MACROSYSTEM MODELING IN SYSTEM ANALYSIS

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Contents

Summary

In this chapter results of the equilibrium and nonequilibrium theory of the macrosystems and their applications in system analysis are described. The macrosystem is a system with stochastic and deterministic properties. The theory of the equilibrium states of macrosystems contains the methods of the mathematical modeling, which use the principle of entropy maximization above a feasible set. The mathematical models of the macrosystems with Fermi, Einstein and Boltzmann states are proposed. For computer realization of these models the multiplicative algorithms are developed. This part of the theory of macrosystems contains the methods of estimation of the parametric sensitivity of the macrosystem models.

The theory of the nonequilibrium states of macrosystems contains the methods of the mathematical modeling, which use the idea of division of movements and the principle

of the local equilibria.

The applications of the macrosystems theory in urban planning, regional development and dynamics of biological communities are considered.

1. Introduction

Trends in science since the 1980s clearly indicate efforts towards integrating knowledge and scientific disciplines, which reveal themselves in studying various objects of nature and human society from holistic positions. These tendencies follow from completely objective processes.

Attempts of holistic analysis are usually combined under the name of systems approach, and the objects approached in this way are called systems. There are many different definitions of these terms, but here we are interested only in system's acquisition of new properties, untypical for its elementary parts. There are many theoretical and applied problems in which we meet systems, containing a lot of elements and behavior of these systems significantly differs from behavior of their elements. For example, gas particles, occupying a given volume, move randomly, but they result in a gas reaching deterministic temperature value.

Resources exchanges in the market economy are in most part stochastic, but the multiple acts of individual exchanges result in deterministic equilibrium state.

Consumption of food in biological community is a nondeterministic process, but the multiple acts of individual consumption result in the equilibrium state, which may be described in terms of deterministic parameters.

These examples deal with systems of diverse origin. But they underline the important common feature: all these systems consist of a large number of elements and behave themselves in a way, drastically different from their individual elements.

The systems consisting of a large number of elements and behaving themselves in deterministic way, while their elements have nondeterministic type of behavior are called macrosystems. The classical example of macrosystem is given by L. Boltzmann (1879). It is a thermodynamical system consisting of a large number of particles which interact with each other in random way. Here non-deterministic type of elements behavior is described in stochastic terms. For many physical, technical and biological systems stochastic models of non-deterministic behavior are quite adequate. At the same time, extending of the macrosystems approach to the social and demo-resource systems leads to significant modernization of the stochastic models of behavior and to appearance of other mathematical models of non-deterministic behavior (see S.Meerkov (1979) and V. Levchenkov (1982)).

This chapter deals with the main results of the macrosystems theory and its applications in urban planning, regional development, biological dynamics, and chemical kinetics.

2. Examples

2.1 Economical Exchange

The systems, elements of which are producers and consumers simultaneously, are considered. Each element of system uses products of other elements for the production of its own product. In the situation of a market economy products exchange among elements is nondeterministic. However, over long time periods the system as a whole has deterministic characteristics (periods of equilibria).

2.2 Passenger Transport Flows

In the book of A. Wilson (1979) the urban transport network that links places of residence and places of work is studied. Choice of each resident pair "residence-work" is nondeterministic. This choice depends on a lot of factors, and the cortege of these factors and decision rules are different for residents and are known imprecisely. Because the choice of "residence-work" trip is of nondeterministic nature passenger transport flows in network give rise to a large number of such choices. The distribution of flows is deterministic in certain interval of day.

2.3. Chemical Kinetics

Consider the closed network of chemical reactions of catalytic type (hypercycle), where the same catalyst is used for activating all the reactions (see M.Eigen and P.Shuster (1979). The catalytic reaction has two stages. The complex "product - catalyst" form on the first stage and the product of chemical reaction in given complex forms and the catalyst becomes free on the second stage. Then molecules of the catalyst are distributing between reactions of hypercycle. It is natural that the distribution process is random, but the state of hypercycle network as a whole is characterized by the deterministic parameters - amount of reactions products.

2.4. Image Reconstruction

There is the problem of image reconstruction because the measured image is distorted by noises. If the image is described by the function of density of electron flow, which determines light intensity of display screen, then the measured image may be characterized by random distribution of this density. It means that numbers of electrons, hitting on each elementary cell of screen are random values. The procedure of reconstruction intends to decrease the random factors as much as possible and to obtain the deterministic distribution of density on the screen, which resembles the true one.

3. Equilibrium States of Macrosystems

3.1. Phenomenological Scheme

Consider an abstract macrosystem consisting of Y non-distinguishable elements with stochastic type of behavior. These elements can occupy the states in the subsets of close states $S_1, \dots, S_m$.

The states of the elements of these sets may belong to one of the following types: a)

Fermi-states (each state may contain only one element); b) Einstein-states (each state may contain any number of elements). The important characteristic of subsets S_n is their capacity G_n, that is the number of states included into S_n.

Elements of a macrosystem may occupy any state from subsets S_n in a random and independent way. For any fixed subset S_n an element has just two possibilities: to occupy any state of S_n with prior probability a_n and to miss this subset with the probability $1-a_n$.

A distribution of all Y elements over the states from subsets $S_1,\ldots,S_m$ is a *microstate*. The microstate is the list containing element's number and the number of subsets occupied by these elements. A distribution of the number of elements over subsets $S_1,\ldots,S_m$ is a *macrostate*. Macrostate is characterized by the collection $N_1,\ldots,N_m$, where N_n is a number of elements occupying corresponding subset S_n $(n \in \overline{1,m})$. If these subsets contain Einstein-states, the numbers of elements occupying states are not constrained by the capacities of subsets. Therefore if the overall number of elements Y is fixed, then $0 \le N_n \le Y$, $(n \in \overline{1,m})$.

If the subsets $S_1,\ldots,S_m$ contain Fermi-states then the numbers of elements occupying states are constrained from above by the capacities of subsets, i.e. $0 \le N_n \le G_n$ $(n \in \overline{1,m})$. Hence, if elements number Y is fixed, Fermi-states may be realized only if certain relations between subsets capacities $G_1,\ldots,G_m$ and prior probabilities $a_1,\ldots,a_m$ hold.

The distribution of macrosystem elements over the subsets $S_1,\ldots,S_m$ takes place subject to certain constraints, implied by some balance relations between $N_1,\ldots,C_m$ and consumption of resources of different types.

The simplest example of the balance constraint is given by the constant number of elements $(\sum_{i=1}^{m} N_i = Y)$. The typical resource constraint in a thermodynamical system is given by the conservation of total energy of elements; in transportation system the typical resource constraint is given by the constancy of average trip cost.

According to the definition, both macrostates and microstates are not unique. But observations of macrostates for various macrosystems show, that the equilibrium state corresponds to the unique macrostate $N^0 = (N_1^0,\ldots,N_m^0)$ which is called below *the realizable macrostate*.

3.2. Stochastic Characteristics of Macrostates

Consider the macrosystem without constraints. The absence of constraints means that the macrosystem does not interact with the external world (no exchanges of elements, energy, resources). In this case the set N includes all possible macrostates. Such macrosystems are called isolated.

The random mechanism of distributing macrosystem elements over states in the subsets $S_1,\ldots,S_m$ generates the set N of feasible macrostates characterized by the vector $N=\{N_1,\ldots,N_m\}$. The coordinates of this vector - the numbers of elements occupying subsets $S_1,\ldots,S_m$ - may with non-zero probability take any values between 0 and Y for Einstein-states and between 0 and G_n for Fermi-states.

Since the sets $S_1,\ldots,S_m$ do not intersect and the stochastic distribution mechanism is such that the elements behavior is independent, the probability distribution function is

$$P(N)=\prod_{n=1}^{m} P_n(N_n), \tag{1}$$

$$\sum_{N_1,\ldots,N_m=1}^{Y} P(N_1,\ldots,N_m)=1, \tag{2}$$

where $N=\{N_1,\ldots,N_m\}$, $P_n(N_n)$ - probability of N_n elements occupying the subsets S_n with capacity G_n (all other elements stay outside of S_n).

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Biographical Sketch

Yuri S. Popkov, Doctor of Technical Sciences, Professor, born in 1937. He graduated from the Moscow Energy Institute, worked in the Institute of Control Problems. From 1976, he works in the Institute of Systems Analysis of the Russian Academy of Sciences. In the time of his activities, he worked his way up from engineer to assistant director of the scientific work. In 1964, he maintained the thesis for a Candidate's degree and in 1971, the thesis for the degree of Doctor. His sphere of scientific interests centers on problems of the mathematical investigation of complex dynamic systems that have a dual nature - a stochastic and a deterministic one. He developed the methods of investigation of the so-called systems effects arising in multiple element systems. These results provided the basis for the development of methods of functional-spatial modeling of urban, transport, and regional systems. Many of these methods have found application in the practical activity for the formation and implementation of the Master Plan of Moscow development.

Yu.S. Popkov is the author of more than 120 scientific papers and 9 monographs. In 1999 he was given the rank of the Honored Scientist of the Russian Federation. Yu.S. Popkov is a corresponding member of the Russian Academy of Sciences and Member of the New York Academy of Sciences and the International Academy of Information Processes and Technologies.