuli. Therefore, the history of mathematics parallels the history of mechanics.

Although he was in his forties, when the treatise; *Prilosophiae naturalis principia mathematica*, Newton had formulated most of the axioms earlier and, by the age of 23, had developed calculus and the binomial theorem. In 1693, he heard for the first time that the calculus was becoming well known on the Continent and that it was commonly attributed to Leibniz (1646-1716). Newton and Leibniz were initially on cordial terms. Each recognized the others merits and neither suspected that the one had stolen any part of the calculus from the other.

Later, in 1712, when even the man in the street—the zealous patriot who knew nothing of the facts—realized vaguely that Newton had done something tremendous in mathematics (more, probably, as Leibniz said, than had been done in all history before him), the question as to who had invented the calculus became a matter of acute national jealousy, and all educated England rallied behind its somewhat bewildered champion, howling that his rival was a thief and a liar.

"Newton at first was not to blame for this nationalistic attitude. Nor was Leibniz. But as the British sporting instinct presently began to assert itself, Newton acquiesced in the disgraceful attack and himself suggested or consented to shady schemes of downright dishonesty designed to win the international championship at any cost—even that of national honor. Leibniz and his backers did likewise. The upshot of it all was that the obstinate British practically rotted mathematically for all of a century after Newton's death, while the more progressive Swiss and French, following the lead of Leibniz, and developing his incomparably better way of merely *writing* the calculus, perfected the subject and made it the simple, easily applied implement of research that Newton's immediate successors should have had the honor of making it." [8]

Newton's laws or general statements are credited to be the beginning of classical mechanics but according to Truesdell [7], he made no attempt to form his laws in any mathematical expressions. These mathematical expressions were later developed by Euler. Leonard Euler (1707-1783) was the most prolific mathematician in history and Truesdell considered him to be the greatest mathematician of all time. Bell [8] wrote; "Even total blindness during the last seventeen years of his life did not retard his unparalleled productivity; indeed, if anything, the loss of his eyesight sharpened Euler's perceptions in the inner world of his imagination." In 1776, Euler introduced the integral forms of the principles of linear momentum and moment of momentum.

The concept of stress took about a century to develop. James Bernoulli around the end of the 17th century introduced the concept of tension in a flexible line and in 1739 John Bernoulli introduced the concept of an internal force, in hydraulics. Euler later introduced the fully general idea of internal pressure. The concept of shear stress may date back to Newton's "Principia" in the statement; "The resistance arising from the want of lubricity in the parts of a fluid is, other things being equal, proportional to the velocity with which the parts of the fluid are separated from one another." Parent in1713 and Coulomb in 1773 introduced the concept of shear stress in beams. Du Buat's

"*Principes d'hydraulique*", published in 1779 considered friction of water against the walls of a stream bed and the viscosity of the fluid. Coulomb's work on friction also contributed to the concept of shear stresses. Augustin-Louis Cauchy (1789-1857) presented the general concept and mathematical theory of the stress tensor in 1823 and 1827.

Cauchy's childhood fell in the bloodiest period of the French Revolution. Schools were closed and men of science were left to starve or carted off to the guillotine. To escape this danger, the senior Cauchy moved his family to the village of Arcueil. Arcueil was a southern suburb of Paris, 5.3 km from the center of Paris. Arcueil adjoined the estates of the mathematician Marquis Laplace and the chemist Count Claude-Louis Berthollet. Laplace, a mathematical physicist and Lagrange, a pure mathematician, were the two leading French men of science of the 18th century. They both escaped the guillotine because they were requisitioned to calculate trajectories for the artillery and to help in directing the manufacture of saltpeter for gunpowder. Laplace was struck by the young boy, Cauchy, as he discovered his phenomenal mathematical talent. Cauchy senior was elected Secretary of the Senate in 1800 and young Cauchy shared his study. He frequently saw Lagrange, who later would say of Cauchy "this young man will supplant all of us." During the last nineteen years of his life Cauchy produced over 500 papers on all branches of mathematics, including mechanics, physics, and astronomy. He introduced rigor into mathematical analysis and was surpassed in intellectual productivity only by Euler and Cayley. The development of theories of deformation and kinematics of continuous media occurred during this century.

In 1776, Euler introduced and interpreted the tensor of rate of deformation or stretching and with minor changes would yield the description of infinitesimal strain constructed by Cauchy. Finite-strain theory was almost completely developed by Cauchy from 1823 to 1841 and his theory of infinitesimal strain was developed from the finite-strain theory. As will be discussed in later chapters, the kinematics and kinetics of a continuum has been developed in terms of spatial description, sometimes called an Eulerian description and a material description, sometimes called a Lagrangian description. The names Eulerian and Lagrangian for the spatial and material coordinates, respectively, are historically incorrect. [5]. A material coordinate labels the initial coordinates of the continuum at time zero and the spatial coordinate is the coordinate at time, *t*. The description of the continuum can then tell where a material coordinate is at any time, or if a spatial coordinate is picked, the continuum can be described by what material coordinate is at that point in space. Therefore, the material coordinates may be taken as a function of the spatial coordinates and time, or the spatial coordinates may be taken as a function of the material coordinates and time. Cauchy's deformation tensor was developed in terms of material coordinates and the Green's (1841) deformation tensor is in terms of spatial coordinates.

Water's importance to all civilizations has led to concentrated study of its properties and behavior of fluids. The early centers of civilization in Egypt, Mesopotamia, India and China developed machines for irrigation and to supply water. Archimedes (287-212 BC) is considered as the father of *hydrostatics* (study of pressure) for his law of buoyancy. The Roman soldier and engineer Sextus Julius Frontius (first century BC) was an inspector of the aqueducts and the public fountains in Rome and wrote an extensive treatise on practical *hydraulics* (study of the motion of fluids). He noted that the amount of water flowing through an orifice in a given interval of time depended upon the size of the orifice and on the depth of the orifice below the surface of the water in the reservoir. Torricelli (1608-1647) postulated that the velocity v of the water is proportional to the square root of the depth h. Daniel Bernoulli (1700-1782) mathematically expressed Torricelli's theorem as $v = \sqrt{2gh}$.

Leonardo da Vinci (1425-1519) gave a complete formulation of the law of the flow of currents. "All motion of water of uniform breadth and surface is stronger at one plane than at another according as the water is shallower there than at the other." Castelli (1577-1644) formalized this concept as the velocity-area law: $vA = \text{constant}$, where v is the velocity of the water and A is the cross-sectional area of the flow. Later Euler would develop this idea of continuity of flow for an incompressible fluid. Daniel Bernoulli introduced the term *hydrodynamics* in 1738 combining the fields of hydrostatics and hydraulics and formulated *Bernoulli's principle*, relating the velocity of flow at a point in a pipe to the pressure. Euler, in 1755, derived the fundamental equations of hydrodynamics using the concept of a fluid particle. A fluid particle is considered an infinitesimal body, small enough to be treated mathematically as a point, but large enough to possess such physical properties as volume, mass, density, and so on. [9]. He developed the equation of continuity, which expresses the conservation of matter and starting from Newton's second law derived the general equations for the motion of an ideal fluid, Euler's equations of motion.

During the centuries before the Wright brothers' first flight in 1903, researchers were concentrating on fluid flow of water, an incompressible fluid. Very few were interested in air flow as human flight was considered impossible. Since water and air are both fluids, many of the concepts of hydrodynamics are applicable to aerodynamics. Bernoulli's Principle that the fluid's pressure decreases as the fluid's velocity increases is applicable to both water and air. Euler's equations accurately represent both compressible and incompressible flow of any fluid. However, neither Euler nor anyone else had been able to solve these equations during the 17th and early 19th centuries. Although Newton had stated that the motion of a fluid gradually communicates itself to the rest of the fluid due to "defectus lubricitatis (lack of slipperiness) viscous flow was not considered until the beginning of the 19th century. The word viscosity comes from the Latin word *viscum* for mistletoe. The mistletoe berries produced a viscous glue which was used to entangle birds (lime birds).

Poiseuille (1707-1869) was interested in the circulation of blood in capillary vessels. Using small glass capillaries, he experimentally observed that the quantity of liquid discharged in unit time was proportional to the pressure times the radius of the capillary raised to the fourth power and inversely proportional to the length of the capillary. Wiedemann (1826-1899) in 1856, as well as Hagenbach in 1860, determined that the proportionality constant was $\frac{\pi}{8\eta}$, where η was called the viscosity of the fluid. The first scientist to theoretically use the property of viscosity in the fundamental equations of hydrodynamics was the French civil engineer Navier (1785-1836) in a memoir read in 1822. His theory was that any pressure tends to reduce the distance between the molecules of the fluid. He introduced an explanation of Newton’s *lack of slipperiness* by a dissipative term of viscosity using a viscosity coefficient . Stokes (1819-1903) rediscovered viscosity without using a molecular theory and history calls the important equations of fluids, the Navier-Stokes equations. Poisson (1781-1836) derived an equation for the motion of solid bodies and fluids with a second constant in addition to the viscosity. Until the 1900s most researchers related the second constant to the shear viscosity by using Stokes’ relation.

The 19th century’s industrial revolution gave rise to the development of a new science, thermodynamics. Thermodynamics emphasized the irreversible character of macroscopic phenomena, such as heat dissipation. During the same period, the kinetic theory of gases and developments in chemistry argued for a discrete description of matter. However, the motion of these materials is governed by continuum mechanics. Maxwell and Boltzmann addressed this paradox in the late 1800s. Boyle (1637-1691) stated that the world works like a clock; once energized after creation, would run forever. Descartes (1596-1650) stated that the total amount of motion in the world would remain constant. Huygens (1629-1695) worked on the problem of collisions and claimed that the vector sum of the product of mass and velocity remains unchanged after collision even though there was a dissipation of energy. It was argued that if the macroscopic objects lose some motion after a collision, the motion is transferred to the invisible particles of the objects and this motion of the invisible particles emerged as heat. This “world-machine” is often attributed to Newton as his dynamics was a reversible theory.

Fourier (1768-1830) interest in the problem of the cooling of the Earth led to his development of the equation of heat conduction, an irreversible equation. Carnot (1796-1832) focused his research on the question of the limited efficiency of the steam engine and on the existence of dissipation but more important stated that work and heat are two different expressions of energy. The first law of thermodynamics stated the energy of an isolated system remains constant (conservation of the internal energy in a closed system.). Clausius (1822-1888) introduced entropy as a measure of the quantity of work lost during the transfer of heat from a hot to a cold body. The second law of thermodynamics states: the entropy of an isolated system always tends to increase. The word entropy comes from the Greek words $\varepsilon\nu\varepsilon\rho\gamma\varepsilon\iota\alpha$ energy) and $\tau\rho o\pi\eta$ (transformation). Boltzmann (1844-1906) worked on the kinetic theory of gasses and on the discrete (non-continuum) character of matter. He proposed a more general definition of entropy in terms of probabilities of molecular arrangements. [9] In 1966, Truesdell [10] would write: “As mechanics is the science of motions and forces, so thermodynamics is the science of forces and entropy. What is entropy? Heads have split for a century trying to define entropy in terms of other things. Entropy, like force, is an undefined object, and if you try to define it, you will suffer the same fate as the force-definers of the seventeenth and eighteenth centuries; either you will get something too special or you will run around in a circle.”

Despite discoveries by the scientists supporting the atomic hypothesis of mater, others still rejected it. During the early part of the 20th century, the controversy between the proponents of the kinetic theory of Boltzmann and the continuum mechanists was alive and well. Einstein (1879-1955) work on the Brownian motion supported the kinetic theory. It is beyond the scope of this chapter to follow this debate through the first half of this century. In the 1950's Truesdell emerged as the greatest champion of the continuum theory.

Clifford Ambrose Truesdell III (1919-2000) should be known as the godfather of continuum mechanics. When he was in his early 30s during the early 1950s, he had established himself as the world's best-informed person in the field of continuum mechanics including the classical theories of fluid mechanics and elasticity and also the newer attempts to describe mathematically the non-classical behavior of materials. Clifford Truesdell has been called many things; "a singularity among all prominent scientist-scholars of the twentieth century"; "an extraordinary figure of 20th century science"; "an egotist"; "a curmudgeon"; but no one has ever criticized his pre-eminence in the development of continuum mechanics in the decades following the Second World War. Truesdell received B.S. degrees in Physics and Mathematics in 1941 from the California Institute of Technology, an M.S. in Mathematics in 1942, a Certificate in Mechanics from Brown University in 1942, and a Ph.D. in Mathematics from Princeton in 1943. After serving at the Radiation Laboratory at MIT, U.S. Naval Ordnance Laboratory and the U.S. Naval research Laboratory, he was a Professor of Mathematics at Indiana University from 1950 to 1961. The remainder of his career, he was a Professor of Rational Mechanics at The Johns Hopkins University until retirement in 1989.

Truesdell mastered Latin perfectly and was fluent not only in his native language, English, but also in French, German and Italian. In 1951, he founded with T. Y. Thomas a journal to serve the growing fields of mathematical continuum mechanics and the analysis of nonlinear partial differential equations, the Journal of Rational Mechanics and Analysis. He had persuaded his colleagues to use the phrase "rational mechanics" as it was introduced and defined by Newton. The dictionary definition of "rational" is "having reason or understanding." After a dispute with the head of the department, Truesdell was removed as Editor and the journal was renamed the Journal of Mechanics and Mathematics (later became the Indiana University Mathematics Journal). Siegfried Flugge, the Editor of the *Handbuch der Physik*, convinced Springer-Verlag to take over publication of a journal under Truesdell's sole Editorship. The new journal was named the Archive for Rational Mechanics and Analysis. Although Truesdell published 26 monographs and 268 papers, the relationship with Flugge led to the two most important publications for continuum mechanics of that time.

In 1960, Encyclopedia of Physics (Handbuch der Physik) Volume III/1 *Principles of Classical Mechanics and Field Theory* [5] was published. This nine hundred and two page volume has three sections: *Classical Dynamics* by John L. Synge, *The Classical Field Theories* by C. Truesdell and R. A. Toupin and *Appendix. Tenser Fields* by J. L. Ericksen. In 1965, Encyclopedia of Physics, Volume III/3: *The Non-linear Field Theories of Mechanics* by C. Truesdell and W. Noll [11] was published. Truesdell also wrote three sections on fluid mechanics in Volume VIII/1-3 for the Encyclopedia of Physics from 1959 to 1963. The February 8 to February 12, 1960 lectures by Truesdell to the Field Research Laboratory of the Socony Mobil Oil Company, Inc. were tape recorded and transcribed and published in 1961. [12] Truesdell writes in a forward to this publication: "No one knows how badly he speaks until he is confronted with a transcript…In addition to chagrin for the style, I feel regret that into the course of ten lectures I could not find skill to compress a greater portion of the recent work on rational mechanics.” He states in The Classical Field Theories; “This treatise is intended for the specialist, not the beginner. Necessarily it presents the foundations of the field theories, not as they appeared in the last century and linger on in the textbooks, not as the experts in some other domains may think they ought to be presented, but as they are cultivated by the specialist of today.” Such was the man, Clifford Truesdell. He was an outstanding scholar, mathematician and historian and those studying continuum mechanics should read these treatises after study of the introductive texts.

It is not surprising that the 1960s brought a number of excellent introductory text books in the theory of continuum mechanics. Some of which are listed here in chronological: Prager in 1961 [13], Eringen in 1962 [14], Frederick and Chang in 1965 [15], Sedov in 1965 [16], Scipio in 1967 [17], Calcote [18] and Leigh [19] in 1968, and Fung [20] and Malvern [21] in 1969. These books introduced hundreds of young students to the field and greatly aided in the understanding of subsequent courses in elasticity, fluid mechanics and viscoelasticity.

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Biographical Sketch

Robert William Soutas-Little (Robert Wm. Little prior to marrage to Patricia Soutas in 1982) graduated with a Ph.D in Theoretical Mechanics (Minor in Physics and Mathematics) in 1962 from the University of Wisconsin. From 1955 to 1957 he worked as a Design Engineer, Allis-Chalmers Manufacturing Company. In 1957 he became an Instructor at Marquette University, USA, where he was until 1959. In 1959, he joined the University of Wisconsin and later in 1963 the Oklahoma State University as an Assistant Professor. In 1965 he moved to Michigan State University where he has been a Professor since 1970 and has served as Chairman of the Department of Mechanical Engineering and the Department of Biomechanics as well as Director of the Biomechanics Evaluation Laboratory,and the Biodynamics Research Laboratory. He is presently Professor Emeritus of the Department of Mechanical Engineering at that University. He has published over 100 refereed Journal Papers and Chapters in Books as well as 6 Books in the fields of Statics, Dynamics, and Elasticity including the classic monograph "Elasticity", re-issued in 1999 as a Dover paperback.