MATHEMATICAL MODELS OF PHYSICAL RELIABILITY THEORY

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Keywords: reliability, structural safety, structural integrity, safe domain, safety factor, load-strength model, cumulative model, Poisson's model, random process, up-crossing, down-crossing.

Contents

Summary

Fundamentals of reliability theory applied to mechanical engineering systems are discussed. The so-called physical reliability theory is developed to include into the consideration causes and effects of mechanical failures such as damage, fracture, and fatigue. Both time-independent models and time-dependent models based on the theory of random processes are discussed.

1. Failures in Mechanical Systems

Design of machines and structures is one of the oldest applications of engineering sciences. Unique engineering systems such as the long span bridges were designed and constructed during the last two centuries demonstrating not only a high level of engineering decision but also a high accuracy of structural analysis. This accuracy was provided with successful development of elasticity theory, structural mechanics and other branches of applied mechanics.

The primary elements of reliability theory could be found in the classical methodology of safety factors. The ratio of the design strength r to the design load s presents, in a certain degree, the reliability level of a structural component. These design parameters are in fact the fractiles of random variables, even if the probabilities of their occurrence are not discussed at all. An understanding of the probabilistic nature of safety factors was possible only in the first third of the 20th century. In this connection, the names of M. Maier, N.F. Khotsialov and N.S. Streletsky have to be mentioned. Their ideas were developed after World War II. Many norms and standards, especially in civil engineering, are based, at least implicitly, on the probabilistic interpretation of safety factors. In particular, the partial safety factors corresponding to various random variables were introduced to substantiate the fractile interpretation in codified design. The next step was taken at the end of the 1950s when the theory of random processes was primarily applied to reliability problems in combination with the concepts of system reliability theory.

Mechanical engineering systems are usually composed both of purely mechanical and electrical or electronic components. The reliability evaluation for non-mechanical parts may be performed within the framework of system reliability theory. Physical reliability theory is to be employed when we consider the mechanical parts of a system.

The behavior of engineering systems essentially depends on their interaction with the environment as well as on the character and intensity of the operating processes. To predict reliability and durability of machines and their parts, it is necessary to consider the processes of deformation, fatigue, wear, corrosion, damage accumulation and fracture under loading, temperature, and other actions varying in time. To evaluate reliability measures of a machine as a whole, it is not sufficient to know reliability measures of its elements. Many machines and mechanical engineering systems are too large, produced in small amounts, or too expensive to expect that a sufficiently reliable statistical information can be obtained from laboratory or field tests. It is the reason why reliability analysis in mechanical engineering is mostly based on the models of mechanics of materials and structures with the use of statistical data concerning material and action properties.

There are many modes of mechanical failures. One of them is fracture, i.e. the formation and development of cracks until the dimensions which are considered unsafe from the viewpoint of structural integrity. Other modes are fatigue, corrosion and wear. Sometimes the occurrence of excessive elastic or elastic-plastic deformations is considered as a mechanical failure. Excessive vibrations and noise phenomena also are the modes of mechanical failure.

The current state of mechanics of materials and structures as well as of numerical methods allows prediction of the behavior of mechanical systems with comparatively high confidence if the input data are sufficiently reliable. The material and action properties are considered as random in the reliability theory of mechanical systems. Therefore, the behavior of an item is presented as a random process.

The conditions of normal, up state operation of a system are formalized as restrictions on the parameters of the system behavior. Failures and attainments of limit states are interpreted as first excursions of a random vector characterizing the current state from certain up state domain into the space of down state parameters. This domain is frequently referred to as the *safe domain,* and the reliability with respect to the failures of mechanical nature is named *structural safety*. By the way, the latter term is frequently used in civil engineering in the sense of reliability with respect to mechanical failures. Perhaps, this term arises from the term *safety factor* that is until now present in many norms and standards. On the whole, the modern reliability theory for structures and mechanical engineering systems is based on the synthesis of the mechanics of solids and structures and the theory of random processes.

2. General Concepts of the Physical Reliability Theory

Consider a mechanical item functioning and interacting with other items and the environment. The current state of the item at any time instant t is described by the set of parameters forming the state vector **u**. In contrast to analytical mechanics, the terms *state* and *state parameter* are applied not only to generalized displacements but to any variables characterizing the quality of item's performance. The state vectors are considered as the members of a certain space U termed as *state space*. The dimension of this space depends on the choice of the item's scheme and the more detailed schematization is used, the higher is the dimension. The time t could be not only the calendar time, but also the operating time or any other parameter to count the current age of an item. For simplicity, we assume later that t is a continuous parameter with magnitudes from the segment $[t_0, t]$. Usually one assumes $t_0 = 0$. Any history of the item is characterized by a trajectory $\mathbf{u}(t)$ in the space U.

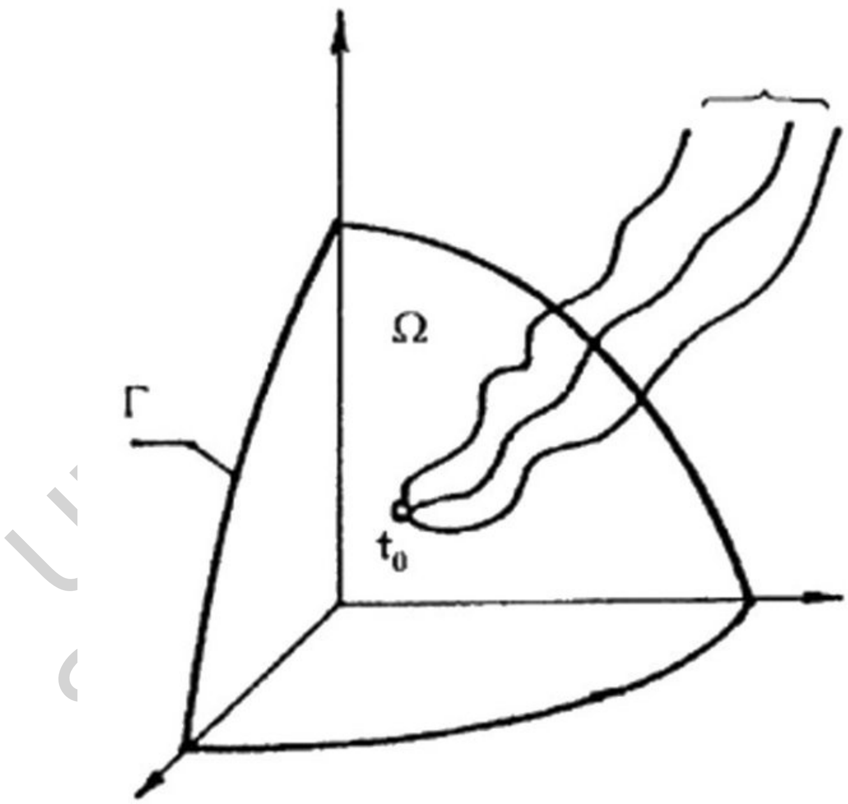

Figure 1. A failure as the first excursion of a random process out of an admissible domain.

The requirements of reliability, efficiency, and safety of an item result in certain restrictions put on the components of vector **u**. The bounded open set of admissible magnitudes of the vector **u** form the domain Ω in the state space U. The term *safe domain* is frequently used along with the term *admissible domain*, although the excursion out of this domain not necessary signifies the loss of safety. When this domain is given with the inequality $g(\mathbf{u}) > 0$, the function $g(\mathbf{u})$ is named *state function*. The boundary Γ of the admissible domain is usually named *limit surface*. The states on this surface not necessarily correspond to the limit states when the standard terminology of system reliability theory is applied. In general, one ought to be attentive when the terms from civil engineering are transferred to other branches of engineering, and vice versa.

The behavior of an item on the design stage is considered as a random process. Therefore, the trajectory $\mathbf{u}(t)$ in the state space U is random, and the first excursion of the process $\mathbf{u}(t)$ from the admissible domain Ω is a random event. This excursion is to be interpreted as the first failure (Figure 1). By silent agreement, it is usually assumed that at $t = t_0$ the state of the item belongs to the admissible region. Hence, the probability of non-failure operation of the item in the segment $[t_0, t]$ is equal to the probability that the vector $\mathbf{u}$ stays in the admissible region during this segment:

$$R(t) = \mathbf{P}\left\{ \mathbf{u}(\tau) \in \Omega;\ \tau \in \left[t_0, t \right] \right\}. \tag{1}$$

The same equation may be re-written in the terms of state function as follows:

$$R(t) = \mathbf{P}\{ g[\mathbf{u}(\tau)] > 0;\ \tau \in [t_0, t] \}. \tag{2}$$

As an example, consider an instrumentation block in a container subjected to random vibration. Both the displacements of the block with respect to the container, and its absolute accelerations may be dangerous (or at least undesirable) from the viewpoint of the item's operation. Denote the field of the relative displacements $\mathbf{u}(\mathbf{x},t)$ and the field of absolute accelerations $\mathbf{a}(\mathbf{x}, t)$. Let the restraints on the displacements and accelerations are given with limit magnitudes u_* and a_*. Then the admissible domain is defined as

$$|\,\mathbf{u}(\mathbf{x},t)\,| < u_*, \quad |\,\mathbf{a}(\mathbf{x},t)\,| < a_* \tag{3}$$

Here $\mathbf{x}$ is the reference vector for the points in the volume occupied by the block. In this example, the state vector in equation (1) is a function both of time t and the reference vector $\mathbf{x}$. It means that the state is characterized by a random field, and the problem of reliability becomes a problem of the applied theory of random fields. However, if the restrictions are put only on the displacements and accelerations at certain points of the block $\mathbf{x}_1, \ldots, \mathbf{x}_m$, inequalities (3) take the form:

$$\begin{aligned} &|\,\mathbf{u}(\mathbf{x}_k, t)\,| < u_*, \\ &|\,\mathbf{a}(\mathbf{x}_k, t)\,| < a_*, \quad k = 1, 2, \ldots, m \end{aligned} \tag{4}$$

In the latter case, the probability (1) of non-failure operation is introduced as follows:

$$R(t) = \mathbf{P}\left\{ \begin{array}{c} \sup\limits_{\tau} |u(x_k, \tau)| < u_*, \ \sup\limits_{\tau} |a(x_k, \tau)| < a_* \\ k = 1, 2, ..., m; \quad \tau \in \left[t_0, t \right] \end{array} \right\} \qquad (5)$$

Note that the conditions in Eq.(4) result in a non-analytical safety function $g(\mathbf{u})$. Therefore, the use of the definition given in (2) could appear more difficult than the direct definition (1) without any limitations on the analytical properties of the admissible domain.

Similar to (1) and (2), the reliability measures for repairable items are introduced. Any failure is interpreted as an excursion from the admissible domain, and any recovery as a return to this domain. The reliability of non-failure operation of an individual item is introduced in the same way. The information on the item's behavior during its previous life has to be taken into account in this analysis. Let t_k be the last observation instant, and $\mathbf{u}(t_k) \in \Omega$. Then the probability of non-failure operation in the segment [t_1, t] is defined as

$$R(t \mid T_1) = \mathbf{P}\{\mathbf{u}(\tau \mid T_1) \in \Omega; \tau \in (t_1, t]\} \qquad (6)$$

Here symbol T_1 denotes the amount of diagnostic information accumulated on the segment [t_0, t_1]; the vertical line is the symbol used for conditional probabilities.

The above concepts are applicable for components and blocks of engineering systems as well as to large engineering systems, including life support and infrastructure systems. The nature and significance of failures may be various. Some failures are only obstacles to an efficient use of a system, other failures result in a temporary interruption of the service, third ones correspond to the attainment of limit states (in the sense of the summed service life). The components of a system usually interact in the sense of the system reliability theory (see *System Reliability Analysis*).

Let the boundary Γ be consisted of several boundaries with equations $g_k(\mathbf{u}) = 0$, $k = 1, \dots, m$. If the elements form a system in series configuration, the system fails even if one of the elements fails. In this case the admissible domain Γ in equation (1) is given by conditions $g_k(\mathbf{u}) > 0$, $k = 1, \dots, m$ (Figure 2a). In the case of parallel configuration the domain of *non-admissible* states is defined by conditions $g_k(\mathbf{u}) < 0$, $k = 1, \dots, m$. This case is illustrated in Figure 2b.

The evaluation of the probability of non-failure operation (*reliability function*) may be considered as the central problem in reliability theory. Most of other reliability measures are connected with the reliability function $R(t)$. To assess the reliability of a system at the design stage, it is necessary to compare the evaluated measures with the specified ones. If the admissible domain in equation (1) corresponds to the limit states in the sense of lifetime expiration, equations (1) and (2) allow evaluation of the lifetime distribution function, lifetime fractiles, and other durability measures. The solution of the problem opens a way for the multi-factorial analysis of engineering systems. This analysis may include finding optimal parameters of a system, optimal service and maintenance regimes, etc.

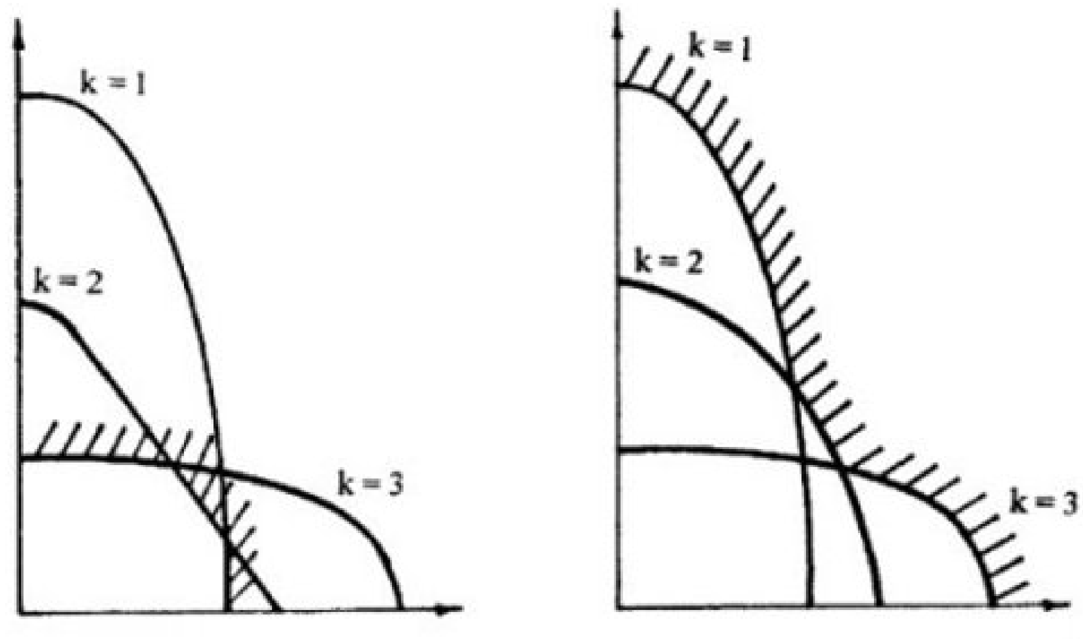

Figure 2. Admissible domains in the state space: (a) series configuration; (b) parallel configuration.

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Biographical Sketch

Bolotin Vladimir V. was born on March 29, 1926, Tambov, Russia. He graduated from the Moscow Institute of Railway Engineers as the Civil Engineer (Bridges and Tunnels) in 1948. He received from the same Institute the Degree of Candidate of Sciences in 1950 and the Degree of Doctor of Sciences in 1952.

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