BEARING CAPACITY OF ICE

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Contents

Summary

This chapter presents the basic definitions, models, and approaches related to the problems on evaluation of the bearing (or carrying) capacity of floating ice under the action of short-term and long-term static loads, and moving loads. The question of ice cover breakage is also discussed with special attention to a possible role of the crack system (radial and arc-wise) occurring at a certain stage of ice cover loading. Several model problems for an ice beam resting on water are considered in details to illustrate the main mechanical effects inherent to ice cover at static loading, while the appropriate effects in the case of ice plates are discussed mainly qualitatively. The mechanisms of ice fracture, which have essential influence on the ice cover bearing capacity, are illustrated. Some relevant historical remarks are also given.

1. Introduction

The ice cover of rivers, lakes and offshore sea areas is increasingly used for the storage and transportation of various objects. The indigenous population of the northern regions has for a long time utilized the ice cover for on-ice transportation in the winter-spring period. Economic activities in cold climate regions led to the creation of ice crossings and roads. The world's first railway ice crossings were built in the late XIX century in

Russia. For the first time the track was laid on the ice cover of the Volga River near the town Sviyazhsk (Kazan). This operation was carried out annually from 1892 to 1913. The successful experience contributed to the creation of similar crossings in other regions, in particular, on the Volga River near Saratov (1895), Lake Baikal (1904), as well as on the Irtysh River in Omsk, on the River Cole in the construction of the Murmansk railway, etc. (e.g., Figure 1). The creation of the on-ice railway crossings were preceded by serious engineering research and development relating to both the study of the deformation and strength properties of ice and floating ice, and the mechanical behavior of ice cover under moving load (wagons rolling, locomotives pass with the full train).

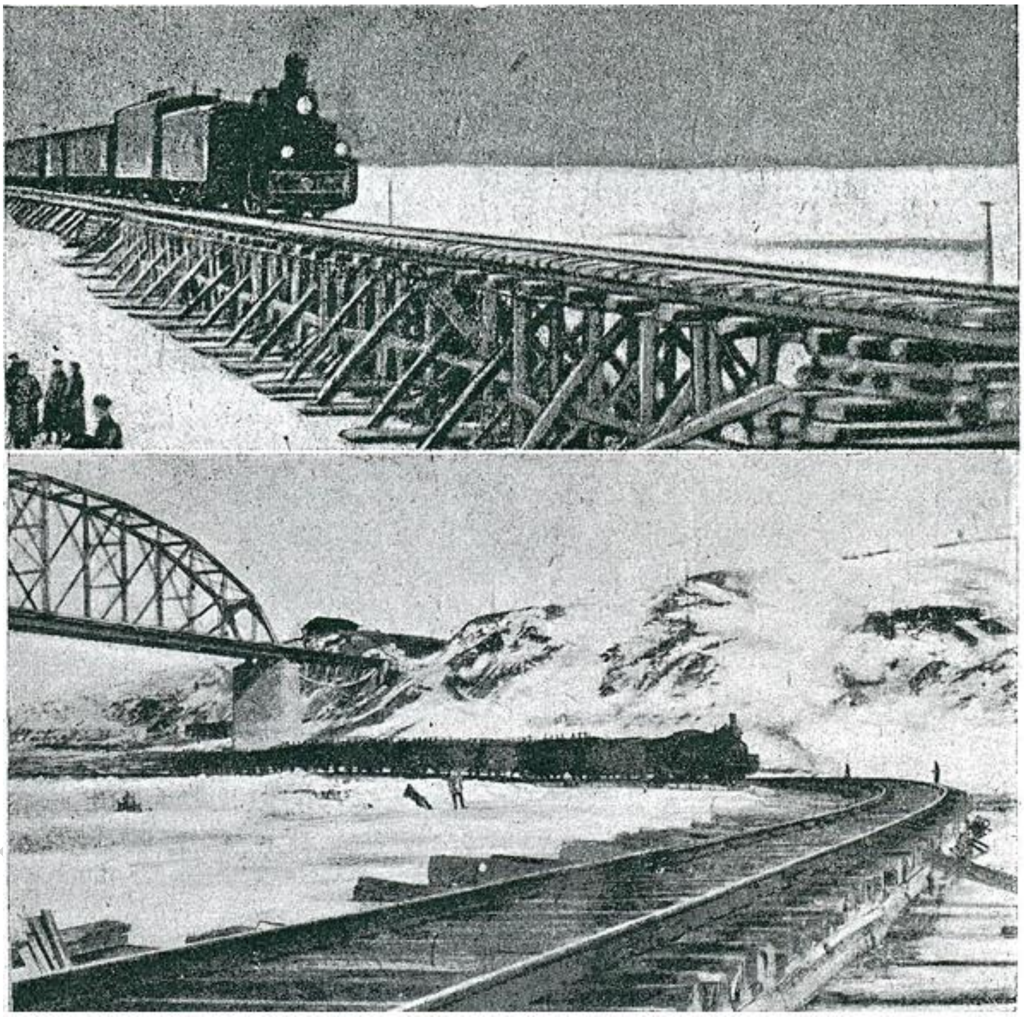

Figure 1. Railway crossings built on ice in early 1900s in Russia (Nagrodskiy, 1935).

Despite the imperfections of the testing systems used by engineers to measure the elastic moduli and tensile strength of ice in-situ and on samples, their results enabled the development of a number of schemes of laying a track on an ice cover, with a special attention to the zone of contact between the ice and the banks, and led to recommendations to the construct the railway ice crossings. E.g., the track did not go perpendicular to shore line but in a curve starting with a small angle, and cracks were cut to avoid problems with thermal expansion later on. The results of these research efforts have been published in a number of papers that have become a rarity. Among them, we note a special issue of the Proceedings of the Scientific and Technical Committee of the People's Commissariat of Transport of the USSR [Issue 84, 1929], where there are also links to other publications. In this issue of greatest interest are the papers by Sergeev (1929) and Bernshteyn (1929). Note, that in the latter paper, in particular, the results of the Hertz theory (Hertz, 1884) on the deformation of an ice plate resting on a hydraulic foundation under the action of a concentrated load on the ice surface was used. This theory became a classic and was later developed in numerous publications on the evaluation of the bearing capacity of ice cover, taking into account factors such as the geometry of the loading area, the mode of loading (short-term and long-term static loading, impacts, and moving loads), the boundary conditions on the outer contour of ice sheet, and the ice salinity with its thermodynamic characteristics.

The fundamentals and the technology of ice roads and crossings under moving loads (road, railroad, and horse-drawn transportation) were developed during the World War II, as well as the creation of on-ice landing fields for airplanes. In the history a special place is occupied by the “Road of Life”, laid on the ice cover of Ladoga Lake, which played a key role in the defense of Leningrad during the siege supplying the population with food and other essential goods (Figure 2). The results of the studies that led to creation of the “Road of Life” were described, in particular, in the books by Bregman (1943) and Golushkevich (1947), and many journal publications (e.g., Kobeko, 1946a; Ivanov, 1946; Kobeko, 1946b; Shapiro, 1942&1943).

Figure 2. The Road of Life on the ice of Lake Ladoga during the Leningrad Siege.

After the war, there were growing needs for research on ice cover for economic development of the northern territories, as well as for opportunities of storage and transport of military equipment. This stimulated extensive experimental and theoretical research on the mechanics of ice and ice sheets, aimed at developing models and methods for calculating the bearing capacity of the ice cover, and on the development of regulations for the creation of ice roads, ice crossings, ice platforms, ice islands and other ice objects exposed to loads of different nature. Significant achievements have been attained in many countries, especially in Canada, China, Finland, France, Japan, Norway, Russia, Sweden, UK, and USA.

Substantial progress in the use of a floating ice cover for support and transportation of heavy cargo has been made in Canada and the United States in the 1970s and 1980s in connection with the development of mineral deposits in Northern Quebec, the Canadian Arctic and Northern Alaska (Masterson, 2009; Mesher, 2008). Thus, according to the data given in the review of Masterson (2009), 40 oil and gas wells have been drilled from the landfast ice in the Canadian Arctic from 1974 to 1986. The longest winter ice road was 568 km long connecting Tibbitt Lake and Contwoyto Lake and crossing 65 smaller lakes in the Canadian Arctic (Mesher, 2008). This road was operated annually from 1979 to 1998 and it provided transportation of cargo to the mines and minerals extracted into the opposite direction. Interestingly, on that road the first attempt was made to use mobile radar to monitor the thickness of ice.

In recent years, interests in ice crossings, roads and islands to support drilling rigs rose again in connection with the planned development of oil and gas resources on the continental shelf of the Russian Arctic. Thus the development and improvement of regulations for the design of ice roads and the creation of ice islands continues.

In this chapter we present the status of understanding of issues related to the bearing capacity of ice cover under static (short-term and long-term) loading, and under mobile loading during cargo transportation. Furthermore, we give a discussion of the role of cracks occurring in a loaded ice cover and providing the residual load-bearing capacity of the ice cover until its breakage. Information about some of the existing regulations to evaluate the bearing capacity of ice cover is also given. Generalization of representations on mechanics of ice and ice cover, in particular, on bearing capacity of floating ice sheets, is given in (Doronin, Khesin (1975), Bogorodskiy, Gavrilo (1980), G.D.Ashton (1986))

2. Bearing Capacity of Ice Cover under Static Loads

2.1. Ice Strength under Static Loads

The bearing capacity is a property of floating ice to resist fracturing under the action of various loads. It is expressed as the maximum force (load) applied to the ice cover over a period of time when the ice sheet is able to withstand without break-through. The bearing capacity of ice cover is essentially influenced by its geometric parameters, ice properties, loading time, and the nature of the load. Usually three ice loading regimes are separated:

1) Static, when the inertial forces can be neglected;
2) Dynamic, when effects of inertia are essential, and deformations are mainly determined by the elastic properties of the ice, while the energy dissipation is caused by the inelastic properties;

3) Long-term loading mode, in which the strain is mainly determined by the time-dependent (viscous) properties of ice.

Ice cover, for most of the static problems with a relatively small short-time load, can be considered as a homogeneous elastic plate resting on an elastic foundation. We separate the carrying capacity problem into the formation of first cracks and into the full bearing capacity related to attaining a break-through load. The appearance of cracks can be dangerous in some technical applications, although the bearing capacity of the ice cover at the same time remains far from the limit. The full bearing capacity is reached when a break-through of the ice occurs accompanied by separation of the loaded region by breakthrough from the surrounding ice field.

The experiments generally confirm the feasibility of the model of a plate on an elastic foundation for the ice bearing capacity problem. If the loading changes slowly, the elastic foundation is hydraulic type (Winkler foundation), as the water pressure depends on the deflection of ice plate in the given point independent on the deflections in other points. In rapid actions, the hydrodynamic pressure field in the water drastically changes the elastic foundation, and the ice cover deformation also depends on time. If the loading time is much less than the relaxation time of ice, ice plate behavior is ideally elastic. In contrast, creep properties are inherent at long-term loading.

The time scale of each regime is determined by many factors, such as the phase composition of ice (pure ice, brine, gas), temperature, nature of the stress state (static or dynamic), etc. Capillary – porous structure of saline ice reduces its compressive and tensile strength. The filling of capillaries by brine which can transform a change of the capillary volume at compression into a side thrust, also leads to decreasing the compressive strength of ice (Goldstein, Osipenko (2010)).

At uniform horizontal compression of ice cover its effective strength does not coincide with the averaged strength of the through thickness layers because of a difference of limit strains in layers with varying brine content (Goldstein, Osipenko (2014)). The brine content in sea ice is varied in the range from 4 to 60 per cent by volume, such that the ice compressive strength decreases from 6-7 MPa to 1.5-2MPa (see Timko, Frederking (1990), Bogorodsky, Gavrilo (1980)). Most significant strength reduction occurs at the brine volume ~ 15%.

Natural ice cover is usually non-homogeneous, and its elastic properties vary spatially. At evaluation of the bearing capacity, it can be considered homogeneous, if the dimensions of the inhomogeneities are smaller than the linear scale of the ice deflection map. Larger scale non-homogeneities, for example long cracks, polynyas and hummocks, need to be taken into consideration. The anisotropy of the ice causes a little distortion to the overall picture of the impact of external forces. But in practice an ice plate is taken as a polycrystalline body, considered statistically isotropic in the horizontal plane.

Vertically, there is an essential change in the ice properties, especially for sea ice, which is characterized by a layered structure and the presence of gradients in salinity and temperature through the thickness of ice. An ice plate having this kind of anisotropy is called transversely isotropic (Anderson, 1961). Such a plate may be considered as isotropic elastic material with the effective density and stiffness calculated by integration over the plate thickness. Numerous laboratory and field experiments have shown that before the appearance of the first crack the bearing capacity of ice cover is determined by its bending (flexural) strength. Note that the ice plate deflection is essentially smaller than its thickness. The main scenarios of the bearing capacity violation at quasi-static loading can be obtained by considering a beam model, in which the ice cover is assumed to be a homogeneous isotropic elastic beam of unit width (or a plate) loaded by a vertical concentrated load or a linear load. Let us consider some of the scenarios for the stage of loading until the first cracks are formed.

2.2. An Infinite Beam under the Action of the Linear Force

Let us consider an ice beam supported along its entire length (Figure 3) by an elastic foundation (water) such that the foundation provides at each point a reaction force on the beam proportional to the deflection of the beam at the point, y . The proportionality coefficient k is the coefficient of the elastic foundation. The assumption of proportionality between the reaction and deflection represents an approach close enough to the real conditions. The coefficient of proportionality k, called as the "modulus of subgrade reaction", was introduced first by Academician Fuss in 1801 (Rzhanizin, 1982). Thus, the intensity of the reaction at each point of the foundation equals the force ky .

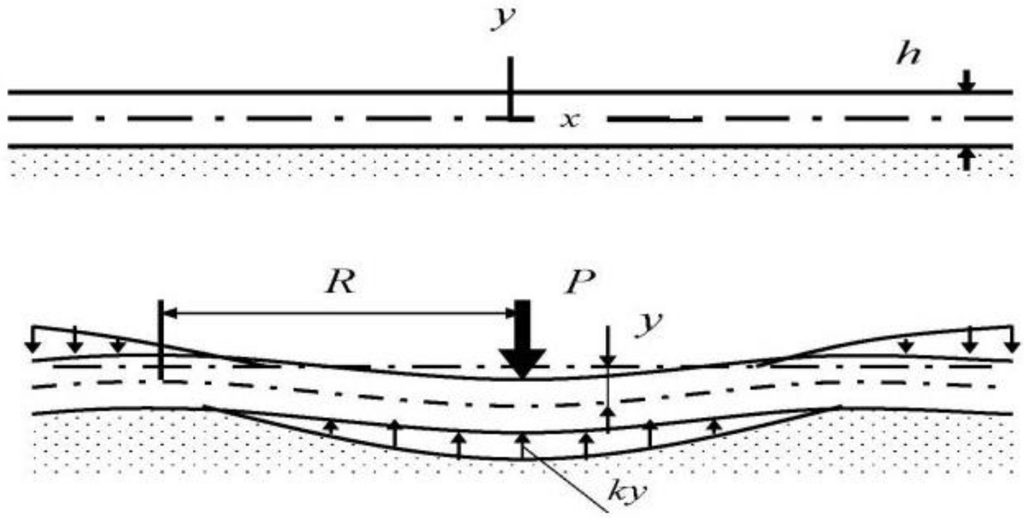

Figure 3. Bending of floating ice beam under the action of a concentrated force

The foundation provides a reaction at the beam deflections both downwards and upwards. In practice, the problem of a beam on an elastic foundation also arises in civil engineering, e.g., in design of foundations for various buildings and roads, and calculation of the load transfer to the ground. Such a beam is statically indeterminate, since the static condition related to the sum of the foundation reactions does not allow us to determine the distribution of the reactions along the beam, and hence to calculate the bending moments and transverse forces. Since the reaction rate at each point is associated with the beam deflection, for solving the problem one needs first to search for the equation of the bending line of the beam, $y = f(x)$, and then to calculate the bending moment and the transverse force. Let us obtain the equation of the bending line of a beam having a constant (rectangular) cross-section of unit width resting on an elastic foundation and loaded by a concentrated force P (Figure 3).

The origin is at the point of the force action; the *x*-axis is directed to the right, and *y*-axis is vertical, positive upward. The direction of loads action up will be considered positive. Let us write the differential equation of the curve

$$EI\frac{d^2y}{dx^2} = M(x) \ , \tag{1}$$

where $I = bh^3/12$ is the moment of inertia of ice beam of unit width ($b=1$), h is the thickness of the beam, E is the elastic modulus of ice, y is the deflection of the beam, and M is the bending moment. Since $M(x)$ is unknown, we shall use the interrelation between the deflection and load which is obtained by differentiation twice of the previous equation

$$EI\frac{d^4y}{dx^4} = q(x) \tag{2}$$

where $q(x)$ is the intensity of the distributed load acting on the beam in the section with abscissa x. This load on the beam is the reaction of the elastic foundation. Its intensity is proportional to the deflection; it is positive when the ice beam is displaced down, and vice versa. Therefore, this load has a sign opposite the sign of deflections. The coefficient of the elastic foundation equals $k = 10^4 \text{ N/m}^2$ (for freshwater, where ρ_w is the water density). Then

$$EI\frac{d^4y}{dx^4} = -ky \tag{3}$$

The general solution of Eq. (3) has the form

$$y = e^{\alpha x}\left(A\sin\alpha x + B\cos\alpha x\right) + e^{-\alpha x}\left(C\sin\alpha x + D\cos\alpha x\right) \ ; \ \alpha = \sqrt[4]{\frac{k}{4EI}} \ ; \tag{4}$$

The problem of bending of an infinitely long beam loaded with a concentrated force (Figure 4) is solved most simply. Let us define the constants A, B, C, D. Since the resulting reaction of the foundation, equal to the force P, must be finite, the deflection of the beam at points infinitely distant from the point of application of force must be zero. The second term in Eq. (4) vanishes at $x \to \infty$ because of the factor $e^{-\alpha x}$, while the first term can vanish then only if $A = B = 0$. Hence,

$$y = e^{-\alpha x}\left(C \sin \alpha x + D \cos \alpha x\right) \tag{5}$$

Since the problem is symmetric and the transverse force is known in a beam section at the point of the load application, we find that

$$C = D = -\frac{P}{8EI\alpha^3} \tag{6}$$

Hence the deflection of the beam and the bending moment are given by

$$\begin{aligned} y(x) &= -\frac{P}{8EI\alpha^3} e^{-\alpha x}\left(\sin \alpha x + \cos \alpha x\right) \\ M(x) &= -\frac{P}{4\alpha} e^{-\alpha x}\left(\cos \alpha x - \sin \alpha x\right) \end{aligned} \tag{7}$$

These solutions show fairly rapidly decaying deflection wave with the distance from the loaded zone. An analysis given by Eqs. (1-8) relates to the one represented by Hetenyi (1946) where was also corrected the expression for the deflection obtained by Biot (1937).

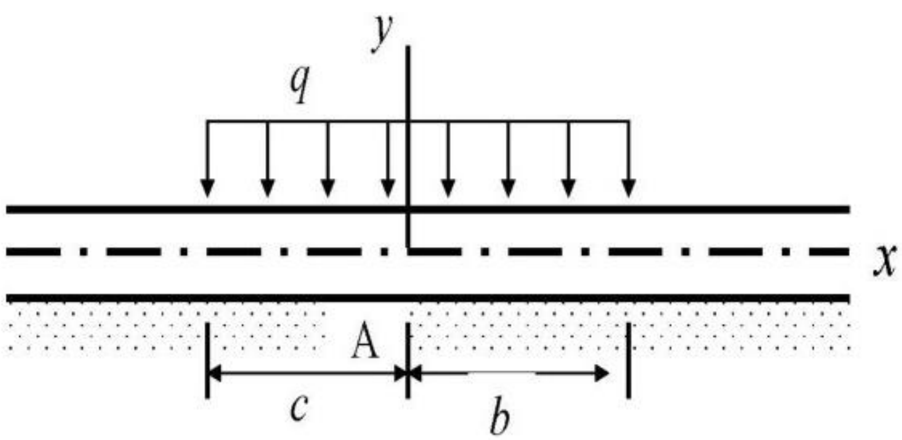

Figure 4. Bending of floating ice beam under the action of a load distributed on a finite interval

The maximum deflection and bending moment are observed under the point of the load application and are equal to

$$y_{\max} = \frac{P}{8EI\alpha^3} \quad ; M_{\max} = \frac{P}{4\alpha} \tag{8}$$

respectively. The maximum tensile bending stresses occur at the same section of the beam on its bottom surface and are equal to

$$\sigma_{\max} = \frac{3P}{2h^2\alpha} \tag{9}$$

The parameter α characterizes the relative stiffness of the ice plate. The quantity $R(3\pi/4)\alpha^{-1}$ (Figure 3) determines the size of the deflection bowl of the ice beam.

In the case of a uniformly distributed load q (Figure 4), one can write, by incorporating the law of superposition and using Eq. (7)

$$dy_0 = -\frac{qdx}{8\alpha^3 EI} e^{-\alpha x}\left(\cos\alpha x + \sin\alpha x\right)$$

$$y_0 = \int_0^b dy_0 + \int_0^c dy_0 = -\frac{q}{2k}\left(2 - e^{-\alpha b}\cos\alpha b - e^{-\alpha c}\sin\alpha b\right) \tag{10}$$

The maximum deflection and bending moment are attained in the centre of the loaded area.

It is widely believed that the bearing capacity of the ice is determined mainly by a small difference of ice and water densities that supports the ice afloat. According to the available measurements (Bogorodskiy, 1980), the density of pure ice at a temperature of 0°C is 916.8 kg/m^3, while the density of sea ice is a function of salinity and temperature and varies from 920 to 950 kg/m^3. The density of seawater is also affected by these parameters and ranges from 1020 to 1030 kg/m^3. Density difference providing buoyancy to ice is about 80 kg/m^3.

The above buoyancy mechanism is provided by displacement of water at a deflection of a loaded thin ice plate (before occurrence of through cracks). Let us estimate its role on the example of an infinite beam under the action of a concentrated load P (Figure 3). According to Eq. (7), the lifting force, P_{i}, determined by the deflection of the beam on the interval from the loading area to a distance x equals

$$P_{\mathrm{i}}(x) = -2\rho_{\mathrm{w}} g\int_0^x y(x)dx = \frac{P\rho_{\mathrm{w}} g}{4EI\alpha^4}\left(1 - e^{-\alpha x}\cos\alpha x\right) = P\left(1 - e^{-\alpha x}\cos\alpha x\right) \tag{11}$$

The second term in the brackets on the right rapidly decreases as the distance x increases.

Assuming that $E \approx 6$ GPa, $k = 10$ kPa for freshwater ice at a temperature of –10°C we obtain $\alpha = 0.0669/h^{3/4}$ (α in 1/m and h in m) for a floating rectangular beam of unit width. Figure 5 shows the change of the size of the region providing buoyancy because of the beam deflection (Eq. 11). Similar results can be obtained using the estimate of the deflection bowl, R. Bowl deflection increases with increasing ice thickness.

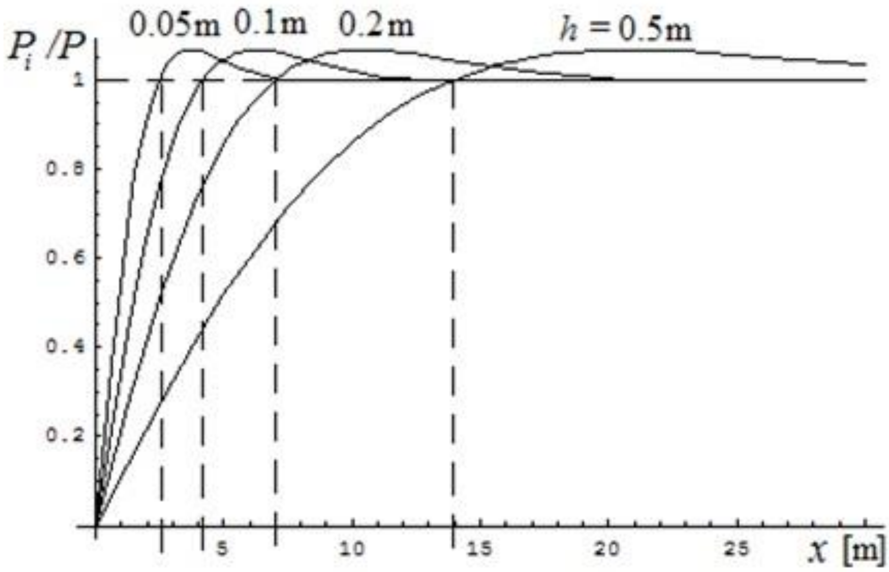

Figure 5. The lifting force created by the bending of the ice cover in the vicinity of the load. h is ice thickness and P_{i} / P is the ratio of the lifting force P_{i} to the applied force P

For this example, we estimated the bearing capacity of an ice beam to correspond to the appearance of cracks according to Eq. (9), setting $s_{\max} = \sigma_{\mathrm{t}} \approx 1$ MPa. The dependence of the ultimate load on the ice thickness is given in Figure 6. According to these estimates, the load of body weight (about 800 N) is compensated at the beam thickness of 4 cm. At this thickness, the size of the deflection bowl is about 2–2.5 m. Thus, the volume of an ice beam part supporting the load approximately equals 0.2 m^3. This volume even at full immersion creates a lifting force due to the density difference between water and ice five times less than that required.

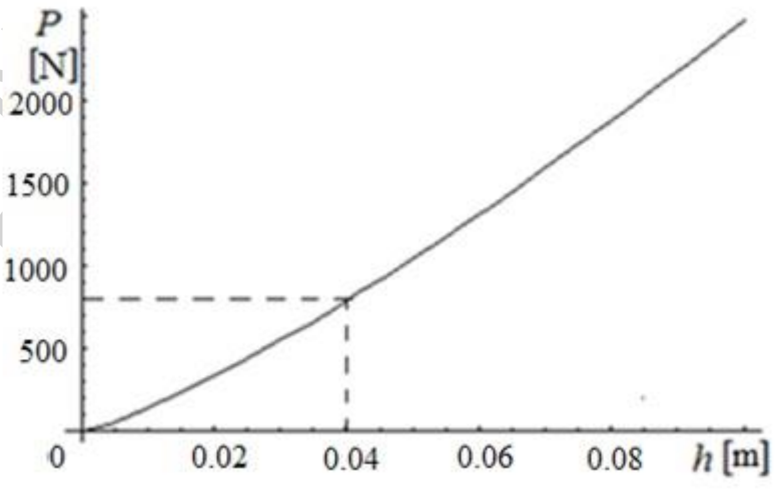

Figure 6. The maximum transverse load on the floating ice beam of unit width before the appearance of cracks.

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Masterson, D.M. and Smith, T. (1991) Creep and settlement of ice structures. A case history. In: Ice-structure interaction. IUTAM Symp. St. John's, Canada, 1989 (Ed. by S.Jones, R.F.McKenna, J.Tillotson, I.Jordaan). Springer-Verlag, 37-54. [The experience of drilling of wells by BP Canada Inc. during the late winter of 1986 in the Cold Lake region of northern Alberta, Canada, is discussed. The characteristics of drilling rigs and time are given.]

Masterson, D.M. (2009) State of the art of ice bearing capacity and ice construction. *Cold Regions Science and Technology* 58, 99-112. [Most recent review on the status of science of bearing capacity of ice]

Mesher, D., Proskin, S. (2008) Ice road assessment, modeling and management. 7th International Conference on Managing Pavement Assets 2008, pp. 1-11, Canada. [The results of measurements and observations performed on the Tibbitt to Contwoyto Ice Road, situated north of Yellowknife in Canada's Northwest Territories (the world's longest heavy haul ice road) are presented. The possibilities of using the data and experimental technique for providing the safety of transportation on ice road are discussed]

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Nevel, D.E. (1966) Time dependent deflection of a floating ice sheet. CRREL Research Report 196. [Effects of viscoelasticity of ice on time dependent deflection of ice cover under constant load are analyzed within the framework of the Maxwell model]

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Panfilov, D.F. (1961) On the determination of the carrying capacity of an ice cover for loads of long duration. Izvestiya Vysshikh Uchebnykh Zavedeniy Ministerstva Vysshego i Srednego Spetsial'nogo Obrazovaniya SSSR. *Stroitel'stvo i Arkhitektura,* 6, 47-57. [A method for accounting for nonlinear deformation of ice at evaluation of the bearing capacity of an ice cover under the action of long term loading is described. The method implies using of the appropriate empirical dependences]

Peary, R.E. (1910) *The North Pole: Its discovery in 1909 under the auspices of the Peary Arctic Club*. Frederick Stokes, New York, 373p. [Very impressive description of the Peary expedition to the North Pole]

Rzhanitsyn, A.R. (1982) *Civil engineering*. Moscow. Vysshaya Shkola. 400p. [The textbook on the Civil engineering]

Sergeev, B.N. (1929) Creation of winter crossing of wagons on the ice and behavior of a layer of ice under load. *Trudy Nauchno-Tekhnicheskogo Komiteta Narodnogo Komissariata Putei Soobshcheniya*, 84, 5-35. [The results of field experiments and engineering estimates performed for design and construction of an ice road for river crossing by railway wagons are described accounting for ice deformation]

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Shukhov, V.G. (1977) *Selected works. Civil engineering*. Ed. A.Yu. Ishlinsky. Moscow: Nauka, 193p. [A collection of selected papers by the famous Russian engineer V.G. Shukhov]

Shimansky, Yu.A. (1938) Conditional meters of ice performance of ships. *Trudy AARI*, 130, 125p. [An analytical solution of the problem on banding of a semi-infinite floating beam under the action of transverse and longitudinal loads is given]

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Steenis, K. van., Hicks, F.E., Hrudey, T.M. and Beltaos, S. (2003) Modelling creep deformation in floating ice. *Can. J. Civ. Eng.* 30, 28-41. [A two-dimensional axisymmetric finite element model was suggested to model the behavior of a homogeneous, transversely isotropic, infinite ice cover under long-term loading. The efficiency of the model was confirmed by 33 sets of long-term load test data from large-scale field experiments in different conditions]

Sodhi, D.S. (1995) Breakthrough loads of floating ice sheets. *Journal of Cold Regions Engineering*, 9, 4-22. [An analytical approximation of the critical load which determines the bearing capacity of the ice plate is given]

Sodhi, D.S. (1996) Deflection analysis of radially cracked floating ice sheets. Proc. Int. Conf. on Offshore Mechanics and Arctic Engineering. Florence, Italy, June 16-20, 4, 97-101. [On the basis of the finite element analysis of the problem of deflection of an ice sheet with radial cracks the author determined critical length of these cracks. The numerical estimates agree with the data of full-scale and small-scale experiments]

Sodhi, D.S. (1998) Vertical penetration of floating ice sheets. *Int. J. Solids Structures*, 35(31-32), 4275-4294. [The experimental results related to evaluation of the floating ice compressive strength in tests of beams with fixed ends are presented]

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Svec, O.J., Frederking, R.M.W, (1981) Cantilever beam tests in an ice cover: Influence of plate effects at the root. *Cold Regions Science and Technology* 11: 93-101. [An analysis of difference between the tensile and bending strengths of ice cover is given]

Svec, O.J., Thompson, J.C. and Frederking, R.M.W. (1985) Stress-concentrations in the root of an ice cover cantilever: Model tests and theory. *Cold Regions Science and Technology* 11: 63-73. [The effects of the stress concentration near the cantilever beam root in field tests are analyzed both theoretically and experimentally. It is shown that stress concentrations can be relieved by adjusting the geometry of the concentrators and beam]

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Timoshenko,S., Woinowsky-Krieger,S (1959) *Theory of plates and shells*. McGraw-Hill, 580p. [Classical text book on the theory of plates of shells. A representation of the stress state in a floating ice cover under the loading on a circular region is, in particular, given]

Van Steenis, K., Hicks, F.E., Hrudey, T.M. and Beltaos, S. (2003) Modeling creep deformation in floating ice. *Canadian J. Civil Engineering*, 30, 28-41. [Modeling of inelastic deformation of ice cover at a power creep law is performed]

Umansky, A.A. (1933) *Analysis of Beams on Elastic Foundation* M.: Stroyisdat.112p. [Classical book on solving the problems on beams resting on an elastic foundation]

Westergaard, H.M. (1947) New Formulas for Stresses in Concrete Pavements of Airfields, *Transaction ASCE*, (1947), 687-701. [The problem of deformation of a plate on elastic foundation is solved]

Wyman, M. (1950) Deflections of an infinite plate, *Canadian Journal of Research*, Section A, vol. 28, №3, 293-302. [One of the first largely cited works on the deflection of floating ice in western literature]

Zubov, N.N. (1942) *Basics of road design at the ice cover.* Gidrometeoizdat, 74p. [The methods for calculation of ice cover strength under the action of a surface load are given accounting for an influence of ice thickness temperature, salinity of the ice plate and snow cover. Various timbered constructions aimed at increasing the ice cover bearing capacity are discussed]

Zuev, V.A., Gramuzov, E.M., Knyazkov, V.V. (1982) Bearing capacity of the ice cover under shear stress. In interuniversity collection: *Theory and strength of icebreaking ship*. *Gorky*, 5-13. [The question on the bearing capacity of ice cover is considered on model problems from the point of view of its fracture by ice breaking ships]

Biographical Sketches

Robert V. Goldstein (born in 1940 in Moscow, Russia) received PhD degree in physical mathematical sciences (the Rayleigh waves resonance phenomena in elastic bodies) in 1968 from the Institute for Problems in Mechanics of the USSR Academy of Sciences and degree of Dr. of Physical Mathematical Sciences (fracture mechanics of large scale structures) in 1983 from the Institute for Problems in Mechanics of the USSR Academy of Sciences. From 1965 till now he is working in the same Institute (last 25 years as the Head of Laboratory on Strength and Fracture of Materials and Structures). One of his main research fields is mechanics of ice and ice cover.

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Nikolai M. Osipenko (born in 1942 in Ilansk, Russia) received PhD degree in technical sciences (fracture toughness of rocks) in 1972 from the Institute of Physics of Earth of the USSR Academy of Sciences. He is working in the Institute for Problems in Mechanics of the USSR Academy of Sciences as senior researcher since 1980. His main area of scientific interests is related to fracture mechanics and mechanics of ice and ice cover.