ELEMENTARY FLUID DYNAMICS

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Keywords: Fluid Mechanics, Statics, Kinematics, Navier-Stokes, Mass Conservation, Momentum balance, Bernoulli equation, Boundary layer, Moody, Compressible flow, point sources, lift, drag, energy machine, pump, fan, wind generator, propeller, Pelton turbine.

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Summary

This is an introductory chapter to the science of Fluid Mechanics; it is a self contained text and it is written with the purpose of explaining definitions, concepts and laws of Fluid Mechanics to such a level of practicality so as the reader will be able either to solve problems of Fluid Mechanics of engineering interest or to have the necessary background for a more in depth study of Fluid Mechanics.

The text covers all introductory aspects of Fluid Mechanics, giving emphasis on Fluid Mechanics of everyday life, such as flows in pipes or around bodies and less emphasis in high speed flows, where compressibility effects or supersonic flow regions are present. For the latter cases however the basic background knowledge is presented and the reader has the necessary tools to proceed further.

The text consists of six sections, starting from the basic definitions and concepts of what is a fluid and then by applying known laws of mass and momentum conservation, the laws of Fluid Mechanics are established; whenever it is thought appropriate examples are presented. Emphasis is given in the fourth section on the viscous flows inside ducts or around bodies and the distinction between the two classes of flows, laminar and turbulent, is clarified. In section five, the theory of superposition of elementary flows is presented for inviscid fluids and complex flow fields are created by the application of this theory. In the last section the laws of Fluid Mechanics and the knowledge presented in the previous sections are applied in studying energy flow machines, such as fans, pumps, hydraulic turbines, wind turbines and propellers, machines which form the basis of the energy cycle of the modern society.

1. Fundamental Concepts of Fluids

1.1 What is Fluid Mechanics?

Fluid Mechanics is the branch of science which, with the use of Mathematics and Experiments studies the motion of fluids under the action of external and internal forces. The fluid can normally be in two kinetic states, either in motion (flow in pipes, flow around cars or airplanes, flow in arteries or vanes, air flow in the atmosphere) or no-motion, static (fluid in a reservoir); the first kinetic state, fluid in motion, is studied by the science of Fluid Dynamics whilst the second, the fluid being static, by the science of Hydrostatics. In Fluid Mechanics the external forces act on the fluid either on the surfaces which define the volume of the fluid body (the surface could be impermeable or permeable to fluid molecules) or remotely on the fluid mass; for the latter force there is no need for any contact between the fluid and the force origin. Such forces are gravitational force, electrostatic or electromagnetic force or acoustic force; these forces can be- referred to as forces per unit fluid mass or per unit fluid volume. The forces acting on the (external) surface defining the fluid volume can have direction normal and tangential to the surface and are, called normal forces (compression and expansion) and tangential or shear forces.

1.2. What is a Fluid?

All material substances consist of molecules which can move in space and time; the molecules attract each other (at distances from a fraction of the mean distance between molecules to some tens of the mean molecular distance) and repel each other in the form of elastic collisions (at distances smaller than a fraction of the mean molecular distance). Depending on the number of molecules packed in a given volume, material substances can be divided in two categories, solids and fluids. In solids the molecules are closely packed, have a fixed position in space, Figure 1, and vibrate around their equilibrium position with small amplitude compared to their mean molecular distance; the molecules in crystals are located at the corners of notional geometrical three dimensional structures (cubes, pyramids). In solids the intermolecular bonds are so strong that the solid has a fixed shape and volume. In fluids the molecules are not closely packed and they are separated by greater distances than in solids; the attraction forces are not strong enough to keep these in fixed positions in space and hence move freely in space, Figure 2; therefore the molecules cover distances larger than the mean intermolecular distance before these collide with each other and change path direction.
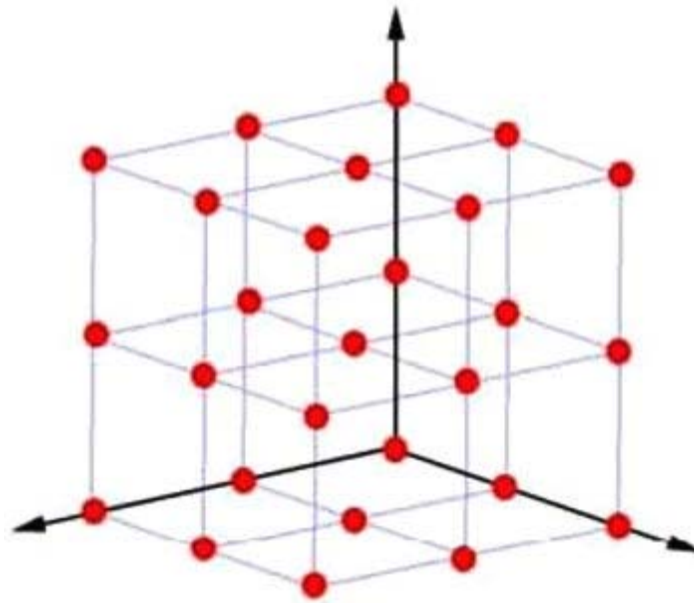
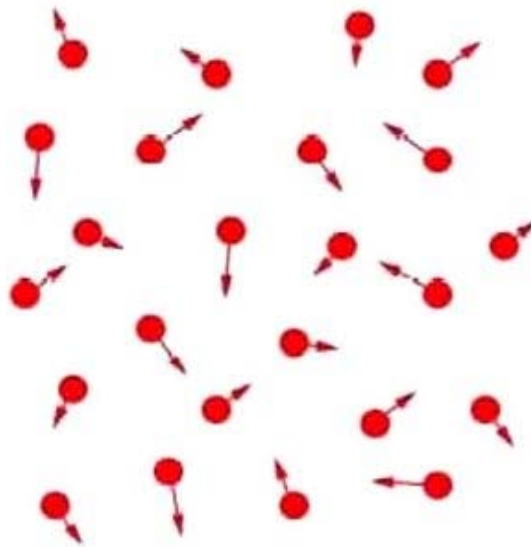Figure 1. Location of molecules fixed in space (crystals)

Figure 2. Molecules moving in space (fluids)

The fluids are further divided into liquids and gases depending on the degree of the kinetic state of their molecules; to obtain a sense of magnitude of the mean distance between molecules, for air at standard conditions (one atmosphere of pressure and at a temperature of 0° degrees Celsius), the mean intermolecular distance is approximately 3nm (3 nanometers=3.10^{-9}m) and the mean free path of the molecule (the average distance a molecule travels in straight line between two collisions) is approximately 25 times the intermolecular distance. The number of molecules per mole of the gas is expressed by the Avogadro number N equal to 6.10^{23}; therefore 1m^3 of air at standard conditions has 3.10^{25} molecules, a very large number beyond our comprehension; even in a cube of space of 1mm side there are 3.10^{16} molecules present and in a cube of even

1μm side, there are 3.10^7 molecules. It is also found that within approximately 10 collisions with other air molecules, the air molecule loses its initial kinetic characteristics and reaches a statistically equilibrium condition. As the molecules move randomly in space colliding with each other, the fluids do not posses their own volume (i.e. their own "shape") by themselves but occupy the volume (maybe partly or whole) which is available to them (e.g. a bottle, or a river bed) and take the shape of their containment.

The above description of the nature of a fluid is based on a microscopic view of the material structure which has been developed out of the kinetic theory of gases the last 100 years or so.

1.3. Continuum

Mathematics is the universal common language which we have in expressing quantitatively the laws of physics. Mathematics however uses the concept of continuous functions with continuous derivatives and so on, in expressing these physical laws, a concept which requires that the values or the entities expressing physical quantities as velocity, density, etc, should be continuous functions in space and time. With the microscopic model of the fluid presented above (&1.2) based on the discrete nature of molecules, the kinetic state and condition of each fluid should be characterized by the velocities and the number of molecules at every time and position in space. Therefore a function expressing a physical quantity such as the velocity at a particular point in space and time could not be continuous because the point in space cannot be occupied at all times by a molecule. Engineers and most scientists are not interested in length scales of the order of molecular distances but distances of the engineering scale which could nevertheless be small, for example the size of a micrometer. Then the statistical approach of averaging over a region of finite volume ΔV is employed because within this finite volume of space, but infinitesimally small compared to engineering length scales, there is an enormous number of molecules over which the statistical approach can be employed. Within this volume ΔV there is a large number of molecules having a specified velocity; it is said that the fluid mass contained within the volume has a (vector) velocity $\vec{v}$ which is the average (vector) velocity of all the molecules $\vec{v}_i$ which exist within the volume at the particular time,

$$\vec{v} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i} \tag{1}$$

where m_i is the mass (or the number) of molecules within the volume having velocity $\vec{v}_i$ and $\sum_i m_i$ is the total mass (or the number) of the molecules within the volume (i.e. the mass of the fluid contained within the volume).

A similar definition is used for the fluid density. The fluid density is the mass of fluid contained in the unit volume,

$$\rho = \frac{\sum_i m_i}{\Delta V} \qquad\qquad (2)$$

The value of the physical quantity defined as above, such as the density ρ, converges to a value representative of the fluid density at a particular point as the size of the volume ΔV over which the property is averaged becomes smaller and smaller, but always containing a large number of molecules for a statistically significant estimate of the average. Therefore "small" which for engineering science is effectively infinitesimally small means a linear dimension of say 10 times the mean free path of a molecule. Even if the volume ΔV is taken smaller, say a linear dimension equal to the mean free path of the air molecule at standard conditions the error which could be made in the calculation of density would be around 0.8%.

The continuum hypothesis is employed in the science of Fluid Mechanics and it allows the introduction of the concept of the fluid element or fluid particle which is a an infinitesimally small element of fixed mass with properties such as density, temperature, representing the local properties of the fluid; this hypothesis is valid due to the large number of molecules within the length scales of scientific interest. However there are cases where the continuum hypothesis is not valid as in the case of very low pressure or temperature, airflow in the outer atmosphere, flow of gas across a shock wave and flow within devices of the nanometer length scale. In these cases the continuum hypothesis is not valid and different approaches for the study of such phenomena are employed. A dimensionless number, known as Knudsen number (usually abbreviated to Kn), is defined as the ratio of the typical linear scale of the flow to the free molecular path. When the Knudsen number is of the order of unity, then the flow can not be regarded as continuum, otherwise for $Kn \gg 1$, which is almost true in cases of engineering interest, the flow can be treated as a continuum.

1.4. Physical Properties of a Fluid

Fluids have distinctive characteristics, some of which are quantified by numerical values called physical properties; such physical properties are, among others, the fluid density, temperature, compressibility, viscosity and surface tension.

1.4.1. Density

Fluid density varies substantially from liquids to gases (due to the different number of molecules packed in a given volume) and has dimensions expressing mass per unit volume. In the international system of units, the unit of density is kg m^{-3}. The density of a liquid is a weak function of pressure and temperature whilst in gases the density depends strongly on pressure and temperature.

1.4.2. Temperature

Temperature characterizes the random translational kinetic energy of the fluid molecules, so that the higher the kinetic energy of the molecules the higher the temperature of the fluid. The mean kinetic energy of the molecule is

$$E = \frac{1}{2}m\overline{v_{\text{rel}}^2} \tag{3}$$

where $\overline{v_{\text{rel}}^2} = \sum_i \vec{v}_{i\text{rel}}.\vec{v}_{i\text{rel}}$ is the mean squared magnitude of the relative velocity of each molecule ($\vec{v}_i$) to the bulk fluid velocity ($\vec{v}_i$), $\vec{v}_{i\text{rel}} = \vec{v}_i - \vec{v}$ and m the mass of the molecule. According to the kinetic theory of gases this energy is equal to $3/2kT$, where k is the Boltzmann's constant (Boltzmann 1844-1906), equal to $k = 1.38 \times 10^{-23} \text{JK}^{-1}$. Therefore temperature is equal to

$$T = \frac{2k}{3}\frac{1}{2}m\overline{v_{\text{rel}}^2} \tag{4}$$

In the international system of units temperature is measured in Kelvin but usually use is made of the Celsius temperature scale which has a linear relation to Kelvin scale

$$T = \theta + 273.15 \tag{5}$$

where θ is the temperature of the fluid in degrees Celsius.

1.4.3. Pressure

The molecules of a fluid are in a continuous motion; if the molecules hit a material surface, let us say a solid boundary, then these rebound and retain their kinetic energy but change their velocity normal to the surface. This change of the normal component of the momentum of all molecules striking a unit surface in unit time is due to a reacting force from the boundary to the fluid which is called pressure; the frequency of the impact of the molecules on the surface is so high (order of 10^{10} Hz) that they can not be distinguished as individual impacts; this pressure is expressed for a gas by Eq. (6a),

$$P = \frac{\delta N}{\delta t}2mv_{\text{n}} \tag{6a}$$

Where m is the mass of the molecule, v_{n} is the mean velocity component normal to the surface of all molecules striking the surface and $\delta N/\delta t$ is the number of molecules striking the unit surface per unit time. Assuming that v_{n} is equal for reasons of isotropy to $v_{\text{n}} = \frac{v}{\sqrt{3}}$, the kinetic theory of gases gives the following relation for the pressure,

$$P = \frac{Nmv^2}{3} \tag{6b}$$

where N is the number of molecules per unit volume and $v = \sqrt{\overline{v_{\text{rel}}^2}}$. The pressure is then a characteristic property of the fluid and is related to the kinetic energy of molecules

(fluid temperature) and the number of molecules per unit volume; these three quantities are related by the so called "equation of state" of the fluid. For a gas the equation of state is written as

$$\frac{P}{\rho} = \frac{R}{MW}T \qquad (6c)$$

where P is the gas pressure, T the temperature, ρ the gas density, MW the molecular weight of the gas and R a universal gas constant equal to 8314 J.gmole^{-1}.K^{-1}. The pressure as defined above characterizes the force per unit surface which the fluid- in a compressive way-acts normally on not only any solid "external" surface surrounding the fluid volume but on any notional surface within the fluid volume which instantaneously can be regarded as a material surface.

1.4.4. Compressibility

Compressibility is a fluid property which indicates the way in which the volume a fluid occupies changes under the action of compression forces. A fluid volume V under the action of a change of a compression force δP undertakes a change of volume equal to δV ; then the compressibility factor is defined as

$$K = -\frac{1}{V}\frac{\delta V}{\delta P} \qquad (7a)$$

Similarly the modulus of elasticity E is defined (to the inverse of the compressibility factor) as

$$E = -\frac{\delta P}{(\delta V / V)} = \rho\frac{\delta P}{\delta \rho} \qquad (7b)$$

The compressibility K varies for different fluids; a value for water is K=510^{-5}atm^{-1}, corresponding to a modulus of elasticity E equal to $E = 20000$ atm; this compressibility factor for water indicates that for a change of pressure of 1 atm the percentage change of water volume is only 0.005%. For gases the compressibility factor, using the equation of state for gases and assuming that the change of volume takes place under isothermal conditions, using expression (7a) leads to expression (7c), or under isentropic conditions, to expression (7d),

$$K_{\mathrm{T}} = \frac{1}{P} \qquad (7c)$$

$$K_{\mathrm{S}} = \frac{1}{\gamma P} \qquad (7d)$$

This expression indicates that for air at 1 atm, the compressibility factor is equal to K_{T}=1atm^{-1}, indicating that air, as for all gases, is 20000 times more compressible than

liquids and in fact in most cases liquids are treated as incompressible fluids.

1.4.5. Viscosity

The Dynamic viscosity (μ) is a fluid property which indicates the resistance that the fluid offers to rate of change of shape or to motion under the action of shear forces. It arises in real fluids from molecular motion (mainly in gases) and intermolecular attraction (mainly in liquids). When the fluid is in motion, then at every point within the fluid there is a velocity gradient in the direction normal to the fluid velocity, Figure 3; layers of fluid particles move in parallel directions but with different velocities; however the fluid elements comprising these particles have instantaneous velocities in the direction normal to the fluid particle's velocity and hence as these cross the notional common interface between any two adjacent layers, transfer the component of fluid particle momentum from a region of higher to a region of lower velocity and vice versa.
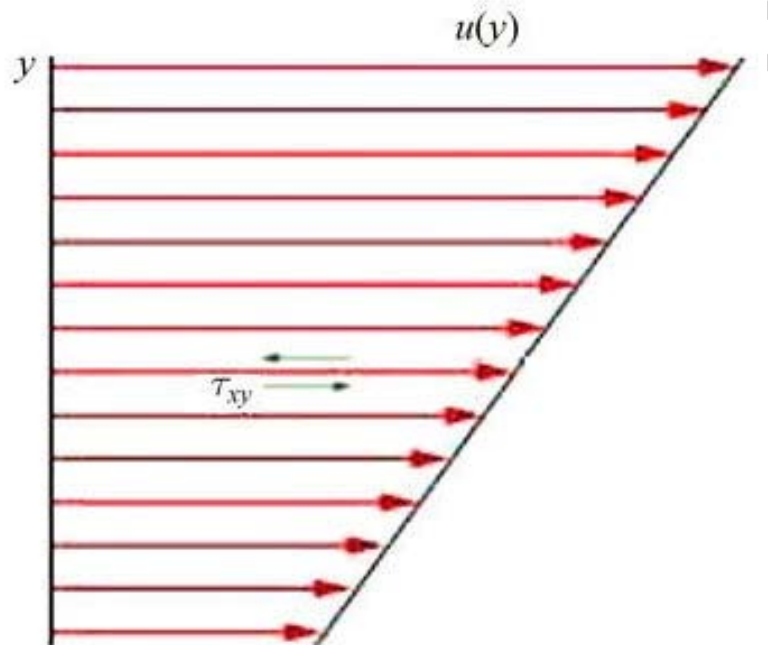

Figure 3. Velocity gradient in a flow of fluid particles gives rise to shear stress τ_{xy}

This loss or gain of momentum which the molecules experience in the direction of the fluid velocity averaged over the great number of molecules crossing the fluid layer interface under consideration, manifests itself macroscopically as a drag force of the lower velocity fluid on the faster one and vice versa; this force is called viscous force and according to Newton's principle the so called "viscous" force per unit area, which is called shear stress, acting between the two layers of the fluid moving in parallel but with different velocities is proportional to the fluid velocity gradient with the constant of proportionality being the fluid dynamic viscosity μ,

$$\tau_{xy} = \mu \frac{du}{dy} \tag{8}$$

Therefore a shear stress can develop within the fluid only in the presence of velocity gradients. This is also an alternative definition of a fluid, namely that a fluid is a material substance which is set in motion under the action of shear stresses.

From the above constitutive law, dynamic viscosity has units of [μ]=Pas. Frequent use is made of the kinematic viscosity, which is the ratio of the dynamic viscosity to the fluid density with units [v]=m^{2}s^{-1},

$$v = \frac{\mu}{\rho} \tag{9}$$

The symbol τ_{xy} expresses the shear stress which acts in x direction and is located on the plane with y constant. The dynamic viscosity of the fluid is a property which depends on temperature and pressure.

1.4.6. Surface Tension

Surface tension is a fluid property which is pertinent in the inter-phase region between two immiscible fluids A and B, Figure 4 or between a liquid and a solid. On a molecule in fluid A the surrounding molecules exert attraction forces within a sphere of influence around the molecule; this sphere of influence has a radius of few molecular distances and the attraction forces are negligible beyond this distance. Since there is isotropy in the fluid the attraction forces result to a zero attraction force on the particular molecule. This situation however does not hold if the particular molecule is near the inter-phase between the two immiscible fluids, say fluid A liquid and fluid B air, as more molecules of fluid A (than fluid B) exert attraction forces on the molecule. In this case there is an attraction force acting on the molecule towards the interior of fluid A. This force becomes a maximum when the molecule is on the inter-phase and gives rise to a force called surface tension σ, with dimensions of force per unit length. This force acts normally on any length on the surface of the inter-phase and performs work, stored as surface energy, during any displacement of the line on which the surface tension applies. In order for the inter-phase to be in stable equilibrium, the surface must take such a shape so as to minimize its surface energy, i.e. an initial flat surface will be curved with radius of curvature such that equilibrium of forces will prevail. In Figure 5 a liquid spherical droplet of radius R is shown in equilibrium in a surrounding of pressure P_a. Equilibrium of forces on the surface of the droplet, results in the Eq. (10)

$$P_b = P_a + \frac{2\sigma}{R} \tag{10}$$

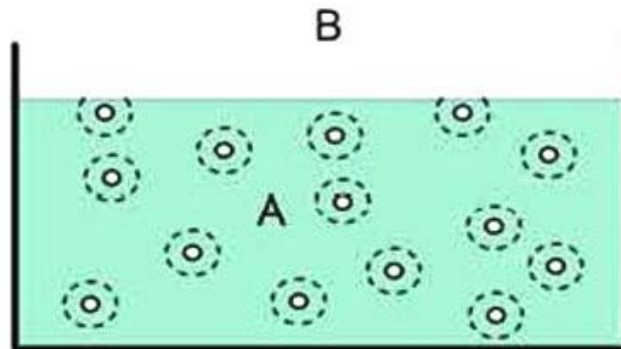indicating that the pressure P_b inside the droplet is higher than the surrounding pressure P_a.

Figure 4. Sphere of influence of attraction forces on molecule A
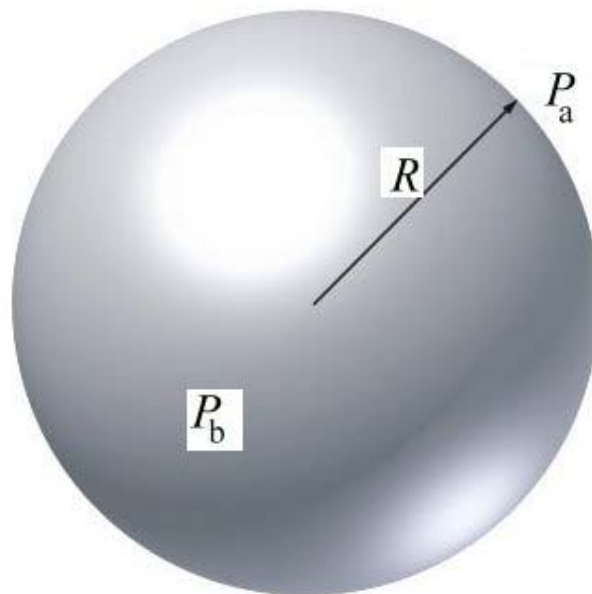

Figure 5. Spherical droplet in static equilibrium with the surrounding

Table 1 gives values of various physical properties for some fluids at 1 atm pressure and temperature of 20°C.

Fluid	Density ρ (kg/m^3)	Dynamic viscosity μ (Pas)	Compressibility K_T (atm^{-1})	Surface tension σ (N/m) (liquid to air)
Water	998.2	1.002x10^{-3}	5x10^{-5}	0.073
Normal heptane	879	4x10^{-4}	1.49x10^{-5}	0.03
Air	1.204	1.82x10^{-5}	1	-
Hydrogen	0.083	8.5x10^{-6}		-

Table 1. Physical properties of some typical fluids

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Biographical Sketch

Dr George Bergeles holds a diploma in Mechanical and Electrical Engineering (1968) from the National Technical University of Athens, an M.Sc. (1973) and a Ph. D (1976) degree in Heat Transfer Engineering from the Imperial College of Science, Technology and Medicine, and a D.Sc. (Eng) (1994) degree from the University of London. He has been an Eisenhower Fellow for Greece (1983), and a Fellow of the UK Institution of Mechanical Engineers since 1988. He is Professor of Fluid Mechanics in the Department of Mechanical Engineering of the National Technical University of Athens and has been Visiting Professor in King's College London and is currently Honorary Visiting professor at City University, London.

Professor Bergeles has extensive experience in education, research and industry, has published more than 80 papers in academic journals, and has supervised many Ph.D. researchers. His engineering contributions include those to the design and manufacture of equipment for Wind Energy, Industrial Aerodynamics, Air Pollution and Combustion Engineering. The design aspects have been facilitated by the development of grid generation techniques and finite-difference and finite volume methods for solving the Navier-Stokes equations in curvilinear coordinates with automatic local grid refinement, and supported by experiments in the largest subsonic wind tunnel in Greece.