LIFE CYCLE PROCESSES FOR MODEL DEFINITION AND DEPLOYMENT

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Summary

The concept technology of mathematical modeling is introduced as union of mathematical models composition, identification, verification, and exploitation. This sequence is treated as stages of the models life cycle. Forming and evolution of the mathematical models structure is described as the result of models exploitation and verification (validation). Connection is pointed out between mathematical models structures and consumption structure.

1. Technology of Mathematical Modeling

All basic notions are used in this paper: mathematical model, independence hypothesis, endogenous and exogenous characteristics of mathematical model, mathematical model identification, validation, exploitation. These are also treated in *Fundamentals of Mathematical Modeling for Complex Systems*. Therefore it is recommended that this topic be studied before the present article. Nevertheless a brief interpretation of these notions will be presented here for the convenience of readers. The 'mathematical model' is treated as system of equations describing the characteristics of phenomena (processes, systems). These characteristics are divided into two classes: endogenous (or inner, or prognosis, or phase) characteristics of the model and its exogenous (or outer, or parametric) characteristics. The property 'independence' is usually connected with the division of characteristics into groups of endogenous and exogenous ones: the endogenous characteristics are significantly influenced by their exogenous characteristics, while from a practical point of view the reverse influence does not take place. A mathematical model is called 'closed', if its endogenous characteristics may be defined by the model as soon as its exogenous characteristics are known. The process of determination of exogenous characteristics of mathematical model is called identification of the model. The process of determination of exogenous characteristics by the model after exogenous characteristics have been determined is called exploitation of the model. . Verification of conditions ensuring correctness of model's prediction is called validation of the model.

Thus for prognosis by a mathematical model the following actions are to be performed.

- composition of the model;
- verification of model closure;
- Development of a procedure for calculating values of endogenous characteristics of model and other characteristics, which may be formulated in the terms of models characteristics (endogenous and exogenous), which are interesting from a practical point of view provided that the values of exogenous characteristics are known;
- Determination of its exogenous characteristics, (i.e. to identify model);
- to verify the model i.e. to check up the framework in which forecasting by means of the model is correct from a practical point of view;
- exploitation of the model, i.e. to obtain prognosis by the model.

The set of these actions will be referred to as technology of mathematical modeling and each action will present a stage of this technology.

It is necessary to discuss in more detail the problem of verification (validation) of mathematical models i.e. the problem of checking their adequacy, because this problem is closely connected with the subject of this paper. Any mathematical model is adequate (i.e. it prognosis of the values of endogenous characteristics or prognosis of properties formulated in their terms is acceptable from a practical point of view) within some limits and under some conditions. The most important part of checking the adequacy of mathematical model consists in discovering these limits and conditions. Stages of composing the model and determining the values of its exogenous characteristics represent some ideas about the adequacy of a model. However, only exploitation of the model can present a more or less complete set of conditions, which are necessary for the model to give prognosis, acceptable from a practical point of view. Only within the process of exploitation of model, prognosis of the model may be compared with real development of phenomena (processes, systems) under consideration.

2. Life Cycle of Mathematical Model

If mathematical models are regarded as tools of a prediction of development of phenomena (processes, systems) or its properties, their similarity to any other engineering system used in the process of production or consumption becomes obvious. Any engineering system is characterised by 'life cycle' and by the stages of this cycle. Typical stages of life cycle of some engineering system are:

- working out general (common) concept, i.e. definition of the basic technical and operational characteristics;
- design;
- fabrication experimental samples and tests;

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- preparation for exploitation;
- serial manufacturing and exploitation;
- termination of serial manufacturing, removal of engineering system from exploitation and its replacement by a new, more perfect engineering system.

The life cycle of certain engineering systems may not contain the exploitation stage, in case that on some earlier stages it was found out that for some reasons it is not rational to use it. Moreover, for many engineering systems the stage of the general (common) concept, which is the initial stage of their life cycle, is at the same time the final stage of this cycle. It happens frequently enough that the engineering system which is undergoing a certain stage of the life cycle (particularly — the stage of tests), is returned back because mistakes were found in the earlier stages. Typical time separation between the first stage and the last stage of its life cycle will be called its 'time of life cycle'.

Observing the process of design, manufacture, and exploitation of engineering systems within the period of time which exceeds its time of life cycle one witnesses the process of forming engineering systems structures. The structure is formed as a result of special variants of engineering system and then as a result also of universal variants that may arise. Complexity of the structure arising in some field is practically limited, because the more complex the structure of an engineering system the greater are the complications that arise in the process of its exploitation.

There exists a profound analogy between engineering systems and mathematical models: the above listed stages of technologies of mathematical modeling are the stages of their life cycle. The stage of composing mathematical model and stage of checking the closure of model may be considered the analogue of the stage of the general (common) concept and the stage of design of an engineering system. The stage of development of procedure for calculation of values of its endogenous characteristics on condition that the values of exogenous characteristics are known is similar to the stage of testing experimental prototype of an engineering system. The stage of identification and verification of model is similar to the stage of preparation to exploitation of engineering systems. Mathematical models can return back to the earlier stages of its life cycle exactly as engineering systems can.

In just the same way as in the case of engineering systems observing the process of life of mathematical model within the time greatly exceeding its time of life cycles one can watch the process of forming mathematical model structures. The reason for forming these structures is the same as the reason for forming structures of mechanisms. The exploitation of any mathematical model specifies borders of its applicability and causes necessity of specifications of the model, and occurrence of special variations of the model, which is simpler than the initial model, and is used in a more narrow area. At the same time, there appear more generalised variations of models; they are more complex than the initial ones, but the borders of their applications are extended. As a result there appears a structure (system) of mathematical models, which are used for prognosis of development of phenomena (processes, systems) in some areas exactly as any tools are used in any other sphere of activity. It is important to note that structures of mathematical models used for prognosis of development of some phenomena (processes, systems) contain valuable information about the phenomena (process, system).

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Biographical Sketch

Yury Pavlovsky is Professor, Associated Member of Russian Academy of Science, head of Department 'Simulation Systems' of Computing Center of Russian Academy of Science. He received his doctorate degree in Mathematical-Physical Sciences from Moscow Physical-Technical Institute, in 1964.

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