THERMOELASTICITY

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Contents

Summary

This chapter is concerned mainly with the basic problems of the linear theory of thermoelasticity. Beginning with the basic laws of thermodynamics, there follows a treatment of the constitutive equations and the derivation of the equations of nonlinear thermoelasticity. The next part of this work is devoted to the linear thermoelastodynamics. First, some basic theorems are established. Then, an investigation of thermoelastic waves is presented. The work concludes with a study of the theory of thermoelastic equilibrium. Relevant examples which illustrate the theory are given throughout the text.

1. Introduction

The theory of thermoelasticity is concerned with the interaction between thermal field and the elastic bodies. The study of thermoelasticity was begun by Duhamel (1837) and Neumann (1885) who postulated the equations of the linear thermoelasticity for isotropic bodies. These equations have been justified by Biot (1956) on the basis of irreversible thermodynamics. A derivation founded on modern continuum thermodynamics has been given by Eringen (1967). The theory of thermoelasticity is of great importance. The published work in thermoelasticity is so large that it is not possible to do justice to all contributors by mere mention of their names. An account of the historical development, as well as references to various contributions, may be found in the monographs by Green and Adkins (1960), Boley and Weiner (1960), Truesdell and Toupin (1960), Nowacki (1962), Carlson (1972), Day (1985), Ieşan and Scalia (1996).

In this work we present a short account of the linear theory of thermoelasticity. The exposition of nonlinear thermoelasticity is presented to provide a base for the linear theory. The reader interested in the nonlinear thermoelasticity is referred to the books by Racke and Jiang (2000) and Ieşan and Scalia (1996).

The present work consists of three main parts. In the first part (Sections 2-5) we focus attention to the derivation of the equations of thermoelasticity. The second part of this article (Sections 6-10) contains a study of the dynamic theory of thermoelasticity. In the last part we investigate some problems of the theory of thermoelastostatics.

To review the vast literature on applications and special problems is not our intention; considerations of space and time have caused extensive selection to be made. The illustrations included are examples considered relevant to the purpose of the text.

The assumptions of zero initial stress and uniform reference temperature are crucial to the development of the classical linear thermoelasticity. Thermoelasticity of bodies with initial stresses and non-uniform reference temperature is not considered here. The reader interested in these subjects will find a full account in the works of Knops and Wilkes (1973) and Ieşan and Scalia (1996).

In recent years there has been some interest in thermoelasticity of polar materials and the theories of thermoelasticity with finite wave speeds. For an extensive review of the literature on these theories the reader is referred to the monographs by Nowacki (1986), Chandrasekharaiah (1986), Eringen (1990), Jou *et al.* (1996), Müller and Ruggeri (1998), Ieşan (2004). We make no claim to completeness. It is hoped that the present work gives an accessible treatment of a part of the contributions that have been made to the subject.

2. Preliminaries

In what follows we consider a body that at time t_0 occupies the region B of Euclidean three-dimensional space E^3. We assume, unless specified otherwise, that B is a bounded regular region.

The configuration of the body at time t_0 is taken as the reference configuration. The motion of the body is referred to the reference configuration and a fixed system of rectangular Cartesian axes. We identify a typical particle x of the body with its position $\mathbf{x}$ in the reference configuration. The coordinates of a typical particle x in B are

$x_j\,(j=1,2,3)$. The coordinates of this particle in the position $\mathbf{y}$ at time t are denoted by y_i. We have

$$\mathbf{y}=\mathbf{y}(\mathbf{x},t),\ \ (\mathbf{x},t)\in \bar{B}\times I, \tag{1}$$

where $I=(t_0,t_1)$ is a given interval of time. We assume continuous differentiability of $\mathbf{y}$ with respect to the variables x_i and t as many times as required and

$$\det\left(\frac{\partial y_i}{\partial x_j}\right)>0\ \text{ on } \bar{B}\times I. \tag{2}$$

The configuration of the body at the time t is denoted by B' and is called present configuration. We shall employ the usual summation and differentiation conventions: Latin subscripts (unless otherwise specified) are understood to range over the integers (1,2,3) whereas Greek subscripts are confined to the range (1,2), summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. In what follows, a superposed dot denotes the material derivative with respect to the time. Letter in boldface stand for tensors of an order $p\geq 1,$ and if $\mathbf{v}$ has the order p, we write $v_{ij..k}$ (p subscripts) for the rectangular Cartesian components of $\mathbf{v}$. We say that f is of class $C^{M,N}$ on $B\times(t_0,t_1)$ if f is continuous on $B\times(t_0,t_1)$ and the functions

$$\frac{\partial^m}{\partial x_i \partial x_j \ldots \partial x_k}\left(\frac{\partial^n f}{\partial t^n}\right),\quad m\in\{0,1,\ldots,M\}, n\in\{0,1,\ldots,N\},\quad m+n\leq \max(M,N),$$

exist and are continuous on $B\times(t_0,t_1)$. We write C^N for $C^{N,N}$.

The local form of the conservation law of linear momentum can be expressed as

$$T_{ji,j}+\rho_0 f_i=\rho_0 \ddot{y}_i\ \text{ on } B\times(t_0,t_1), \tag{3}$$

where T_{ji} is the first Piola-Kirchhoff stress tensor, ρ_0 is the mass density at time t_0 and f_i is the body force per unit mass.

If we define the second Piola-Kirchhoff stress tensor S_{ij} by

$$T_{ki}=y_{i,j}S_{kj}, \tag{4}$$

then the local form of the conservation law of moment of momentum reduces to

$$S_{ij} = S_{ji}. \tag{5}$$

We denote by E_{ij} the Lagrangian strain tensor,

$$E_{ij} = \frac{1}{2}(y_{k,i}y_{k,j} - \delta_{ij}) \text{ on } B \times (t_0, t_1), \tag{6}$$

where δ_{ij} is Kronecker's delta.

The local form of the first law of thermodynamics can be written as

$$\rho_0 \dot{e} = S_{ij}\dot{E}_{ij} + \rho_0 S + Q_{j,j} \text{ on } B \times (t_0, t_1), \tag{7}$$

where e is the internal energy per unit mass, S is the heat supply per unit mass, and Q_j is the heat flux associated with surfaces in B' which were originally coordinate planes perpendicular to the x_j-axes through the point x, measured per unit undeformed area.

We assume that f_i and S are continuous on $B \times (t_0, t_1)$, T_{ij} and Q_j are of class $C^{1,0}$ on $B \times (t_0, t_1)$ and continuous on $\bar{B} \times [t_0, t_1)$.

Let $\mathcal{P}$ be a region of the continuum bounded by a surface $\partial\mathcal{P}$ at time t, and suppose that P is the corresponding region at time t_0, bounded by the surface ∂P. We denote by n_i the components of the outward unit normal at ∂P. Let $\mathbf{t}$ be the stress vector associated with the surface $\partial\mathcal{P}$, but measured per unit area of the surface ∂P, and let q be the heat flux across the surface $\partial\mathcal{P}$, measured per unit area of ∂P. Then, we have

$$t_i = T_{ji}n_j, \quad q = Q_j n_j. \tag{8}$$

We denote by T the absolute temperature, which is assumed to be positive. Let η be the entropy per unit mass. We assume that η is of class $C^{0,1}$ and T is of class $C^{2,1}$ on $B \times (t_0, t_1)$. The local form of the second law of thermodynamics can be expressed as

$$\rho_0 T\dot{\eta} - \rho_0 S - Q_{j,j} + \frac{1}{T}Q_j T_{,j} \geq 0. \tag{9}$$

If we introduce the Helmholtz free-energy,

$$\psi = e - T\eta, \tag{10}$$

then the equation of energy can be written in the form

$$\rho_0(\dot{\psi} + \dot{T}\eta + T\dot{\eta}) = S_{ij}\dot{E}_{ij} + Q_{j,j} + \rho_0 S. \tag{11}$$

From (9) and (11) we obtain the following local dissipation inequality

$$S_{ij}\dot{E}_{ij} - \rho_0(\dot{\psi} + \dot{T}\eta) + \frac{1}{T}Q_j T_{,j} \geq 0. \tag{12}$$

3. Constitutive Equations

A thermoelastic material is defined as one for which the following constitutive equations hold

$$\begin{aligned}
\psi &= \hat{\psi}(E_{mn}, T, T_{,k}, x_r),\\
S_{ij} &= \hat{S}_{ij}(E_{mn}, T, T_{,k}, x_r),\\
\eta &= \hat{\eta}(E_{mn}, T, T_{,k}, x_r),\\
Q_i &= \hat{Q}_i(E_{mn}, T, T_{,k}, x_r).
\end{aligned} \tag{13}$$

We suppose that the functions $\hat{\psi}, \hat{S}_{ij}, \hat{\eta}$ and $\hat{Q}_i$ are of class C^1 on their domain. In the case of homogeneous bodies the constitutive functions do not depend on x_r. Clearly, the constitutive equations (13) satisfy the principle of material frame-indifference.

Let us study the restrictions placed on the constitutive functions by the second law. We introduce the notation

$$\sigma = \rho_0 \psi. \tag{14}$$

In view of (13), the inequality (12) becomes

$$\left(S_{ij} - \frac{\partial \sigma}{\partial E_{ij}}\right)\dot{E}_{ij} - \left(\rho_0 \eta + \frac{\partial \sigma}{\partial T}\right)\dot{T} - \frac{\partial \sigma}{\partial T_{,j}}\dot{T}_{,j} + \frac{1}{T}Q_j T_{,j} \geq 0. \tag{15}$$

We assume that σ in (15) is arranged as a symmetric function of E_{ij}. For a given deformation and temperature, the inequality (15) is valid for all arbitrary values of $\dot{E}_{ij}, \dot{T}$ and $\dot{T}_{,j}$, subject to $E_{ij} = E_{ji}$. Thus, in absence of internal constraints, from (15) we obtain (see Coleman and Mizel (1964), Carlson (1972))

$$S_{ij} = \frac{\partial \sigma}{\partial E_{ij}}, \quad \rho_0 \eta = -\frac{\partial \sigma}{\partial T}, \quad \frac{\partial \sigma}{\partial T_{,j}} = 0,$$

and

$$Q_j T_{,j} \geq 0. \tag{16}$$

We conclude that the constitutive equations of thermoelastic bodies are given by

$$\begin{aligned} &\sigma = \widehat{\sigma}(E_{ij}, T, x_k), \\ &S_{ij} = \frac{\partial \sigma}{\partial E_{ij}}, \quad \rho_0 \eta = -\frac{\partial \sigma}{\partial T}, \\ &Q_p = \widehat{Q}_p(E_{ij}, T, T_{,k}, x_m). \end{aligned} \tag{17}$$

In view of (17), the energy equation (11) takes the form

$$\rho_0 T \dot{\eta} = Q_{j,j} + \rho_0 S \quad \text{on } B \times (t_0, t_1). \tag{18}$$

The next result is a consequence of inequality (16).

Theorem 3.1. *The heat flux vanishes whenever the temperature gradient vanishes,*

$$\widehat{Q}_i(E_{mn}, T, 0, x_k) = 0. \tag{19}$$

Proof. Let us consider the function $f(\xi_1, \xi_2, \xi_3) = \xi_i \hat{Q}_i(E_{mn}, T, \xi_1, \xi_2, \xi_3, x_k)$, where E_{mn}, T and x_k are fixed. The inequality (16) shows that f is nonnegative. Since $f(0,0,0) = 0$, the function f has an extremum at $(0,0,0)$. If we impose that $\partial f / \partial \xi_k = 0$ at $(0,0,0)$, then we obtain the desired result.□

This theorem has been established by Pipkin and Rivlin (1958).

4. Equations of the Nonlinear Thermoelasticity

The basic equations of the nonlinear theory of thermoelasticity consist of equations of motion (3), the energy equation (18), the constitutive equations (17) and the geometrical equations (6), on $B \times (t_0, t_1)$, where t_1 is some time instant that may be infinite. The functions ρ_0, f_i and S, and the constitutive functionals $\hat{\sigma}$ and $\hat{Q}_j$ are prescribed. The response functionals $\hat{Q}_j$ are subjected to the restriction (16). To the field equations we must adjoin boundary conditions and initial conditions. In the case of the mixed boundary-value problem the boundary conditions are

$$\begin{aligned} &y_i = \tilde{y}_i \text{ on } \overline{S}_1 \times (t_0,t_1),\ T = \tilde{T} \text{ on } \overline{S}_3 \times (t_0,t_1),\\ &T_{ji} n_j = \tilde{t}_i \text{ on } S_2 \times (t_0,t_1),\ Q_i n_i = \tilde{q} \text{ on } S_4 \times (t_0,t_1), \end{aligned} \tag{20}$$

where $S_i, (i=1,2,3,4)$, are sub-surfaces of ∂B such that $\overline{S}_1 \cup S_2 = \overline{S}_3 \cup S_4 = \partial B, S_1 \cap S_2 = S_3 \cap S_4 = \varnothing$, and $\tilde{y}_i, \tilde{T}, \tilde{t}_i$ and $\tilde{q}$ are prescribed functions. The initial conditions are

$$\mathbf{y}(\mathbf{x},0) = \mathbf{y}^0(\mathbf{x}),\ \dot{\mathbf{y}}(\mathbf{x},0) = \mathbf{v}^0(\mathbf{x}),\ \eta(\mathbf{x},0) = \eta^0(\mathbf{x}),\ \mathbf{x} \in \overline{B}, \tag{21}$$

where $\mathbf{y}^0, \mathbf{v}^0$ and η^0 are given. We assume that

(i) ρ_0 is continuous and strictly positive on $\overline{B}$;

(ii) $\mathbf{f}$ and S are continuous on $\overline{B} \times [t_0,t_1)$;

(iii) $\mathbf{y}^0, \mathbf{v}^0$ and η^0 are continuous on $\overline{B} \times [t_0,t_1)$;

(iv) $\tilde{y}_i$ are continuous on $\overline{S}_1 \times [t_0,t_1)$ and $\tilde{T}$ is continuous on $\overline{S}_3 \times [t_0,t_1)$;

(v) $\tilde{t}_i$ are continuous in time and piecewise regular on $S_2 \times [t_0,t_1)$ and $\tilde{q}$ is continuous in time and piecewise regular on $S_4 \times [t_0,t_1)$.

The mixed problem of thermoelastodynamics consists in finding the functions y_i of class C^2 and T of class $C^{2,1}$ on $B \times (t_0,t_1)$ that satisfy Eqs. (3), (18), (17) and (6) on $B \times (t_0,t_1)$, the boundary conditions (20) and the initial conditions (21).

It is possible to set up more complicated boundary conditions than those considered here. In the case of the *convection condition* on the boundary, the thermal condition is

$$Q_j n_j = h(T - T_\mathrm{e}) \text{ on } \partial B \times (t_0,t_1). \tag{22}$$

Here T_e is the temperature of surrounding medium and h is the heat transfer coefficient.

The exposition of nonlinear thermoelasticity given here is presented to provide a base for the linear theory. The reader interested in the nonlinear thermoelasticity will find a full account in the books by Racke and Jiang (2000) and Ieşan and Scalia (1996).

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Bibliography

Ames K. and Straughan B. (1992). Continuous dependence results for initially prestressed thermoelastic bodies. *Int. J. Engng. Sci.* **30**, 7-13 [This studies the continuous dependence of solution on the data in the linear theory of thermoelasticity with initial deformations].

Barber J.R. and Zhang R. (1988). Transient behaviour and stability for the thermoelastic contact of two rods of dissimilar materials. *Int. J. Mech. Sci.* **30**, 691-704. [This paper presents interesting results in the problem of thermoelasticity for dissimilar materials].

Biot M.A. (1956). Thermoelasticity and irreversible thermodynamics. *J. Appl. Phys.* **27**, 240-253. [This presents a thermodynamic basis for the equations of linear thermoelasticity].

Boley B.A. (1980). Thermal stresses: A survey. In "*Thermal Stresses in Severe Environments*" (ed. P.H. Hasselman and R.A. Heller), 1-12. New York. Plenum Publ. Corp. [This presents a bibliography of the papers concerning the theory of thermoelasticity].

Boley B.A. and Weiner J.H. (1960). *Theory of Thermal Stresses*. New York: John Wiley. [This monograph presents a detailed analysis of the basic problems of linear thermoelasticity].

Carlsaw H.S. and Jaeger J.C. (1948). *Conduction of Heat in Solids*. Oxford: Clarendon Press. [This presents an analysis of various interesting problems concerning the propagation of heat in solids].

Carlson D.E. (1972). *Linear Thermoelasticity*. Handbuch der Physik, vol. VI a/2 (ed. C. Truesdell), 297-345. Berlin-Heidelberg-New York: Springer-Verlag. [This presents some important problems of the linear theory of thermoelasticity].

Chadwick P. (1960). Thermoelasticity. The Dynamical Theory. In *Progress in Solid Mechanics* (I.N. Sneddon and R. Hill), vol. 1, 263-328. Amsterdam: North Holland Publ. Co. [This presents a detailed analysis of the problem of harmonic plane progressive wave solutions].

Chadwick P. and Powdrill B. (1965). Singular surfaces in linear thermoelasticity. *Int. J. Engng. Sci.***3** , 561-595. [This presents a study of propagation of waves in thermoelastic continua].

Chadwick P. and Sneddon I.N. (1958). Plane waves in an elastic solid conducting heat. *J. Mech. Phys. Solids* **6**, 223-230. [This presents a detailed study of the propagation of plane harmonic waves in a thermoelastic body].

Chandrasekharaiah D.S. (1986). Thermoelasticity with second sound: A review. *Appl. Mech. Rev.* **39**, 355-376. [This presents a bibliography of papers devoted to the theories of thermoelasticity with finite wave speeds].

Chiriță S. (1982). Uniqueness and continuous data dependence in dynamical problems of nonlinear thermoelasticity. *J. Thermal Stresses* **5**, 331-346. [This presents continuous dependence results in nonlinear thermoelasticity for heat-conducting materials].

Chiriță S. (1982). Uniqueness and continuous dependence results for the incremental thermoelasticity. *J. Thermal Stresses* **5**, 161-172. [This presents some basic theorems in the theory of thermoelasticity with initial heat flux].

Coleman B.D. and Mizel V.J. (1964). Existence of caloric equations of state in thermodynamics. *J. Chem. Phys.* **40**, 1116-1125. [This establishes restrictions placed on elastic materials by the second law].

Dafermos C.M. (1968). On the existence and the asymptotic stability of solutions to the equations of linear thermoelasticity. *Arch. Rational Mech. Anal.* **29**, 241-271. [This appear to be the first paper to present an existence result in thermoelastodynamics].

Dafermos C.M. (1979). The second law of thermodynamics and stability. *Arch. Rational Mech. Anal.* **70**, 167-179. [This presents a continuous dependence result in the nonlinear thermoelasticity of nonconductors of heat].

Danilovskaya V.Y. (1950). Thermal stresses in an elastic half space due to a sudden heating of its boundary [in Russian]. *Prikl. Mat. Mekh.* **14**, 316-318. [This presents the solution of the uncoupled problem for an half space].

Day W.A. (1985). *Heat Conduction Within Linear Thermoelasticity*. Springer Tracts in Natural Philosophy. Vol. 30, New York: Springer-Verlag. [This presents mathematical aspects of the linear thermoelasticity and numerical solutions].

Deresiewicz H. (1958). Solution of the equations of thermoelasticity. *Proc. 3rd. U.S. National Congr. Appl. Mech.*, Brown University, 287-291. [This presents a solution of the displacement-temperature equations for isotropic and homogeneous bodies].

Duhamel J.M.C. (1837). Second mémoire sur les phénomènes thermomécanique. *J. École Polytechn*. **15**, 1-57. [This presents on purely empirical grounds, the equations of thermoelasticity isotropic bodies].

Eringen A.C. (1967). *Mechanics of Continua*. New York: John Wiley and Sons. [This presents a modern derivation of the equations of the nonlinear thermoelasticity].

Fichera G. (1972). *Existence Theorems in Elasticity*. Handbuch der Physik, vol. VI a/2 (ed. C. Truesdell), Berlin-Heidelberg-New York: Springer-Verlag. [This presents existence results in the linear elastostatics].

Galka A. (1965). Singular solutions of thermoelasticity. *Bull. Acad. Polon. Sci., Ser Sci. Techn.* **13**, 523-529. [This presents singular solutions and applications of the reciprocal theorem].

Green A.E. and Adkins J.E. (1960), *Large Elastic Deformations*. Oxford: Clarendon Press. [This work presents the equations of nonlinear thermoelasticity under some restrictive assumptions].

Gurtin M.E. (1964). Variational principles for linear elastodynamics. *Arch. Rational Mech. Anal.* **16**, 34-50. [This presents variational characterizations of the solutions of the boundary-initial-value problems of linear elasticity].

Gurtin M.E. (1972). *The Linear Theory of Elasticity*. Handbuch der Physik, vol. VIa/2 (ed. C. Truesdell), Berlin-Heidelberg-New York: Springer-Verlag. [This treatise gives an exhaustive presentations of the linear elasticity].

Hetnarski R.B. (1961). Coupled one-dimensional thermal shock problem for small times. *Arch. Mech. Stosow.* **13**, 296-306. [This studies the problem of Danilovskaya (1958) in the coupled thermoelasticity].

Hetnarski R.B. and Ignaczak J. (2004). *Mathematical Theory of Elasticity*. New York: Taylor and Francis. [This book presents a detailed analysis of the problems of linear elasticity and thermoelasticity].

Ieşan D. (1966). Principes variationnels dans la théorie de la thérmoelasticité couplée. *An. Şt. Univ. "Al. I. Cuza" Iaşi*, Matematică **12**, 439-456. [This presents variatonal theorems in linear thermoelastodynamics].

Ieşan D. (1967). Sur la théorie de la thermoélasticité micropolaire couplée. *C.R. Acad. Sci. Paris* **265** A, 271-275. [This presents the first proof of reciprocal theorem which avoids the use of Laplace transform and the assumption of null initial data].

Ieşan D. (1980). Incremental equations in thermoelasticity. *J. Thermal Stresses* **3**, 41-56. [This presents a theory of thermoelasticity with initial stresses and initial heat flux].

Ieşan D. (1987). *Saint-Venant's Problem*. Lecture Notes in Mathematics Vol. 1279. Berlin: Springer-Verlag. [This work is concerned with the problem of deformation of elastic cylinders].

Ieşan D. (1989). Reciprocity, uniqueness, and minimum principles in the dynamic theory of thermoelasticity. *J. Thermal Stresses* **13**, 465-481. [This presents a new method to derive the reciprocal theorem and uniqueness results].

Ieşan D. (2004). *Thermoelastic Models of Continua*. Dordrecht: Kluwer Academic Publishers. [This book is concerned with the basic problems of thermoelasticity for various models of continuous bodies].

Ieşan D. and A. Scalia (1996). *Thermoelastic Deformations*. Dordrecht, Boston, London: Kluwer Academic Publishers. [This work presents a theory of thermoelasticity with initial heat flux].

Ionescu-Cazimir V. (1964). Problem of linear coupled thermoelasticity. Theorems on reciprocity for the dynamic problem of coupled thermoelasticity. I. *Bull. Acad. Polon. Sci. Sér. Sci. Techn.* **12**, 473-480. [This presents the first reciprocal theorem in thermoelastodynamics].

Ionescu-Cazimir V. (1964). Problem of linear coupled thermoelasticity. IV. Uniqueness theorem. *Bull. Acad. Polon. Sci. Sér. Sci. Techn.* **12**, 565-573. [This paper presents a uniqueness result for anisotropic bodies].

Keene F.W. and Hetnarski R.B. (1990). Bibliography on thermal stresses in shells. *J. Thermal Stresses* **13**, 341-540. [This presents some contributions to the problem of thermoelastic deformation of shells].

Knops R.J. and E.W. Wilkes (1973). *Theory of Elastic Stability*, Handbuch der Physik, vol. VI a/3 (ed. C. Truesdell), New York: Springer-Verlag. [This is an important work which presents a study of thermoelastic stability].

Knops R.J. and Payne L.E. (1970). On uniqueness and continuous dependence in dynamical problems of linear thermoelastiicity. *Int. J. Solids Structures* **6**, 1173-1184. [This presents continuous dependence results in a theory of thermoelasticity with initial stresses].

Kupradze V.D. (1965). *Potential Methods in the Theory of Elasticity*. (ed. I. Meroz). Jerusalem: Israel Program for Scientific Translations. [This work presents a study of the boundary-value problems of the theory of elasticity by using the potentials of a single-layer, double-layer and the potential of mass].

Kupradze V.D. and Burchuladze T.V. (1969). Boundary-value problems of thermoelasticity [in Russian]. *Differential Equations* **5**, 3-43. [This presents a study of the theory of steady vibrations for homogeneous and isotropic thermoelastic bodies].

Kupradze V.D., Gegelia T.G., Bashelishvili M.O. and Burchuladze T.V. (1979). *Threee Dimensional Problems of Mathematical Theory of Elasticity and Thermoelasticity*. Amsterdam: North-Holland Publ. [This presents a study of stationary vibrations of thermoelastic bodies].

Lebon G. (1980). *Variational Principles in Thermomechanics*. In "Recent Developments in Thermomechanics of Solids" (ed. G. Lebon and P. Perzyna), 1-94. Wien-New York: Springer Verlag. [This work presents variational characterizations of solutions of the problems of thermoelastodynamics].

Maysel V.M. (1941). A generalization of the Betti-Maxwell theorem to the case of thermal stresses and some of its applications [in Russian]. *Dokl Akad. Sci. USSR* **30**, 115-118. [This presents the reciprocal theorem in thermoelastostatics and applications].

Mikhlin S.G. (1962). *Multidimensional Singular Integrals and Integral Equations* [in Russian]. Moscow: Nauka. [This book presents fundamental results in the theory of singular integral equations].

Muskhelishvili N.I. (1953). *Some Basic Problems of the Mathematical Theory of Elasticity*. Groningen: Noordhoff. [This presents important results in elastostatics and a method to reduce the thermoelastic plane problem to a problem of dislocation].

Navarro C.B. and Quintanilla R. (1984). On the existence and uniqueness in incremental thermoelasticity. *ZAMM* **35**, 206-215. [This presents the first existence result in the theory of thermoelasticity with initial heat flux].

Neumann F.E. (1885). *Vorlesungen über die Theories der Elasticität*. Leipzig: Teubner. [This presents, on purely empirical grounds, the equations of thermoelasticity for isotropic bodies].

Nowacki W. (1962). *Thermoelasticity*. Addison-Wesley: Reading, Mass. [This presents a comprehensive study of various problems of linear thermoelasticity].

Nowacki W. (1964). Green functions for the thermoelastic medium II. *Bull. Acad. Polon. Sci. Sér. Sci. Techn.* **12**, 465-472. [This presents the fundamental solutions of the basic equations of thermoelastodynamics in the case of steady vibrations].

Nowacki W. (1986). *Theory of Asymmetric Elasticity*. Warszawa, Oxford: Polish Scientific Publishers and Pergamon Press. [This monographs discusses some general and particular problems of thermoelastic

Cosserat continua].

Nowinski J.L. (1978). *Theory of Thermoelasticity with Applications*. Alphen aan den Rijn: Sijthoff & Noordhoff International Publishers. [This presents a study of various important problems of thermoelasticity].

Parkus H. (1968). *Thermoelasticity*. Waltham, Mass: Blaisdell Publ. Co. [This work studies basic problems of thermoelasticity].

Pipkin A.C. and Rivlin R.S. (1958). The formulation of constitutive equations in continuum physics. *Div. Appl. Math. Brown University Report.* September (6). [The authors show that the heat flux vanishes whenever the temperature gradient vanishes].

Quintanilla R. (2001). Spatial asymptotic behaviour in incremental thermoelasticity. *Asymptotic Analysis* **27**, 265-279. [This presents spatial decay bounds in incremental thermoelastodynamics for semi-infinite cylindrical domains].

Quintanilla R. and Straughan B. (2000). Growth and uniqueness in thermoelasticity. *Proc. R. Soc. London* **456**, 1419-1429. [This presents uniqueness theorems in theories of thermoelasticity with finite speed thermal waves].

Racke R. and Jiang S. (2000). *Evolution Equations in Thermoelasticity.* Monographs and Surveys in Pure and Applied Mathematics 112, Boca Raton: Chapman & Hall /CRC. [This work studies the mathematical aspects of the theory of thermoelasticity].

Rionero S. and Chiriță S. (1987), The Lagrange identity method in linear thermoelasticity. *Int. J. Engg. Sci.* **25**, 935-947. [This paper presents continuous dependence results in linear thermoelastodynamics].

Sneddon I.N. (1974). *The Linear Theory of Thermoelasticity.* Vienna: Springer Verlag. [The basic equations of the linear theory of thermoelasticity are considered along with thermoelastic waves].

Sneddon I.N. and Lockett F.J. (1960).On the steady state thermoelastic problem for the half-space and thick plate. *Quart. Appl. Math.* **18**, 145-153. [This papers is concerned with the thermoelastic equilibrium of unbounded homogeneous and isotropic bodies].

Sternberg E. and McDowell E.L. (1957). On the steady-state thermoelastic problem for the half-space. *Quart. Appl. Math.* **14**, 381-398. [This paper presents a solution of the thermoelastostatic problem for a homogeneous and isotropic half-space].

Şuhubi E.S. (1975). *Thermoelastic Solids.* In vol. II of the "Continuum Physics" (ed. A. C. Eringen), New York: Academic Press. [This work contains a study of thermoelastic continua including a theory with finite wave speeds].

Tauchert T.R. and Hetnarski R.B. (1986). Bibliography of Thermal Stresses. *J. Thermal Stresses* **9**, Supplement, 1-128. [This presents a list of papers devoted to the theory of thermoelasticity].

Thomas T.Y. (1961). *Concepts from Tensor Analysis and Differential Geometry.* New York: Academic Press. [This presents fundamental results from tensor analysis and differential geometry].

Truesdell C. (1984). *Rational Thermodynamics.* New York: Springer-Verlag. [The reader can find an extensive discussion of the assumption of the symmetry of conductivity tensor].

Truesdell C. and Toupin R. (1960). *The Classical Field Theories.* In vol. III/1 of the Handbuch der Physik (ed. S. Flügge), Berlin-Heidelberg-New York: Springer Verlag. [This treatise is the first to write the laws of thermodynamics with the heat supply terms indicated].

Weiner J.H. (1957). A uniqueness theorem for the coupled thermoelastic problem. *Quart. Appl. Math.* **15**, 102-105. [This work presents the first uniqueness result in thermoelastodynamics].

Wilkes N.S. (1980). Continuous dependence and instability in linear thermoelasticity. *SIAM J. Math. Anal.* **11**, 292-299. [This presents continuous dependemce results in the dynamic theory of thermoelasticity].

Zorski H. (1958). Singular solutions for thermoelastic media. *Bull. Acad. Polon. Sci. Sér. Tech.* **6**, 331-339. [This presents a solution of the displacement-temperature equations for isotropic and homogeneous bodies, and singular solutions].

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