MACROECONOMIC GROWTH MODELS

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Contents

Summary

Here the results of modeling of the economic growth are presented using the approaches suggested by Cobb, Douglas and Ramsey to modern studies. The axiomatics and ways of inclusion in the production function of industrial and other factors - capital and labor, technological level, natural resources and their price, human capital, solvent demand are described.

The equations of dynamic of the physical capital, technological level, investment demand, human capital are presented.

The tasks for closed and open of the economy are considered. Decisions in respect of macromodels are disussed, for which term of the balance, stability and optimality are simultaneously executed. Profound interpretation of the formal results of modeling is given. Requirements to economic strategies of the stable development are indicated.

1. Introduction

Macromodels of economic growth describe dynamics of such main factors of national economy as gross domestic product, national income, main production assets, savings, investments, consumption of the population and state, volume of employment, technological level, used natural resources, distribution GDP on payment of a transactions, profit and taxes, and also, other economic factors.

These models allows us to study mechanisms of economic development, to evaluate the consequences of the various economic strategies and to build the prognoses of growth. With the help of models the sources of growth and their relative contribution to growth rates stable condition of a dynamic equilibrium, alternation of phases of accelerated and slow development, economic rises and falls, optimum modes and strategies of growth, stability of growth in conditions globalization and approximation to global restrictions are investigated.

The short-term, intermediate term and long-term models of economic growth differ on an initial axiomatic, structure of variables and equations, and also on statements of problems. The short-term models are usually created as a dynamic system of econometric equations with a quarter or annual pitch and are used for forecasting smooth inertial dynamics factors. Such prognoses are created for 1 year and are used for drawing up of public finance. Econometric models with an annual pitch used for construction of a five-year prognoses in planned economy are also known. The intermediate term models are for forecasting and choice of the strategy of development in the near future for 4-5 years. The long-term models are intended for a research of conditions and properties of long economic growth for various suppositions about a behavior of the consumer, manufacturer and state, i.e. for researches of the various economic strategies. The intermediate term and long-term models are further considered.

The initial axiomatics for the construction of macromodels takes into account structural and technological features of production (interchangeability and proportions between production factors, connection of different-age technologies with the different-age equipment, expansion of period of investment), behavioral stereotypes in production (maximization of the profit on a microlevel, competitive medium), behavioral consumer stereotypes (maximization of integrated discount consumption, maximization of consumption per capita), institutional and social performances of society (degree of monopolizing, distribution of the incomes).

The axiomatic setting also depends on the problem, which is put by the contributor, what factors of economic growth he/she wants to take into account, what component of general development he wants to study, and so on. The models differ on the kinds of production functions, which describe dependence GDP on production factors. The technologies, natural resources, human capital concern the capital, transactions, and production factors. The model can describe an open or a closed economy, i.e. by taking into account or ignoring the interaction with the world market. The models also differ under orders of the investments. The investments can be as an outcome of a solution of an optimization problem or be set as a certain function describing the behavior of the

investors. The model can describe only one main trajectory of development - trend, and can alongside with the trend, show oscillatory or cyclical solutions.

Often a macromodel is created not for a concrete solution and concrete prognosis of further development, and for installation of properties of economic growth and study of possible responses of economy on various managing and external effects. Usually at the center of attention of the contributors there are rates of economic growth and employment level, norm of accumulation, paces of technological progress, flight of capital abroad, oscillatory business cycles of various durations, response of national economy to the behavior of the world and internal markets, response to devaluation of national currency and inflation, response to modifications in the distribution of the incomes and tax system.

A macromodel of economic growth usually is a system of ordinary differential or difference equations, of balance and econometrics. Sometimes in the right members of such systems variables with lagging arguments, and also external and control actions are included. If the macromodel as the system of the differential equations is closed (amount of endogenous variables equal to the number of the equations), for the specific scripts of exogenous variables on a model the Cauchy problem can be included. If in the right members of the differential equations appropriate control actions are included in the optimization problem. In such optimization problems integrated discount consumption or consumption per capita for infinite period is usually maximized.

2. Main Equations

The macromodel of economic growth usually includes the following equations. The equation of dynamics of GDP is set as usual or rate of production function. The dynamics of all production factors involved in the production function is set either with the help of equations or with the help of scripts. The equation of connection between savings and investments reflects their equality in the closed economy and difference in the magnitude of foreign trade balance in an open economy. The equation of connection between commissioning of production assets and previous investments describes technological features of investment. Usually it is the equations of dynamics for production assets, volume of employment, technological level, human capital, specific resources of GDP and price of natural resource. The task of distribution of GDP for payment of transactions, profit and taxes allows evaluation of the dynamics of effective demand on a home market and dynamics of norm of the savings process. The investment demand is set with the help of a special function or is the solution of an optimization problem. In open economy it is necessary to set dynamics of export and import. Finally, for consideration of short-term and intermediate term processes it is necessary to take into account connection between rates of economic growth and devaluation of national currency, which increases competitiveness of economy. The grain size of the description depends on those problems, which are supposed to be investigated, and on parameters, which are necessary for prediction.

2.1. Production Functions

The production functions in macromodels of economic growth differ with the

considered factors, their properties, and the methods of accounting for these factors.

2.1.1. Account of Capital and Labor

The first functional dependence between the national income Y and production factors - capital K and labor L - was suggested by Cobb and Douglas in a 1928 as

$$Y = AK^{\alpha} L^{1-\alpha}, \tag{1}$$

where elasticity under the capital $\partial \ln Y / \partial \ln K = \alpha$ - share of the capital in the income, elasticity on labor $\partial \ln Y / \partial \ln L = 1 - \alpha$ - share of a labor in the income, A – a factor, which afterwards is to be considered as a function of time $A(t)$ and to connect to scientific and technical progress, (see Douglas P.H., Cobb C.W., 1928). The function of the Cobb - Douglas linearly is homogeneous in capital and labor, is equal to zero, if even one of the factors is absent, and has positive first partial derivative and negative second partial derivative under the factors. Production factors have the property of substitution; i.e. the same level of production can be reached for different K and L. All these properties of Cobb-Douglas production function correspond to the theoretical suppositions about the features of production and the role of production factors.

Leontiev, Solow and other authors have offered an alternative PF, having different properties (see. Kleyner G.B. (1986); Stoleru L. L. (1969)). For example, for Leontiev PF

$$Y = \min\{\alpha K, \beta L\} \tag{2}$$

the linear homogeneity on production factors is executed, but there is no interchangeability. In PF CES, also linearly homogeneous under the factors,

$$Y = A\left[\alpha L^{-\beta} + (1-\alpha) K^{-\beta}\right]^{-\frac{1}{\beta}} \tag{3}$$

the elasticity of a mutual replacement of the factors is constant.

Alongside the usual PF, rate or differential PF can be used. They represent GDP growth rate as the weighted sum of growth rates of production factors, i.e. as the differential equation. For example, we shall accept four hypotheses. GDP - linearly homogeneous function under the capital and labor. GDP - linearly homogeneous function of scientific and technical progress $A(t)$. In economy on a microlevel the profit is maximized, therefore the partial derivative of GDP with respect to labor is equal to the rate of the salary w. The rate of the salary w - function of productivity of labor Y/L. These hypotheses are noted formally as

$$Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L, Y = \frac{\partial Y}{\partial A} A \frac{\partial Y}{\partial L} = w, w = (1-\alpha)\left(\frac{Y}{L}\right)^{\beta} \tag{4}$$

The system (4) is equivalent to the differential equation

$$\frac{dY}{Y} = \left[1 - \left(\frac{Y}{L}\right)^{\beta-1}(1-\alpha)\right]\frac{\dot{dK}}{K} + (1-\alpha)\left(\frac{Y}{L}\right)^{\beta-1}\frac{\dot{dL}}{L} + \frac{\dot{dA}}{A} \qquad (5)$$

This equation has the first integral. For constant parameter α and $\beta = 1$ integral is obtained as Cobb-Douglas function. For $\beta \neq 1$ integral is obtained as CES function. Thus, the differential equation for PF is the more general and convenient form of an entry. Let us remark also, that in function CES the parameter $1-\alpha$ can no longer be interpreted as a share of the incomes of labor, and in Cobb-Douglas function, obtained as an integral, the factors of elasticity are constant and should not vary, though some authors assume that these factors are variable.

2.1.2. Account of Scientific and Technical Progress

Distinguish three main types НТП, which can be presented as follows

$$Y = A(t)\, f\left[A_K(t)K, A_L(t)L\right] \qquad (6)$$

The parameter $A(t)$ reflects increase of general efficiency of production, with (technology augmenting) Hicks neutrality. The parameter $A_K(t)$ reflects increase of efficiency of capital, with (capital augmenting) Solow neutrality. The parameter $A_L(t)$ reflects increase of efficiency of a labor, with (labor augmenting) Harrod neutrality. The properties of neutrality are formulated in terms of labor productivity, capital productivity and other parameters, Stoleru (1969). Other kinds of scientific and technical progress (in particular, resources saving) also find the reflection in models (see below). In long-term models STP on Hicks or Harrod is normally used. To select the most scientifically appropriate kind of technical progress for a long-term model of economic growth, in the work of Dubovsky (1989) the same problem of maximization of integrated consumption with various kinds of exogenous progress was chosen. It was found that only solution of the problem with progress in the sense of Harrod gives results close to an economic reality. Therefore the use of STP in the sense of Harrod in models of economic growth is preferable.

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Biographical Sketch

Sergey V. Dubovsky is Assistant Professor, candidate of physical-mathematical sciences, head of laboratory of Institute of the Systems Analysis, RAS, Moscow, Russia. Graduated Moscow Physical-technical Institute and graduate school CAGI. Since 1968 is concerned with mathematical modeling of social-economic systems. Lectures on mathematical economics in the Moscow Physical-technical Institute and Russia University of the Friendship of Folk. Co-author of three monographs: "Mechanics of the space flight" (in Russian, M. Science, 1975), "The Future of the World Economy" (N.Y., Berlin, 1989), "New paradigm of the development of Russia" (in Russian, M. Academia, 1999). Leader and performer 2 domestic and 3 international global projects. Author mathematical models of the scientific-technical progress, of cycle Kondratiev, of non-equilibrium economic growth, of non-stationary inflation, of exchange rate and foreign trade, of Russia social cataclysms, of variational principle economy. Author more than 80 scientific publications. Laureate bonus of Russia Fund of Fundamental Studies for 1998.