STRESS CLASSIFICATION IN PRESSURE VESSELS AND PIPING

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Summary

The definitions and limits of primary and secondary stress categories are taken from the ASME Boiler & Pressure Vessel Code. Primary stresses are defined as those that develop the internal forces and moments needed to equilibrate the applied external boundary forces and moments and body forces. Secondary stresses are those that develop the internal forces and moments needed to satisfy internal or boundary constraints. These definitions are taken as the basic premise for reach of the stress categories. The general and local primary membrane, primary bending, and secondary stresses, and their limits, are presented in a way that is consistent with that premise. Limits on the applicability of the primary stress approach with respect to the wall thicknesses of pressure vessels and piping are given. Examples are provided that explain the basic concepts for the calculation of primary and secondary stresses.

1. Introduction

The main goal in pressure vessel design is to assure safe and satisfactory performance of a vessel or piping. Stress classification provides a vehicle for reaching that goal. It is recognized that different kinds of stress have different degrees of significance and must be held to different limits. The purpose of stress classification is to identify these different kinds of stress. They are placed in stress categories. The classification of stresses into primary and secondary categories separates the issues regarding overall strength, which is of primary importance and therefore referred to the realm of primary stresses, from the issues of local behavior, which is of secondary importance and therefore referred to the realm of secondary stresses.

The stress categories of interest to this chapter are those of primary stress, and its subcategories of general and local primary membrane and bending stress, and the secondary stresses. The category of peak stress relates to the assessment of fatigue failure of metals and is not included in this chapter.
In this chapter, primary stresses will be discussed first in Section 2, followed by a discussion of secondary stresses in Section 3.

2. Primary Stress

2.1. Definition

The basis of the primary stress concept is rooted in the following definition cited in the ASME Boiler & Pressure Vessel Code:

A primary stress is any normal stress or a shear stress developed by an imposed loading that is necessary to satisfy the laws of equilibrium in terms of the external and internal forces and moments.

When applied to the design of a pressure vessel, the important part of the definition is the idea that equilibrium is required between forces and moments on plane sections through the wall of a pressure-retaining boundary and not between stresses at all points through the wall. This is a plate and shell theory concept as illustrated by a plate in Figure 1. It should be noted at the outset that the use of the primary stress approach as defined above implies limitations on the thickness of the wall to which it can be applied.

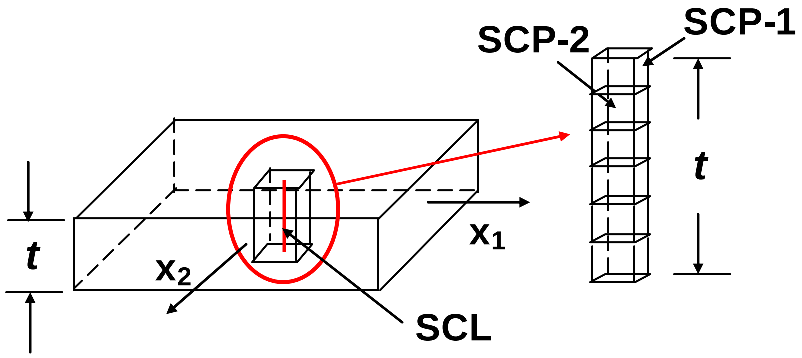

Figure 1. Stack of cubes equilibrated by internal forces and moments

Adopting the terminology of shell theory, a reference surface is defined to represent the shell. The analysis is performed with respect to coordinates on this surface, such as x_1 and x_2 in Figure 1. A normal at a point on this surface will be called in this chapter a *stress classification line*, or SCL. It defines a shell element consisting of a *stack* of infinitesimal cubes (only six shown in Figure 1) between bottom and top surfaces of the shell. The SCL also defines two *stress classification planes* on the sides of the shell element, denoted by SCP-1 and SCP-2 in Figure 1. The internal forces and moments are the membrane, bending, and shear *stress resultants* applied to these two SCPs. The primary stress definition requires that these stress resultants be in equilibrium with the external forces and moments that are applied by an external agency. In simple words, the stress resultants must satisfy the equations of equilibrium of shell theory.

The important point to note is that, according to the definition of primary stress, an SCL defines only two SCPs. There is no SCP-3 normal to the SCL. Primary stresses do not equilibrate each of the individual cubes of the stack along the SCL.

For example, if pressure is applied to the bottom surface of the shell element of Figure 1, primary stress does not recognize any through-thickness normal stress distribution within the stack because there is no external force that it equilibrates. The pressure is equilibrated by the forces on the two SCPs by the shear stress resultants and, for a curved shell, also by the membrane stress resultants. The fact that the bottom cube feels the pressure is not part of the primary stress framework. According to the above definition, the primary-stress mission is to keep only the whole stack in equilibrium, not every individual cube.

2.2. Objective of Primary Stress Approach

The objective of primary stress limits is to prevent the loss of load-carrying capacity of a vessel or piping, which will be referred to as *collapse*. The loading that produces collapse is defined as the *Limit Load* that is defined according to the rules of the theory of limit analysis. The calculation of the Limit Load for complicated geometries is difficult. For practical design purposes, a lower bound to the Limit Load (henceforth lower-bound load) is calculated. This is based on the lower-bound theorem of limit analysis, which can be stated briefly for a 3-dimensional continuum (i.e., 3-dimensional solid elements) as follows:

Theorem (1): Any stress field that is in equilibrium with the applied loading, satisfies the stress boundary conditions and prescribed body forces (i.e., gravity), and lies on or within the yield surface of the material renders the applied loading a lower bound to the

Limit Load.

In this form, the theorem applies to cases in which equilibrium of each individual, infinitesimal cube (see Figure 1) is satisfied. It can also be stated for a 2-dimensional surface with assigned thickness (i.e., beam, plate, and shell elements) in terms of the resultant forces and moments.

Theorem (2): Any field of internal forces and moments that is in equilibrium with the external forces and moments and lies on or within the yield surface written in terms of forces and moments renders the applied loading a lower bound to the Limit Load of the external forces and moments.

In this form, the theorem applies to cases in which equilibrium of only the stack of cubes shown in Figure 1 is satisfied. Equilibrium of each individual, infinitesimal cube is not required for a lower bound load. Since primary stresses are defined in terms of internal forces and moments, Theorem (2) provides the basis for the primary stress approach. Its objective is to produce a lower bound load according to Theorem (2). The following points may be noted:

1. A lower bound according to Theorem (1) applies to a structure of any geometry, including vessel walls of any thickness.
2. A lower bound according to Theorem (2) applies only to thin vessel walls and will approach the lower bound according to Theorem (1) as the wall thickness tends to zero.

This distinction will be encountered in the discussion in Section 2.7.2.2 of the calculated collapse load of a cylindrical shell by elastic-plastic, 3-dimensional continuum analysis and primary stress analysis.

2.3. Primary Stress Subcategories

After the internal membrane, bending, and shear stress resultants are determined from equilibrium, corresponding stresses across the wall, which develop these forces and moments, are calculated. This leads to the following subcategories of primary stress:

1. *Primary membrane* stress
2. *Primary bending* stress
3. *Primary shear* stress

For design purposes, the primary membrane stress is further divided into general primary membrane stress and local primary membrane stress subcategories. A *general primary membrane stress* is one that is so distributed in the structure that no redistribution of stress occurs as the result of yielding. It occurs typically remote from discontinuities and is responsible for the kind of collapse that occurs in a sufficiently long thin cylindrical shell. When the general primary membrane stress reaches the yield surface, higher pressure cannot be equilibrated and collapse occurs.

A *local primary membrane stress* is a primary membrane stress that occurs over a *local* *region*. It is of interest in pressure vessel design when it exceeds the primary membrane stress outside the local region remote from discontinuities. A typical example is a pressurized cylindrical shell with a reduced wall thickness over a part of its length, as shown in Figure 2.

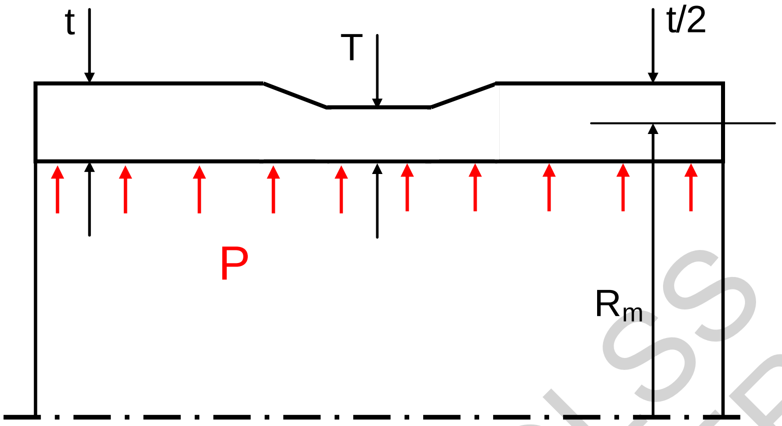

Figure 2. Illustration of occurrence of local primary stress

The length of the *local* region of reduced thickness is important. If it is sufficiently long, then the primary membrane stress inside this region will affect the collapse of the shell in the same way as a general primary membrane stress outside the local region and must be subjected to the same limit, except that the stress must be calculated used the reduced thickness of T. If it is shorter than the length limit given in design codes, then the rise in the primary membrane stress is a local detail that does not influence the collapse of the vessel using the thickness of t. However, if not limited in the latter case, it may produce unacceptable distortion of the shell caused by the reduced section. For this reason, a limit on the local primary membrane stress is applied, but it is higher than that placed on the general primary membrane stress.

2.4. Applicability

It has already been stated in Sections 2.1 to 2.3 above, that primary stress is a concept of shell theory. Furthermore, the names assigned to the primary stress subcategories in Section 2.3, membrane and bending, imply application to a special structure, namely one that has a recognizable *wall*, like that of a shell. In pressure vessel design, that would typically be a wall of a pressure-retaining boundary. All this means that the primary stress approach is applicable to reasonably thin walls, as is shell theory, perhaps no thicker than a shell with a radius/thickness ratio of 25.

For thin walls, membrane and bending stresses define adequately the deformation process, while for thick walls the through-wall normal stress (radial stress in a cylindrical shell) comes into play, which, as stated in Section 2.1, is not a primary stress. A limitation of the primary stress approach to less than a radius/thickness ratio of 25 may not be that prohibitive. Even for low strength carbon pressure vessel steels it would amount to a limit on pressure of about 7 MPa (1000 psi) for a cylindrical shell, which is likely to be higher than the pressures in most applications. The limit would be even higher for high strength steels. A back up to this argument is also given in Section 2.7.2.2 below.

It is important to note that this limit on wall thickness cannot be avoided by using 2-D or 3-D solid elements in FEA instead of shell elements. The wall-thickness limit applies to the wall geometry and not to the method or elements used in the analysis.

If the wall thickness exceeds that of the above limit, there is a practical way to deal with the assessment of collapse, which applies to any wall thickness and arbitrary geometries that have no recognizable wall. That way is to calculate a lower bound load by elastic-plastic FEA. For this FEA, elastic-perfectly plastic (EPP) material law and linear deformation theory must be used. The loading magnitude at which equilibrium can no longer be maintained, typically signaled by an error message that convergence cannot be achieved, gives the highest lower-bound load that can be obtained for the vessel by the FE model and program used in the analysis. If S_m is used for the yield stress of the EPP material law, then, according to Theorem (1) of Section 2.2, the calculated lower-bound load is a conservative allowable load for prevention of collapse.

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Biographical Sketch

Arturs Kalnins joined the faculty at Lehigh University (Bethlehem, Pennsylvania, USA) as an associate professor in 1965 after serving as assistant professor at Yale University (New Haven, Conn.) for five years. He served as professor of mechanics at Lehigh University from 1967 to 2004 and is currently professor emeritus of mechanics. His main research interest has been in the behavior and failure of metal plates, shells, and pressure vessels. He has developed the KSHEL computer programs for the analysis of shells of revolution, and has served as a consultant to many companies in the United State and abroad. Dr. Kalnins also lectured at the University of Mexico, Mexico City (1972); was a Fulbright-Hayes Fellow (1977) at the University of Innsbruck (Tirol, Austria); and gave seminars at Pennsylvania State University, University Park (1977-80), the Petrobras Company in Rio de Janeiro, Brazil (1977-78), and at AREVA NP GmbH in Erlangen, Germany (2009). He is the author/co-author of 124 research papers and one book. Within the activities of the American Society of Mechanical Engineers, Dr. Kalnins joined the Working Group on Shells in 1976 and has served as member of the Subgroup on Design Analysis/Subcommittee on Design and the Working Group on Vessels/Section III since 1987 and 2003, respectively. In this capacity, he developed new design tools for torispherical pressure vessel heads, limit load analysis, and fatigue analysis. Other contributions include three Pressure Vessels and Piping (PVP) Conference tutorials (2000, 2001, 2003) on plastic analysis, shakedown, ratcheting, fatigue analysis, and the finite element method. He has served as topic organizer and session developer at a number of PVP conferences. Dr. Kalnins was honored with two certificates of appreciation (1987, 1999), a certificate of recognition (1989) and an Outstanding Service Award (2005). He is a Fellow of ASME and the recipient of the 2008 ASME PVP Medal.