GAME MODELS OF NEGOTIATION AND ARBITRATION

D. Marc Kilgour
Department of Mathematics, Wilfrid Laurier University, Canada

Keywords: negotiation, bargaining, arbitration, game theory, divide-the-dollar, alternating offer, simultaneous offer, buyer and seller, final-offer arbitration

Contents

1. Introduction and Overview
2. Negotiation Models
2.1. Divide-the-Dollar
2.2. Alternating Offer Divide-the-Dollar
2.3. Simultaneous Offer Buyer and Seller
2.4. Unilateral Deterrence
3. Mediation
4. Arbitration Models
4.1. Conventional Arbitration
4.2. Final-Offer Arbitration
Glossary
Bibliography
Biographical Sketch

Summary

This article has provided a few examples of game models of negotiation and arbitration, showing not only what sort of approaches are typical but also what kinds of conclusions can be drawn. This is an active area of interdisciplinary research, driven in part by the escalating number, range, and complexity of problems being settled by negotiation (including mediation and arbitration), and in part by the practical usefulness of the insights that this research has already produced. By understanding negotiation and arbitration better, we will be able to practice it better, reaching socially-optimal outcomes quickly and reliably.

1. Introduction and Overview

To analyze negotiation using game models, it is helpful to begin with a broad definition. Negotiation is the process by which two or more sides attempt by communication to influence each other's decisions on matters that affect all negotiators. (For other definitions of negotiation, see the other articles in Topics *Approaches to Conflict Resolution* and *Formal Models for Conflict Resolution and Case Studies*) A consequence of this view is that underlying any negotiation is a strategic conflict – an interactive decision problem in which each negotiator must choose one of two or more courses of action, and has an interest not only in its own choice, but also in the choices of others. The eventual outcome of a strategic conflict is its resolution. Mathematical models of strategic conflicts are called games, and game theory is their study. A game model of the underlying conflict can be converted into a game model of the negotiation

simply by including a communication component.

Thus bargaining, by which we mean the process of sending and receiving messages, is an essential feature of negotiation. Messages, which may include offers, statements, questions, and threats, can be explicit or implicit, precise or ambiguous. Bargaining may be governed by rules or constraints, as it is in formal negotiations and diplomacy, or may be unrestrained, like haggling in a market. For the purposes of this study, the defining characteristics of a negotiation are that:

- each participant in a negotiation has a decision to make;
- every participant is made better or worse off according to the decisions of all participants;
- all participants can send and receive messages;
- no participant is obligated to accept any particular resolution.

Negotiator *A* cooperates with negotiator *B* if the course of action eventually chosen by *A* tends to lead to outcomes favorable to *B*. As implied by the fourth characteristic, a negotiator need not cooperate with any other; in particular, an individual can refuse to negotiate, or can drop out of an ongoing negotiation. It follows that negotiators who are rational (in the sense of pursuing their own self-interests) will continue to negotiate only if they have a reasonable expectation of receiving more than the maximum return they could achieve without negotiation. For example, when a seller *A* and buyer *B* negotiate over the price of something that belongs to *A* and might be sold to *B*, they both cooperate when they agree that a particular price is acceptable, for then a transfer can take place. Otherwise, *A* and *B* "walk away." Note in particular that it is reasonable to expect that a negotiation cannot make a party worse off, at least not when there are only two parties. And when two-party negotiations are successful, both parties cooperate.

Of course, the underlying conflict may be clarified, or even changed, during a negotiation. Changes can be positive: discussion of values and principles can sometimes uncover an achievable outcome preferable to all parties, a so-called "win-win" solution. Or they can be negative: not only must negotiators sometimes bear the cost of fruitless negotiations, but also they may threaten each other, and even act on their threats, as for example countries sometimes do in disputes over land or resources.

During a negotiation (typically, a two-sided negotiation), the negotiators or an outside party may employ a mediator, an individual whose job is to assist the sides at identifying and attaining an acceptable compromise (see *Mediation in Environmental Disputes*). Strategically speaking, the mediator has no power, as the outcome is still determined by the decisions of the negotiators. In practice, though, a skillful mediator can make an enormous difference in the outcome of a negotiation, by clarifying the issues, assisting the parties in understanding their own and their opponent's positions, identifying possible compromises, foreseeing and avoiding pitfalls, and moving the negotiators quickly and smoothly toward an acceptable resolution.

Arbitration, on the other hand, is a procedure (actually, there are many variants) that may be implemented when a negotiation fails to achieve cooperation. In fact, negotiation often takes place under rules that impose arbitration if a negotiated

resolution is not achieved within a specified time (see *Arbitration of Environmental Disputes that Cross National Boundaries*). If a dispute is submitted to arbitration, then the arbitrator (or arbitration panel) will determine the resolution after hearing demands and arguments from all parties. Unlike a mediator, an arbitrator has power in a strategic sense because it can determine an outcome. Moreover, an arbitration is not a negotiation, because the resolution determined by the arbitrator is imposed. In particular, dropping out is no longer an option, and a participant in a two-sided arbitration risks a resolution less preferable than the status quo.

Game models are used to study negotiation and arbitration mainly through the modeling and analysis of the strategic implications of specific features of the preferences of individuals and the procedures for reaching a resolution. Unfortunately, game models must usually abstract one or a few specific features from a real-world situation, drastically simplifying the rest, in order to avoid problems of complexity and tractability. In most cases, realistic game models are impossible to analyze. There are systems for constructing and analyzing a comprehensive model of a strategic conflict, but they inevitably simplify the representation. However, one such system, the Graph Model for Conflict Resolution (see *The Graph Model for Conflict Resolution*) is surprisingly practical, and can be applied conveniently using the GMCR II software.

In studying the strategic situation of participants in a negotiation or arbitration, we are inquiring into the behavior of a participant who acts to further its own interest insofar as possible – and into how this behavior reflects the individual's characteristics and strategic options. In particular, we use only non-cooperative game theory; it is possible to apply principles of fairness to negotiations (usually in the absence of models of communication), but this is the province of cooperative game theory. The bibliography contains definitions of all game-theoretic concepts used below, as well as applications of both non-cooperative and cooperative game theory to negotiation.

This article is a sampler of game models of negotiation and arbitration; only a few basic models are described in an attempt to illustrate typical contributions. A range of simple game models of negotiation is described in Section 2, and some basic conclusions drawn. Then, after a few comments about mediation in Section 3, Section 4 proceeds to a discussion of some simple game models of arbitration procedures. Section 5 concludes with a brief summary.

2. Negotiation Models

In the following models, two negotiators, *A* and *B*, are bargaining over how to share or transfer a dollar, or an object worth up to a dollar. Both *A* and *B* are assumed to have utilities that are linear in money and (where appropriate) additively separable.

2.1. Divide-the-Dollar

A and *B* can share the dollar provided they can agree on how to split it. They demand a and b, respectively, where $0 \le a \le 1$ and $0 \le b \le 1$. Each receives its demand if $a + b \le 1$; otherwise, each receives 0. This situation is represented as an extensive game in Figure 1. (Read downward from the top. All of *B*'s decision nodes are grouped in a single

information set, implying that *B* cannot distinguish them; in other words, *B* must choose its demand without knowledge of *A*'s demand.)

Game-theoretic analysis of Divide-the-Dollar yields results that are unsurprising, though perhaps disappointing. The appropriate solution concept is Nash equilibrium; for any x satisfying $0 \le x \le 1$, the demands $a = x$, $b = 1 - x$ constitute a Nash equilibrium. In sum, *any* division $(x, 1 - x)$ of the dollar can be justified game-theoretically.
Divide-the-Dollar is often seen as a prototypical bargaining problem: the two sides can capture a surplus, but only if they can agree on how it is to be shared. If they fail to agree, they will receive nothing. As the analysis shows, these facts do not in themselves imply any constraints on the behavior of rational players. Specific behavioral predictions require more precise information about the negotiation procedure.

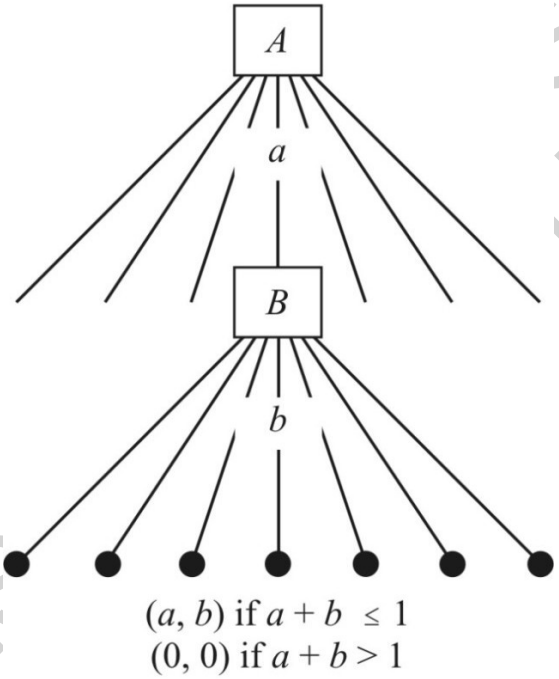

Figure 1: Divide-the-Dollar game

-
-
-

Bibliography

Aumann, R.J. and Hart, S., editors (1992). *Handbook of Game Theory with Economic Applications*. Amsterdam: Elsevier. [A comprehensive reference on Game Theory. See especially Binmore, Osborne,

and Rubinstein's article on Non-Cooperative Models of Bargaining (Ch. 7). For cooperative game theory applications, see Thomson's Cooperative Models of Bargaining (Ch. 35).]

Brams, S.J. and Kilgour, D.M. (1996). Bargaining procedures that induce honesty. *Group Decision and Negotiation*, 5, 239 - 262. [Possible modifications in the Simultaneous Offer Buyer and Seller game that induce the players to make truthful offers, though always at some cost.]

Chatterjee, K. and Samuelson, W. (1983). *B*argaining under incomplete information. *Operations Research*, 31, 835 - 851. [The original treatment of the Simultaneous Offer Buyer and Seller game.]

Kilgour, D.M.. (1994). Game-theoretic properties of final-offer arbitration. *Group Decision and Negotiation*, 3, 285 - 301. [Analysis of the Final-Offer Arbitration model, followed by comparison and interpretation of solutions.]

Osborne, M.J. and Rubinstein, A. (1994). *A Course in Game Theory*. Cambridge, MA: MIT Press. [A text focusing on non-cooperative game theory, containing many bargaining models.]

Raiffa, H. (1982). *The Art and Science of Negotiation*. Cambridge, MA: Harvard University Press. [This is the seminal work on the application of formal models to negotiation.]

Roth, A.E. (1985). *Game-Theoretic Models of Bargaining*. New York: Cambridge University Press. [A good basis for the study of game-theoretic models of bargaining and negotiation.]

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, 50, 97 - 109. [The original presentation and solution of the Alternating Offer Divide-the-Dollar model and generalizations.]

Young, H.P., editor (1991). *Negotiation Analysis.* Ann Arbor: University of Michigan Press. [A collection of applications of formal modeling to negotiation and arbitration. See especially Brams, Kilgour, and Merrill's article on Arbitration Procedures (Ch. 3) and Myerson on Mediation (Ch. 4).]

Zagare, F.C. and Kilgour, D.M. (2000). *Perfect Deterrence.* Cambridge, UK: Cambridge University Press. [A detailed description and solution of the Unilateral Deterrence model and many related models.]

Biographical Sketch

D. Marc Kilgour is Professor of Mathematics at Wilfrid Laurier University in Waterloo, Ontario, Canada, Director of the Laurier Centre for Military Strategic and Disarmament Studies, and Adjunct Professor of Systems Design Engineering at the University of Waterloo. With degrees in Engineering Physics, Applied Mathematics, and Mathematics from the University of Toronto, he has held academic and administrative positions at Wilfrid Laurier University since 1973, with leaves at the University of Waterloo, Graduate Institute for International Studies (Geneva, Switzerland), and Kyoto University (Japan). International awards have supported other research and teaching visits to the USA, France, Japan, and Germany.

Dr. Kilgour's primary research interests lie in decision analysis, at the intersection of mathematics, engineering, and social science. He has applied game theory and related formal techniques to problems in international security and arms control, environmental management, negotiation and arbitration, voting, fair division, and coalition formation, and has pioneered the development of systems for decision support in strategic conflict.

Dr. Kilgour has produced four books and more than 100 refereed articles across a broad spectrum of academic disciplines, including mathematics, operations research, management science, political science, international security, systems engineering, environmental management, economics, social choice, biology, and philosophy. His most recent book is *Perfect Deterrence* (Cambridge University Press, 2000), co-authored with Frank C. Zagare. Dr. Kilgour has been Corresponding Editor (Special Responsibility for Game Theory) of *Theory and Decision* since 1993; also, he is a member of the editorial board of three journals and is active in twelve professional societies. He has received international recognition for both the quality and the interdisciplinary nature of his research.