HISTORY OF MEASUREMENT THEORY

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Contents

Summary

The history of Measurement Theory (MT) can be divided into two periods, the Formation Period and the Mature Theory, where Suppes' foundational work marks the transition from one to the other. The first period consists of two different yet complementary research traditions: the work of Helmholtz, Hölder and Campbell on axiomatics and morphisms on one side, and the studies by Stevens and his school on scales, transformations and invariance on the other. These two lines converge in Suppes' foundational paper "A set of independent axioms for extensive quantities", with which the second, mature, period starts. After this foundational paper MT develops and expands in different ways until it reaches its canonic exposition in the three-volume summa *Foundations of Measurement*. Before delving into the history of MT, we begin with a clarification of some conceptual issues in order to elucidate the nature and goal of MT.

1. Measurement and Measurement Theory

1.1. Measurement as Numerical Representation

Measurement is the assignment of numbers to objects in order to numerically represent properties they have. Importantly, not every property can be numerically represented; only properties called *magnitudes* or *quantities*, which are capable of "more or less" instantiation, can be represented in this way. These are "gradual" properties, i.e. ones that can be exemplified to different degrees; standard examples are *being heavy*, *being large* or *being hard*. If we have two heavy objects, it makes sense to ask which one is heavier than the other; and the same applies to large rods or hard metals. This contrasts with properties like *being a tiger*, or *being a proton*, which are "all or nothing" properties, i.e., ones that cannot be instantiated to different degrees. If we have two protons it does not make sense to ask which one is more proton than the other.

Measurement provides a more fine-grained way of conceptualizing the world than classificatory or comparative concepts; actually it provides *the most fine-grained way*. Suppose you have a diamond in front of you. It is small, sparkling, light, hard, cold and beautiful. If you are asked to be more precise, you could refine your qualitative classificatory concepts and say that it is *very* small, *quite* light and cold, *very, very* hard, and *extremely* beautiful. You could carry on, but however much you refine your adjectives, it seems that you will always be able to do just a little bit better (in almost all cases). By contrast, as soon as you give the measures of the diamond's properties you can no longer be asked to be more precise. Nevertheless, as we know, this is not possible for all its properties. You can say that the diamond's volume is x, its mass y, its temperature z, and even that its hardness is w, but not (up to the now) that its beauty is u. The question is: *why not?*

You also have a piece of chalk in front of you. It is small, dull, light, soft, cold and ugly. Now you can compare it with the diamond relative to the gradual properties they share. Both pieces are small, light, cold and hard, although not to the same degree: the diamond is smaller, lighter, colder and harder. Now you can also be more precise and give, if possible, the chalk's measures: its volume x', its mass y', its temperature z' and its hardness w'. It is worth stressing that, for these measurements to make sense, the numbers assigned must be coherent with the previous qualitative comparisons. That is, if the diamond is qualitatively smaller tan the chalk, then the number x assigned to the diamond's volume must be smaller than the number x' assigned to the chalk's volume. And the same goes for the other gradual properties measured in both objects.

It is also worth noting that, although every measurable property must be gradual, not every gradual property can be measured. For instance, both the diamond and the chalk share the gradual property *beauty*, and we can say that the diamond is more beautiful than the piece of chalk. Yet there is no known way of measuring beauty in a consistent and useful manner. For a gradual property to be measurable the property must satisfy certain conditions, conditions that volume, mass, temperature and even hardness satisfy while beauty doesn't. Measurement theory studies, among other things, what these conditions are.

The question of what conditions make measurement possible goes hand by hand with the question of what use we can make of the numbers obtained in measurements. Once you have measured mass, temperature and hardness of the diamond and the chalk, you may want to use these measures to formulate quantitative statements. So you can say that the mass of the chalk is one hundred times that of the diamond, while its temperature is only twice and its hardness is one-tenth of that of the diamond. The crucial point to realize is that not all these statements are meaningful; while the former statement is meaningful, the latter two are not. Or to be more precise, all statements mean *something,* but only the meaning of the former depends exclusively on *facts about the objects*. The three express a numerical fact (the quotient of masses is 100, the quotient of temperatures is 2, the quotient of hardness is 0.1); but only the first statement expresses something that depends exclusively on the objects' properties; the other statements depend on these properties, but also on the conventions adopted in the construction of the measurement scale. For this reason we say that the first statement is *objective* while the others are not. Statements about quotients of temperature or hardness values are not objective in this sense because such statements may be true if, say, temperature is measured in degrees Celsius but false when measured in degrees Fahrenheit. Yet, there are *other* statements involving temperature that are objective. Although the statement that chalk's temperature is twice diamond's is not objective in this sense, the statement that the difference between the chalk's temperature at noon and at midnight is five times the difference between the diamond's temperature at noon and at midnight is objective. This raises the question of *why some quantitative statements are objective and others are not.* MT answers these questions, by investigating the conditions that make measurement possible and by studying the extent to which we can use the measures obtained to make objective statements about objects.

1.2. Derived and Fundamental Measurement

Measurement, the assignment of numbers to objects to represent their gradual properties, can be derived or fundamental. In derived measurement we obtain the desired value of a magnitude for an object from other values we already have (values of the same magnitude for other objects or of different magnitudes for the same object) and which are related with the unknown value in a specific way. For example, we can measure the mass of a heavenly body using the mass of a rocket, the change in its trajectory when it travels near the heavenly body, and certain mechanical laws that relate these values. Or we can measure an inaccessible distance by measuring another, accessible, distances and angles and then use certain trigonometric formulae that relate them. Derived measurement is by far the most common kind of measurement in scientific practice, but it is immediately clear that measurement cannot always be derived. Since derived measurement makes use of values that are *already measured*, measurement cannot (on pain of circularity or infinite regress) always be derived; it is necessary to begin somewhere without relying on previously measured values. This is what *fundamental measurement* does. Fundamental measurement, although not as common in scientific practice, is absolutely essential for it is "where everything begins". In fundamental, or direct, measurement we obtain the desired values with no previous measurements at all *directly* from qualitative empirical data.

Measurement, derived or fundamental, is possible if the measurement systems, i.e. the physical systems where measurement is performed, satisfy certain conditions These conditions, henceforth referred to as *measurability conditions*, are the subject matter of theoretical investigations, and the theory devoted to studying these conditions is Measurement Theory (MT). In derived measurement, the measurability conditions that make measurement possible are the empirical laws that connect the unknown with the known values (mechanical and trigonometrical laws in the above examples). Since these laws (and definitions) are in general studied within their own quantitative theories (e.g. mechanics and geometry), there is no specific question for MT to address; that is, there is no specific question for MT in connection with measurability conditions for derived measurement. This research is done by common quantitative theories, and there is no autonomous theory devoted to the foundations of derived measurement.

In the case of fundamental measurement the systems and the conditions or laws are qualitative (remember that numerical values "begin" with these). Yet, since the same set of qualitative measurability conditions may correspond to qualitative systems with different physical natures, such conditions are not the subject of any *particular* empirical qualitative theory that focuses on a *specific* magnitude. For example, the measurability conditions for fundamental measurement of mass and length are the same. The conditions under which fundamental measurement is possible are then the proper subject matter of MT, which can then be understood as the theory that investigates the (different groups of) qualitative laws that make an empirical system, *of whatever physical nature*, measurable. Of course, this research is not prior in time to the existence of measurement and scales in science. Although in some fields, like psychology or economics, research in MT has given rise to new kinds of measurement, for the most part MT deals with measurements already known (some of them, like mass or length, centuries ago). Therefore, the priority of MT has to be understood not temporally but conceptually: MT provides the conceptual foundations for, and the ultimate understanding of, direct measurement.

2. The Formation Period

Standard MT is the result of two different and complementary research traditions. The first tradition begins with Helmholtz and continues with Hölder and Campbell, and focuses on comparative combinatorial systems and real morphisms. The second tradition originates in the work of Stevens and his collaborators on scale types, transformations and invariance.

2.1. Helmholtz on Alikeness and Additivity

Although measurement has been used in science before Helmholtz, and some unsystematic comments about measurement can be found him in the works of some philosophers and scientists, Helmholtz is the first one that formulates, addresses, and treats systematically the core question of MT: Why can numbers be applied to things? His essay "*Zählen und Messen erkenntnis-theoretisch betrachtet*" (1887) is the first work in which this question is explicitly formulated. Helmholtz calls the attributes of objects that allow for comparisons *magnitudes*. When these attributes are expressed by numbers, these numbers are the *values* of the magnitudes and the procedure by which we assign these values is the *measurement* of the magnitude. He then formulates the question for the measurability conditions in the following way: "we shall have to investigate in which circumstances we can express magnitudes through [...] numbers" (p. 84).

According to Helmholtz, such an investigation must begin with the notion of *alikeness*. This notion captures certain outcomes of an empirical comparison procedure (e.g. putting objects in a pan balance). The magnitudes that two objects display are alike if: (i) the outcome of the comparison does not change when the order of the objects is inverted, and (ii) both objects always give the same outcome when compared with the same third object. Two alike magnitudes are thus interchangeable in the comparison procedure since, by definition, they give rise to the same outcomes with every other object. Less trivially, and more importantly, Helmholtz points out that alike magnitudes may also be interchangeable in other contexts (e.g. two magnitudes that are interchangeable in a pan balance are also interchangeable in spring balance). These other contexts in which alike magnitudes are interchangeable must then be regarded as different manifestations of the very same attribute; hence we must "characterize the further effects in which alikeness is preserved as effects of that attribute, or as empirically dependent upon that attribute alone" (p. 91).

Note that in Helmholtz's talk, a magnitude is not the property or attribute itself (e.g. mass), but a specific instantiation of the property in an object (e.g. the specific degree to which a heavy object exemplifies the property mass). Yet, with the concept of alikeness at hand, Helmholtz can easily define the notion of *magnitudes of the same type*, that is, of magnitudes corresponding to the same property or attribute: "magnitudes whose alikeness or non-alikeness is to be decided by the same comparison method are termed by us *alike in kind*" (ibd.). He mentions as examples of such kinds weight, length and duration, together with their well known comparison methods, i.e. equilibrium, congruence and simultaneity. These kinds of magnitudes are just what later authors mean by 'magnitude', namely the attribute itself capable of being measured. It is worth noting that, from the very beginning of MT in Helmholtz's work, these properties (e.g. mass) are identified via a specific qualitative comparison method: a (kind of) magnitude is "what shows up" in a particular comparison procedure (e.g. by pan balance).

In order for a magnitude to be capable of measurement, alikeness does not suffice. The comparison method so far considered allows us to establish whether two magnitudes are, or are not, alike, but if they are not alike it does not give us any measure of their difference. According to Helmholtz, if magnitudes have to be *completely specifiable* by numbers, "the greater of the two numbers must be portrayable as the sum of the smaller and their difference" (p. 94) , and for this to be possible, there must be some physical conjunction "between magnitudes alike in kind [expressible] as an addition" (ibd.). We have here the first formulation of the additivity condition; that is, in order for measurement to be possible there must exist a physical way of combining objects that "resembles" mathematical addition. We will see below how it is possible to measure magnitudes for which such an addition-like combination does not exist, but for more than half a century the existence of such physical operation was taken as the only way to measure quantities directly from qualitative data.

What are the conditions that this physical combination of magnitudes has to satisfy for it to "resemble" addition? The first that he mentions is obviously that magnitudes be of the same kind, otherwise we would not be combining magnitudes of the same attribute. The important conditions are the following. The first is a variant of substitutivity: two alike magnitudes are exchangeable in a physical combination; that is, they give rise to alike magnitudes when combined with the same third magnitude. The second is commutativity: two magnitudes combined in inverse order are always alike.

The next step in Helmholtz's search for measurability conditions is to define a *greater than* relation. Note that so far the empirical comparison method (e.g. equilibrium, congruence, simultaneity) establishes only whether two magnitudes are alike, but if they are not alike the method doesn't establish which is greater than the other. Once we have the combination operation, "it now also follows which are greater and which are smaller [...], the whole is greater than the parts of which it is composed" (p. 96). That is, the compound magnitude is greater than both components. This cannot be considered as a definition of *greater than* for *any* two magnitudes, but it suggests one which, though not explicitly formulated, is implicitly used: a magnitude is greater than another one if there is a third magnitude which, when combined with the second one, is like the first. Thus defined, we can now regard Helmholtz's above "characterization" of the greater-than relation as a new measurability condition which has to be added to the ones already listed. This new condition is called *positivity*: the composed magnitude is always greater than both components.

We can summarize Helmholtz's measurability conditions as follows, taking $\sim$ for alikeness, o for the combination operation and $\succ$ for the greater-than relation so defined (note that the second implies $\sim$ transitivity).

(H) In order for magnitudes of the same kind $a, b, c, \ldots$ be capable of measurement, there must exit a comparison method which determines $\sim$ and $\succ$ and a combination operation $\circ$ so that that the following conditions hold:

(He1) $a \sim b$ iff $b \sim a$ — $\sim$ symmetry

(He2) $a \sim b$ iff $(a \sim c$ iff $b \sim c)$ — $\sim$ substitutivity

(He3) $a \sim b$ iff $a \circ c \sim b \circ c$ — $\circ$-monotonicity

(He4) $a \circ b \sim b \circ a$ — $\circ$-commutativity

(He5) $a \circ b \succ a$ & $a \circ b \succ b$ — $\circ$-positivity

This is Helmholtz's answer to the question of how fundamental measurement is possible, which he raises for the first time. To conclude his main contributions to MT, three final remarks are in order. Firstly, although does not Helmholtz address the question of the sufficiency of these conditions for the magnitudes to be *completely specifiable* by numbers, he does explicitly mention, although without proof, that the numbers thus obtained "only have a proportional value" (p. 89). That is, they have no absolute representational value; they only have representational value as far as we express proportions or ratios with them; the numbers so obtained are what later on will be called proportional scales. Secondly, he also mentions that in some cases it is possible to find two different additive operations for which the comparison method that determines the alikeness is the same. He mentions the combination of electric wires in series and in parallel and, curiously, he says that by the first method we combine resistances while by the second we combine conductivities (curiously, since he has said before that if the comparison method is the same the magnitude is the same as well). Finally, he mentions *measurement by components*, or vectorial measurement, as a peculiar type of measurement assuming that it is possible to additively compose magnitudes of different types by means of a single physical operation on the objects and mentions as examples velocity, acceleration, force and color. The type of representation involved here is what below will be called *multidimensional representation*.

2.2. Hölder on Axiomatics and Real Morphisms

Helmholtz is the first to ask the main question on how fundamental measurement is possible and also to answer it by providing a set of conditions that the system must satisfy. Yet, as we have seen, he does not demonstrate that his conditions are sufficient for numerical representation. This was achieved at the turn of the century by Hölder, who was the first one to formally study the necessity/sufficiency of a set of conditions for the numerical representation of a qualitative comparative-combinatorial system (Hölder 1901; only a little later, Huntington (1902) presented a similar result but it was Hölder's work that came to be the point of reference for subsequent research; Wiener (1921) was also heading in the same direction, although much later, including some seminal comments on the problem, we shall see below, of the empirical inaccuracy of the formal conditions).

The qualitative facts necessary for the representation are those that correspond to a certain order relation $\succeq$, determined by a comparison method and a certain combination o among objects displaying the magnitude. The numerical entities assigned to the empirical objects are positive real numbers. The numerical facts which represent the qualitative empirical facts are those involving the grater than relation $>$ and the addition operation + over the real numbers. The representation here consists in a *complete translation*, that is, in an isomorphism. Hölder gives seven conditions, or axioms, that the domain D of objects, the qualitative relation "greater than, or equal to" $\succeq$ and the qualitative operation o must satisfy and demonstrates that these conditions are jointly sufficient for there to be an *isomorphism* from $< D, \succeq, \circ >$ onto $< Re^+, \geq, + >$; that is, for there to be a 1-1 mapping $f : D \to Re^+$ from the domain of objects into the positive real numbers, so that (i) $a \succeq b$ iff $f(a) \geq f(b)$ and (ii) $a \circ b \sim c$ iff $f(a) + f(b) = f(c)$. In this sense numbers "represent" magnitudes, since numbers assigned to objects are such that qualitative $\succeq$-facts among objects are "replicated" by quantitative $\geq$-facts about the assigned numbers, and the same goes for qualitative $\circ$-facts and quantitative $+$-facts. This result is known as Hölder's Theorem.

Among Hölder's conditions there are two new ones worth mentioning with an eye on further developments of MT, namely *solvability* and *Archimedeanity* (also referred to as the Archimedean axiom). Solvability states that we can always "solve" a qualitative difference among objects: if an object is smaller than another, there is a third one that concatenated with the first is equivalent to the second:

(Hö1) If $a \succ b$ then there exists c so that $a \sim b \circ c$ (solvability)

The Archimedean axiom states that no object is "infinitely" greater than another, that is, when an object is greater than another one we can "reach" the greater "replicating" the smaller one (i.e. concatenating the smaller one with itself) a finite number of times

($nb \triangleq b \circ ... \circ b$, n times):

(Hö2) If $a \succ b$ then there exist n $(\in N)$ so that $nb \succ a$ (Archimedean axiom)

These are the most important conditions Hölder introduces. Some of his other conditions are similar to some we have already seen in Helmholtz, and the rest (formally too complicated to discuss here) have almost no empirical meaning. The last-mentioned, empirically unrealistic, conditions make Hölder's Theorem a mainly mathematical result with only marginal empirical significance. Nevertheless, suitably modified (to make it applicable to more realistic situations), it contains the nucleus of most of the subsequent formal analyses of the conditions that make additive measurement possible. In the spirit of Hölder's work, this analysis can be regarded as establishing the conditions that a comparative-combinatorial empirical system must satisfy for there to be an additive morphism (not necessarily *iso*; in general it will be sufficient for it to be a *homomorphism*) of such a system *into* (not necessarily *onto*) the real numbers. This idea has also inspired MT when it expanded beyond these first, paradigmatic measurement systems, and started to deal with non-combinatorial systems and non-additive measurability conditions.

2.3. Campbell on Order and Additivity

N. Campbell is widely acknowledged as the main philosopher of measurement in the first half of the XXth century. Quite surprisingly, he does not mention Hölder's results at all, not even in his most important and monumental work, *Foundations of Science*, written almost twenty years later (Campbell 1920, re-edited as Campbell 1957, from which I quote). The reason for this may be the rather philosophical, not mathematical, orientation of his work. Campbell devotes the whole second part of his book to the study of measurement in science, where we find a systematic study of practically all the questions related to measurement and, among these, the conditions which make fundamental measurement possible. This is what concerns us here.

Campbell characterizes measurement as "the process of assigning numbers to represent qualities" (p. 267) and explicitly address our main question: "Why can and do we measure some properties of bodies while we do not measure others?" (p. 268). His answer basically is that measurable properties of bodies "resemble" the properties of numbers in a specific way yet to be clarified. This resemblance is analyzed, as in Helmholtz, in terms of certain conditions that the empirical system must satisfy.

The first condition is that the property generates, relative to a certain method of comparison, a relation $\succ$ which is asymmetrical and transitive (i.e. a so-called order relation). This relation must be such that if it does not connect two objects, these objects must behave with the others in the same way. When this is the case, then we can regard $\succ$-disconnected objects as alike in magnitude. These three conditions allow for "empirically informative" numerical representation such as the representation of hardness and (if directly measured without the aid of other magnitudes) density. However, this type of representation is not very informative since the difference between the numbers assigned "does not represent the physical difference" (p. 274. For a physical difference to be representable, mathematical addition must have a physical interpretation; that is, there must be a way of combining objects *analogous* to numerical addition. When this can be done, the property may be measured *perfectly and definitively*: "The difference between those properties which can be measured perfectly and definitively, like weight, and those which cannot arises in the possibility or impossibility of finding in connection with these properties a physical significance for the process of addition" (pp. 277-278). Though his view is not always uniform as regards the conditions that physical combination must satisfy for resembling addition, he explicitly mentions positivity, commutativity, associativity, monotonicity and a property that implies solvability and Archimedeanity (but is stronger than their conjunction). We can summarize Campbell's conditions as follows (where $a \sim b \triangleq \text{not } a \succ b \ \& \ \text{not } b \succ a$):

(Ca1) If $a \succ b$ then not $b \succ a$

(Ca2) If $a \succ b$ & $b \succ c$ then $a \succ c$

(Ca3) If $a \sim b$ then for every c: $a \succ c$ iff $b \succ c$

(Ca4) $a \circ b \succ a \ \& \ a \circ b \succ b$

(Ca5) $a \circ b \ \sim \ b \circ a$

(Ca6) $a \circ (b \circ c) \ \sim \ (a \circ b) \circ c$

(Ca7) $a \succ b$ iff $a \circ c \succ b \circ c$

(Ca8) $a \sim b$ iff $a \circ c \sim b \circ c$

(Ca9) If $a \succ b$ then there exists n $(\in N)$ so that $nb \sim a$

It is worth emphasizing that these conditions are intended to be empirical properties of qualitative comparison combinatorial systems. Thus, as for Helmholtz, whether or not a physical combination $\circ$ and a comparison relation $\succ$ satisfy them is a question that only experience can decide.

Three additional issues Campbell deals with are worth mentioning. He acknowledges that some of these conditions can be deduced from known laws of empirical theories without conducting further experiments; for instance, we can deduce from the laws of mechanics that pan balances satisfy some of these conditions. Yet, this does not mean that fundamental measurement is parasitic on quantitative laws, since our belief in the truth of these laws is based on our knowledge that the measurement of weight is possible, and so assumes that these conditions are fulfilled. Secondly, Campbell also raises the question of how arbitrary or univocal a numerical assignment is, given that the property satisfies these conditions. If we consider a single mode of combination, he (informally) demonstrates that every two different numerical assignments that represent $\succ$ by > and o by + are proportional; that is, one is a multiple of the other. Third, like Helmholtz, Campbell notes that there may be more than one mode of combination that satisfies these conditions, and also mentions the combination of wires in series and in parallel. But in this case there is no additional arbitrariness, for these combinations satisfy the conditions relative to different (inverse) $\succ$ order relations. He points out that genuine arbitrariness would obtain if the same relation $\succ$ satisfied the same conditions with different combinations, although he does not mention any (decades later Ellis will call attention to the linear and orthogonal combination of roods for the case of length).

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Biographical Sketch

José A. Díez (Barcelona 1961) studied Philosophy at the University of Barcelona, where he got the Doctored degree (1992) with a dissertation on Measurement Theories. He has been visiting scholar at several universities in Europe (Munich, LSE) and EEUU (NYU, Brown), invited professor in Europe (Italy, Portugal, Germany) and Latin America (Argentina, Mexico, Colombia, Chile), and is now Research

Associate at LSE. He is Professor of Logic, History and Philosophy of Science at the University of Barcelona and founder member of LOGOS Research Group on Logic, Language and Cognition. His main research lines are measurement theories, explanation and representation, semantic approaches and specially Structuralism, and theoretical concepts. He has published *Iniciación a la Lógica* (2002), *Fundamentos de Filosofía de la Ciencia* (1998, co-authored with U. Moulines) and *Filosofía del Lenguaje 1* (2004, with LOGOS), and a number of articles in international journals and collective works.