OCEAN WAVES AND SEA ICE

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Contents

Summary

At the time when we begin writing this chapter, Arctic summer ice is experiencing the lowest areal coverage since the beginning of the satellite record. The depletion of sea ice has far exceeded any model predictions. The reduction of sea ice presents unknown environmental threats even without further human activities in the region. Yet accelerated human activities are inevitable under the pressure for resource development. Among which, the northern routes have already opened for shipping between the Atlantic and the Pacific Ocean. To better manage the Arctic Ocean, synthesis of knowledge is required to prepare predictive tools for evaluating future evolution of this region.

One of the most obvious changes in the warming of the Arctic is the increased open water, especially in the summer. Wind over open water generates waves. The greater the distance that wind can travel, the longer and more intense the wave becomes. In the past, global wave models ignored the Arctic Ocean completely, due to the lack of open water. Now, both the reality of its presence and our need to know its consequence can no longer allow the absence of reliable wave information in this region. Furthermore, as a material, ice is not a rigid cover. It can be manipulated mechanically by the wave action to fracture and raft. The formation of ice cover from supercooled water is also quite different in a wave field than in a quiescent water body. Wave and ice are truly interactive entities.

There is a large body of information concerning the theoretical development of waves under ice covers, particularly in the recent couple of decades. Comparatively, field,

remote sensing, and laboratory studies of this topic are few. On the other hand, the effect of waves on forming and reforming the ice cover is a much less studied topic. In this chapter we will review the most basic theories of waves under ice covers to provide a foundation for those who desire to explore the recent developments. A number of direct observations from field, remote sensing, and laboratory studies of waves under an ice cover will be discussed. The effect of waves on ice is introduced in the second half of the chapter. Some perspectives of this field are given at the end in the Conclusions.

While this chapter covers aspects of wave and ice interactions, there is another chapter in this EOLSS collection under the *Oceanography* Theme which covers more broadly many other issues on sea and ice interactions (Weber, 2008).

1. Introduction

Ocean waves are fascinating. They appear to be perpetual, random, forever changing. But, they are also one of the most fundamental types of mathematical problems. In fact, after removing the nonlinear effects which are often small in most practical cases, mathematically waves are surprisingly simple, elegant and entirely predictable. These predictions from the linearized theory replicate observations with impressive accuracy. Stoker (1957) is an excellent reference of this subject. A schematic of a wave field consisting a moving water body and the atmosphere is depicted in Figure 1. The water flow is assumed incompressible and irrotational. The fluid viscosity is ignored. The governing equation of ocean waves is the Laplace equation

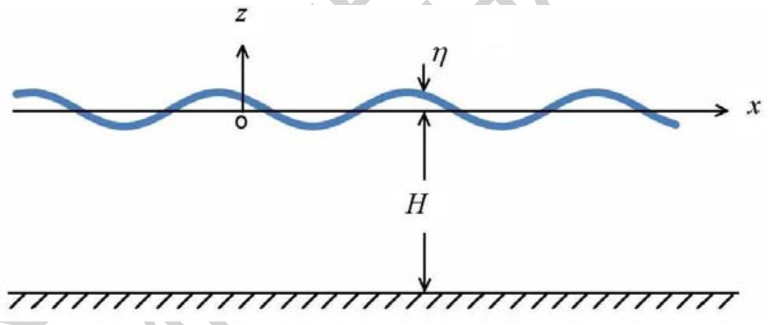

Figure 1. Schematic of a wave field.

$$\nabla^2 \phi = 0 \qquad (1)$$

where ϕ is the velocity potential such that the water particles under the wave motion is described by

$$v_x = -\frac{\partial \phi}{\partial x}, v_y = -\frac{\partial \phi}{\partial y}, v_z = -\frac{\partial \phi}{\partial z} \qquad (2)$$

The coordinate system has the z axis opposite to gravity. The surface profile $\eta(x, y, t)$ is related to the velocity potential through the physical constraint that the water velocity in the vertical direction must be the same as the velocity of the surface profile,

$$-\frac{\partial\phi}{\partial z}=\frac{\partial\eta}{\partial t}-\frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x}-\frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y},\qquad z=\eta\left(x,y,t\right) \tag{3}$$

The above is called the “kinematic” surface boundary condition which means the water particle on the wave surface moves up and down with the surface profile. The “dynamic” surface boundary condition comes from the pressure balance at the air-water interface, i.e. the Bernoulli equation

$$-\frac{\partial\phi}{\partial t}+\frac{p_\eta}{\rho}+\frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right]+gz=0,\quad z=\eta\left(x,y,t\right) \tag{4}$$

where ρ_η is the water pressure at the surface, ρ is the water density. At a horizontal sea bed the vertical velocity must vanish to satisfy the rigid impervious boundary condition,

$$-\frac{\partial\phi}{\partial z}=0,\qquad z=-H \tag{5}$$

In general, due to the nonlinearity in the boundary conditions at the free surface, the above system of equations cannot be solved analytically. Linear wave theory is thus developed under the assumption that the ratio of wave amplitude to wavelength is infinitesimal, hence all nonlinear terms may be dropped. Under this assumption (3) becomes

$$-\frac{\partial\phi}{\partial z}=\frac{\partial\eta}{\partial t},\qquad z=0 \tag{6}$$

and (4) becomes

$$-\frac{\partial\phi}{\partial t}+g\eta=0,\qquad z=0 \tag{7}$$

in which the atmosphere pressure is taken as zero. Assuming a sinusoidal solution in time, the Laplace equation is solved using the standard separation of variables technique, which gives the elemental solutions in terms of the wave number k and angular frequency ω

$$\phi\left(x,z,t\right)=\left(Ce^{kz}+De^{-kz}\right)e^{i\left(kx-\omega t\right)} \tag{8}$$

In which x is defined as the direction of a propagating planar wave, hence the variation in the y direction vanishes. Applying the sea floor condition (5), C and D are related to combine into

$$\phi\left(x,z,t\right)=B\cosh k\left(H+z\right)e^{i\left(kx-\omega t\right)} \tag{9}$$

The dynamic free surface boundary condition (4) serves to relate the wave amplitude A defined by the surface profile

$$\eta = Ae^{i(kx-\omega t)} \tag{10}$$

and the coefficient in the velocity potential

$$B = \frac{igA}{\omega \cosh kH} \tag{11}$$

Finally, the linearized kinematic free surface boundary condition (3) provides the dispersion relation between the wave frequency ω and the wave number k

$$\omega^2 = gk \tanh kH \tag{12}$$

Thus the wavelength $L = 2\pi/k$ and the wave period $T = 2\pi/\omega$ are directly related. The "group velocity", i.e. the speed of wave energy propagation can be obtained by calculating $c_{\mathrm{g}} = \frac{\partial \omega}{\partial k}$. The phase velocity (or celerity) defined as $c = L/T = \omega/k$ is the apparent speed of the wave crest (or trough). In general, $c_{\mathrm{g}} < c$ and only when water depth approaches 0 these two speeds approach each other.

For water of depth greater than half the wavelength, one may approximate the dispersion relation so that

$$L = \frac{g\mathrm{T}^2}{2\pi} = 1.56\mathrm{T}^2 \tag{13}$$

Another useful information is the dynamic pressure inside the water body. Because the fluid is assumed irrotational and inviscid, (4) is valid everywhere. Dropping the nonlinear terms in (4) we obtain the total pressure at any depth z as

$$p = -\rho g z + \rho g A \frac{\cosh k(h+z)}{\cosh kh} e^{i(kx-\omega t)} \tag{14}$$

The second term on the right is called the "dynamic pressure". It asymptotically approaches zero towards the bottom of the sea.

In the field, ocean waves are a combination of many such components which form a continuous spectrum. The energy of each band of component may change due to local wind stress, due to nonlinear interactions between different bands of waves, and due to dissipations such as interactions with the boundaries. Many of these detailed processes for open water waves are still under investigation. The main problem is to expand the theory so that nonlinearities can be dealt with, since most of the practical problems are related to these nonlinear terms.

2. Ice Covers in the Marginal Ice Zone and Basic Models

Before describing how to include an ice cover in the wave theory, we first survey how varied an ice cover can be. Figure 2 shows an example of the entire Arctic as viewed from space. It is difficult to detect any details at this distance, but it is already apparent that the surface texture is significantly heterogeneous. A close-up view of the sea ice may be obtained from ships, helicopters, or airplanes. Figure 3 shows a collection of some of these observations. The composition of sea ice covers differs both temporally and spatially.

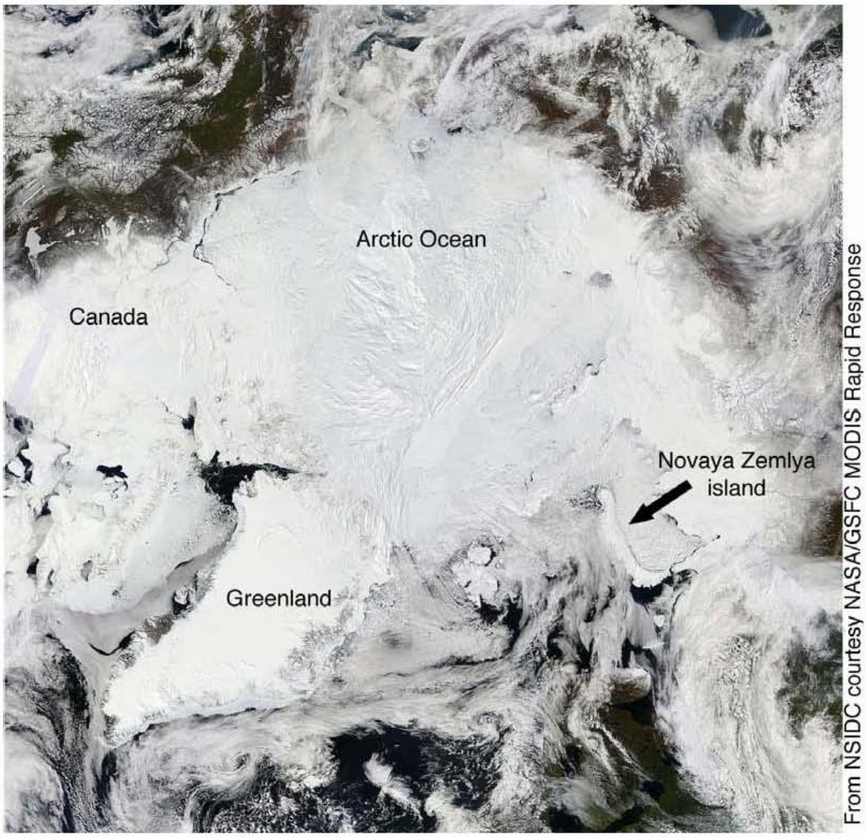

Figure 2. An image of the Arctic ocean taken on May 25, 2009 by the MODIS sensor on the NASA Terra Satellite. (Image/photo courtesy of the National Snow and Ice Data Center, University of Colorado, Boulder.)

Figure 3a. New pancake ice a wave field. Photo taken some time in the 1980s near 63^{o}s 55^{o}e in the Southern ocean. (Courtesy of the Australian Antarctic Division.)

Figure 3b. Arctic sea ice from a 2012 Operation IceBridge aerial survey. Varying thicknesses of sea ice are shown here, from thin, nearly transparent layers to thicker, older sea ice covered with snow. (Courtesy of the National Snow and Ice Data Center, Credit: NASA.)

Figure 3c. A broken ice sheet. Photo taken in 2003 during the ARISE Program from Aurora Australis (Courtesy of the National Snow and Ice Data Center, Credit: Rachel Marsh.)

Figure 3d. Aged broken ice field. Photo taken in 2012 in the Southern Ocean. (Credit: Steve Ackley.) or use A photo of ice floes interspersed with pancake ice. From Healy in the Greenland Sea on a trans-Arctic voyage. (Credit: Don Perovich.)

When newly formed in a quiescent environment, such as in narrow leads from cracked up large ice sheets, the ice cover is smooth. This type of ice is rare in the ocean. Near the ice edge waves agitate the water surface, where a different process called the "pancake ice cycle" takes place (Lange et al., 1989). Ice crystals that form at the air-water interface agglomerate first into a soupy consistency called "grease ice". The accumulation of grease ice eventually freezes into pancake ice which continues to grow in size until waves attenuate sufficiently so that a continuously frozen ice sheet may form. Grease ice obtained its name from the fact that this slurry sheet damps out high frequency waves, renders the surface a smooth appearance similar to a layer of grease on top of water. Pancake ice is named after its resemblance of pancakes. This type of ice is formed after sufficient accumulation of grease ice forces the top layer into air much colder than the water below. The exposed surface freezes. Under the wave agitation the freezing process is limited by the internal stresses that exceed the frozen bonds (Shen et al., 2004). The wave induced collisions among neighboring ice floes erode the rough corners of the floes to form the strikingly circular shape with nearly uniform size. As the wave energy damps out by the existing pancake ice field, these circular floes freeze together. The formed ice sheet keeps growing through both thermodynamic and mechanical transformations that change its physical composition: thermal growth from frozen water underneath, melting and refreezing snow from above, sea water flooding and freezing on top, fracturing due to wave bending, and rafting and ridging due to the external stress field. These processes change the physical properties of an ice cover throughout its entire lifecycle. The mechanical property of an ice cover depends on its physical composition as well as the temperature and salinity. For the same ice cover, in general, the colder it is the more rigid it is. This rigidity also increases with reduced salinity. Hence young sea ice covers are less rigid than the multi-year ice covers. A valuable resource for viewing different types of sea ice covers is the CD-ROM produced by Worby (1999).

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Biographical Sketches

Hayley H. Shen received her B.S. degree in mathematics from the National Taiwan University in 1972, and a Ph.D. in applied mathematics from the University of Iowa in 1976. All of the mathematics was intended to prepare her for her real interest: the physical laws behind the universe. After completing a Ph.D. in Engineering Sciences in 1982 from Clarkson University she began teaching and doing research in the Civil and Environmental Engineering Department at Clarkson University up to the present.

Her research areas include granular materials, in particular, the mechanical laws governing moving granular materials, and sea ice, in particular, the rheology of fragmented ice fields and wave-ice interaction. She started her study in cold regions problem during her visit at the US Army Cold Regions Research and Engineering laboratory in 1983, when the marginal ice zone was intensely investigated under the MIZEX campaign. Her first project in cold regions was to apply the granular materials knowledge to determine the constitutive laws for moving and deformation fragmented ice fields typically found in the marginal ice zone. From there, her work expanded to wave induced ice movement, attenuation of wave energy due to ice interactions, formation of pancake ice, and limiting size and thickness of pancake ice fields. She participated in field studies in the Greenland Sea in 1991 with researchers from the Norwegian Polar Institute and on the Ross Island in 1994 with researchers from the Otago University.

Dr. Hayley H. Shen has been fortunate to have had collaborations with many international scientists and engineers around the world, visited outstanding institutions in many countries. The intellectual journey into the cold regions through these colleagues world-wide has been exhilarating. The physical journeys into the cold regions have been humbling. Dr. Shen is a member of the American Geophysical Union, the International Association of Hydraulic Research, and the Engineering Mechanics Institute.