PLASTIC VERSUS ELASTIC DESIGN OF STEEL STRUCTURES

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Contents

Summary

Plastic design offers several advantages over the traditional elastic design. With plastic analysis, a structure can be designed to form a preselected yield mechanism at ultimate load level leading to a known and predetermined response during extreme events. This has special significance in the context of Performance-Based Design philosophy where it is essential for the structure to deform in a preselected manner to achieve desired levels of performance. Because of this and other advantages, many of the design guidelines and specifications particularly for seismic applications, rely directly or indirectly on plastic design concepts. This chapter presents an overview of plastic design theory and its applications. Key concepts including elastic-plastic behavior at

cross-section, component, and system levels are first presented. Plastic analysis methods including mechanism and incremental load methods are reviewed. A design example is provided to illustrate the contrasts between elastic and mechanism-based plastic design approaches. Finally, factors that affect plastic behavior are addressed.

1. Synopsis of Elastic and Plastic Design Methods

Basically there are two approaches to provide adequate strength of structures to support a given set of design loads: Elastic Design and Plastic Design. Drift checks are also required in actual design practice, but the focus of discussion herein will be limited to strength consideration only.

Elastic design is carried out by assuming that at design loads structures behave in a linearly elastic manner. An elastic analysis is performed by applying the design loads and required internal forces in the structural elements (members and connections) are determined and adequate design strength is provided. Since the element forces are determined based on elastic behavior, the design is governed by elastic stiffness distribution (ratios) among the system elements.

It is commonly understood that most structures designed by elastic method possess considerable reserve strength beyond elastic limit until they reach their ultimate strength. The reserve strength is derived from factors, such as structural redundancy, ability of structural members to deform inelastically without major loss of strength (i.e., ductility), etc. One drawback of using elastic method for designing such structures with ductile members is that the reserve strength beyond elastic limit is neither quantified nor utilized explicitly. But more importantly, the yield state (mechanism) of the structure at ultimate strength level is also not known. The yield mechanism may involve structural members that could lead to undesirable system performance under accidental overloading or extreme events, such as strong earthquake ground motion, blast, impact, etc.

This chapter presents an overview of plastic design concepts and their modern applications in which emphasis is placed on designing the structure with a preselected yield mechanism for enhanced performance under extreme loading. An overview of classical plastic analysis methods as applied to steel frame structures is first provided for reference. A design example is then presented to illustrate the contrasts between elastic and mechanism-based plastic design approaches.

2. Elastic and Plastic Behavior of Structural Members

2.1 Introduction to Elastic-Plastic Behavior

Attempts to systematically utilize and quantify reserve strength to overcome the shortcoming of classical elastic analysis were made as early as 1914 (Heyman 1998). Significant advances were made after the 1930s. The fundamental theorems available in the late 1940s to early 1950s (Horne 1950, Greenberg and Prager 1952) eventually provided a foundation for the widespread acceptance of the theory of plasticity.

Central to the idea of all plastic analysis methods is an implicit assumption that the structure being analyzed is made from ductile materials. Most civil engineering materials possess ductility to a certain degree. However, in this article, the discussion will be limited to steel. Ductile nature of steel makes it one of the most suitable candidates for plastic analysis.

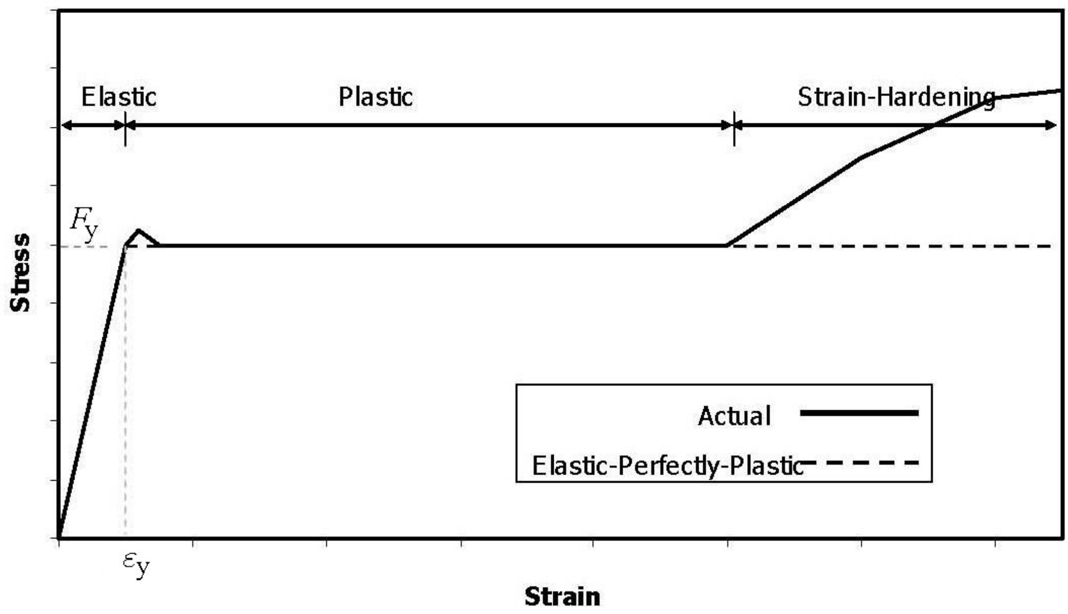

Figure 1. Typical Stress-Strain Diagram of Structural Steel.

A typical stress-strain curve of structural steel is shown in Figure 1. The stress-strain relationship can be largely divided into elastic, plastic, and strain hardening regions. In structural design, it is customary to neglect the strain hardening of the material and to utilize mainly the elastic and plastic parts of the stress-strain relationship. To this end, a simple bilinear approximation is usually adopted. This results in the elastic-perfectly plastic stress-strain model as shown by the dashed line in Figure 1. This model is assumed for all subsequent analyses in this chapter. More complex models can be used in the analyses, if preferred, using the same basic principles. As can be seen from Figure 1, large deformation can occur beyond the elastic limit. This ability to undergo significant inelastic deformation allows a structure made from a ductile material to maintain stable behavior beyond the elastic limit and to redistribute the loads to other parts of the structure that are less stressed. The effect of inherent ductility on the response of a simple structure is illustrated in the following example.

Consider a simply supported wide-flange beam (b_f = 6.06", t_f = 0.605", h = 18.06", $t_w = 0.36$", $S_x = 77.77$ in.^{3}) under a center point load of progressively increasing magnitude as shown in Figure 2. The response of the beam for the entire range of loading up to full plastification will be studied by monitoring the stress and strain distributions of the beam section at mid-span. The analysis is carried out by using the following assumptions:

1) Plane section remains plane implying that the strain distribution is linear.
2) Deformation is small.

3) The material is elastic-perfectly plastic as shown in Figure 1 with F_y =36 ksi and $E = 29000$ ksi.

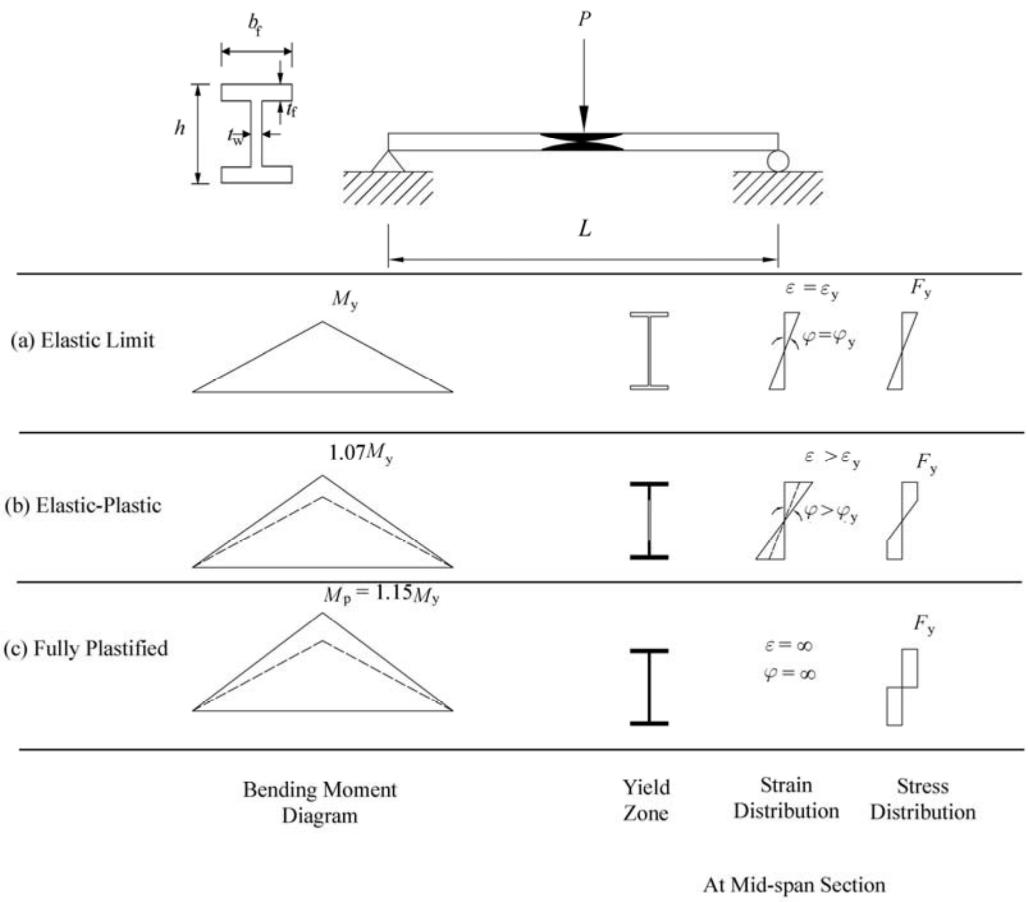

Figure 2. Response of a Simple Beam; (a) Elastic (b) Elastic-Plastic (c) Fully Plastified (Beedle 1961).

The bending moment diagrams for the beam and the strain and stress distributions at the mid-span section for the entire range of loading up to full plastification are shown in Figure 2. The initial response of the beam under loading is elastic. In the elastic regime, the stress and strain distributions as well as the response of the beam are given by the classical beam theory. The limit of the elastic response is reached when the maximum stress in the cross section reaches the yield stress, that is:

$$M_y = F_y S_x = 36 \times 77.77 = 2799.7 \text{ kips} - \text{in} . \qquad (1)$$

where M_y is called the yield moment. The stress and strain distributions at first yield are shown in Figure 2(a). The corresponding yield curvature, φ_y, of the section under consideration is

$$\varphi_y = \varepsilon_y / (h/2) = (36/29000)/(18.06/2) = 1.37 \times 10^{-4} \text{ in}^{-1} \qquad (2)$$

From this point onwards, any further increase in the load will result in the strain at extreme fibers beyond the yield point. However, the stress remains at the yield level since elastic-perfectly plastic material behavior is assumed. Therefore, the contribution of the yielded portions in resisting the applied load remains constant once yielding occurs. The increase in the internal resistance required to counterbalance the additional load is thus delegated or "redistribute" to other portions of the section that are still elastic. The yielding spreads further and further into the elastic zone of the section as the load increases. An example of the strain and stress distributions in the inelastic regime is shown in Figure 2(b).

The theoretical limit of resistance is reached when the entire cross section yields. This occurs when the curvature approaches infinity as shown in Figure 2(c). At this limit, the section is fully plastified and the stress at all points in the section is equal to the yield stress. No further increase in the resistance is possible. The maximum moment that the section can resist is called the plastic moment, M_{p}, and is given by:

$$M_{\mathrm{p}} = \int_{area} F_{\mathrm{y}} y dA = F_{\mathrm{y}} Z_x \qquad (3)$$

where y is the distance from the neutral axis and Z_x is call the plastic modulus

$$Z_x = \int_{area} y dA \qquad (4)$$

For this example beam, with a doubly symmetric section, the plastic moment can be computed by summing the moment resistance from the flanges and the web about the neutral axis.

$$M_{\mathrm{p}} = F_{\mathrm{y}} \left[\left(b_{\mathrm{f}} - t_{\mathrm{w}} \right) t_{\mathrm{f}} \left(h/2 - t_{\mathrm{f}}/2 \right) \right] \times 2 + F_{\mathrm{y}} \left[\left(h/2 \right) t_{\mathrm{w}} \left(h/4 \right) \right] \times 2 \qquad (5)$$

This leads to

$$Z_x = M_{\mathrm{p}} / F_{\mathrm{y}} = \left[\left(b_{\mathrm{f}} - t_{\mathrm{w}} \right) t_{\mathrm{f}} \left(h/2 - t_{\mathrm{f}}/2 \right) + \left(h/2 \right) t_{\mathrm{w}} \left(h/4 \right) \right] \times 2 \qquad (6)$$

$$Z_x = \left[(6.06 - 0.36) \cdot 0.605 \cdot (18.06/2 - 0.605/2) + (18.06/2)(18.06/4) \cdot 0.36 \right] \times 2 \qquad (7)$$

$$Z_x = 89.55 \,\mathrm{in}^4 . \qquad (8)$$

Consequently, the plastic moment is equal to

$$M_{\mathrm{p}} = 36 \times 89.55 = 3223.8 \text{ kips-in} \quad = 1.15 M_{\mathrm{y}} \qquad (9)$$

The response of the section at mid-span as discussed above is best summarized by the moment-curvature plot of the section as shown in Figure 3. The relationship is linear in the initial portion. The onset of yielding is indicated as Point 1 on the plot. This point corresponds to the state of the beam in Figure 2(a). Upon further loading, the moment-curvature relationship now starts to deviate from the straight line and the response enters the inelastic regime. The state of stress and strain distributions as given in Figure 2(b) correspond to Point 2 in Figure 3. The theoretical limit of resistance described by Figure 2(c) is reached as the curvature becomes infinity and the moment-curvature plot approaches the horizontal line corresponding to the plastic moment.

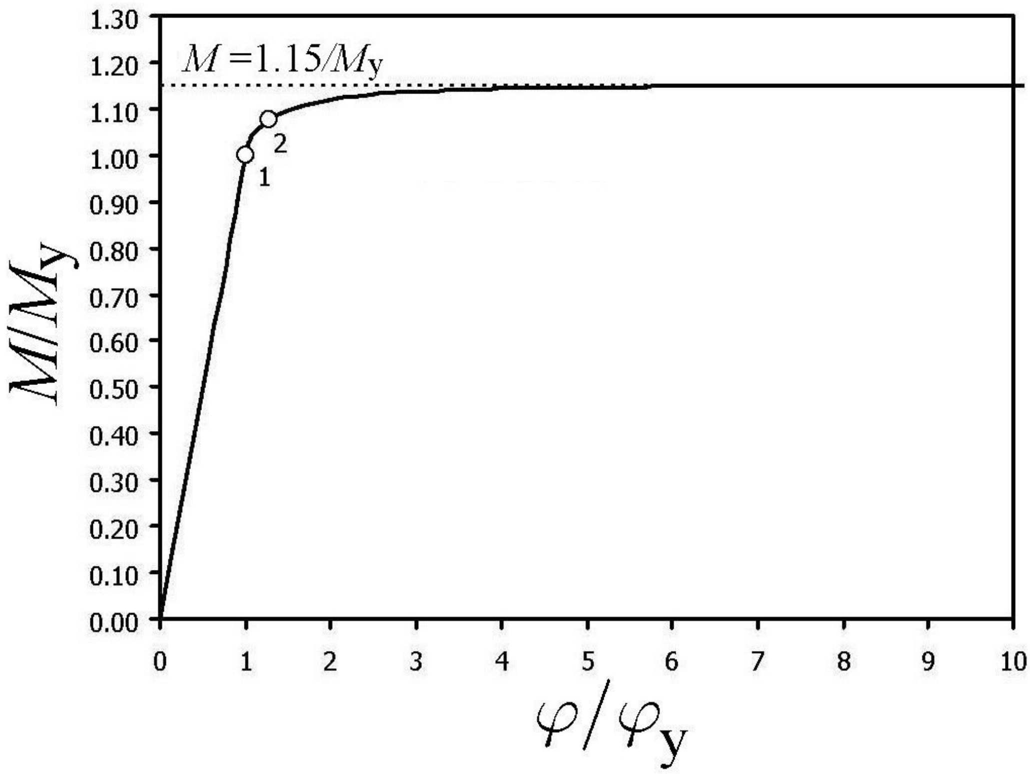

Figure 3. Moment-Curvature Relationship (Beedle 1961).

In the above example, as the section at mid-span approaches its fully plastified state, the curvature increases rapidly with an almost constant value of bending moment. The section thus behaves almost as a hinge at this stage, albeit a hinge accompanying a constant value of moment. This is called the "plastic hinge." Due to the formation of this plastic hinge, an indefinitely large rotation occurs. The structure is said to have formed the "yield mechanism" when enough plastic hinges occur such that no further increase in the loading is possible. For a simple beam, only one hinge is required. The yield mechanism of the example beam and its limit load are shown in Figure 4.

The reserve strength of the structure beyond the yield point appears to be marginal for the example beam ($M_p/M_y = 1.15$). However, for a more complex structure, the redistribution of internal stresses as observed in the previous example also occurs at the member level. Once a plastic hinge forms at one location, the moments are redistributed to other parts of the structure that are still elastic. For this redistribution to occur, the structure must be statically indeterminate to allow for an alternative load path after the first hinge has formed. Depending on the degree of indeterminacy, a number of hinges are required before the yield mechanism can form resulting in significant reserve strength beyond the elastic limit. This reserve strength due to the redistribution of internal forces is what the plastic design methods seek to harness.

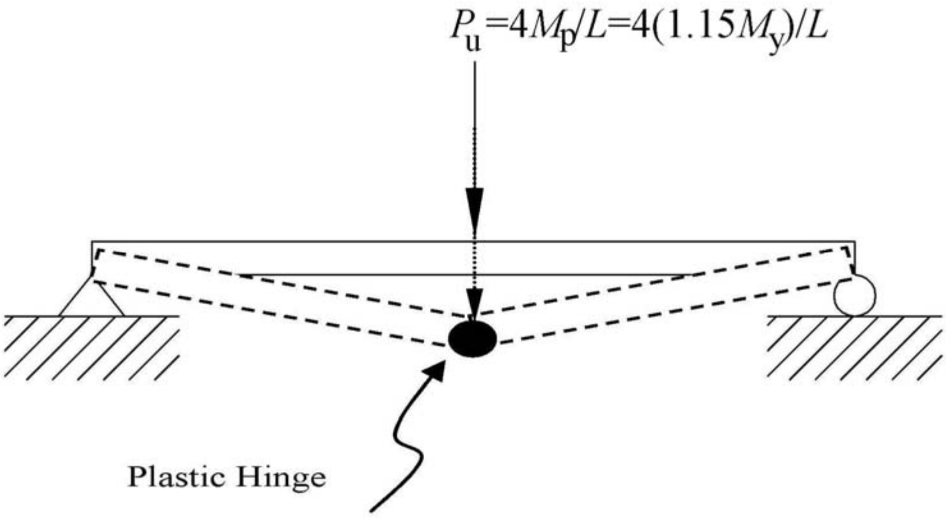

Figure 4. Yield Mechanism of a Simple Beam.

3. Concepts of Plastic Analysis

One goal of plastic analysis and design is to utilize the reserve strength beyond the elastic limit due to the redistribution of internal forces. Therefore, the analysis focuses on the internal forces at the limit level when the yield mechanism forms. Plastic analysis procedures are based on the considerations of equilibrium, yield mechanism, and plastic strength conditions. There are three fundamental plastic theorems regarding theses three conditions as applied to plastic analysis of frames consisting of flexural members. The theorems can be stated as follows (Neal 1977):

1) Upper Bound Theorem (Kinematic Theorem) "For a given frame subjected to a set of loads, the value of load which corresponds to any assumed mechanism must be either greater than or equal to the collapse load."
2) Lower Bound Theorem (Static Theorem) "If there exists any distribution of bending moment throughout a frame which is both safe and statically admissible with a set of loads, the value of loads must be less than or equal to the collapse loads."
3) The Uniqueness Theorem "If for a given frame and loading at least one safe (strength greater than moment demand condition) and statically admissible bending moment distribution (equilibrium condition) can be found, and in this distribution the bending moment is equal to the fully plastic moment at enough cross-sections to cause failure of the frame as a mechanism due to rotations of plastic hinges at these sections (mechanism condition), the corresponding load will be equal to the collapse (ultimate) load"

It should be mentioned that terms such as "ultimate", "failure", and "collapse load" are traditionally used in plastic analysis and design for static loads, where formation of mechanism is indicative of "failure" of structures to carry any further load. In the context of modern seismic design, where plastic analysis plays an important role, structures are expected to form mechanism during strong ground motions. But that does not mean "failure" or "collapse" in the dynamic sense, until the displacements become excessively large. Therefore, the terms "yield mechanism" and "limit load" are more appropriate and have been used herein.

Based on the fundamental theorems stated above, two common analysis methods exist that can be utilized to computed the limit load for a given structure. These two methods are generally referred to as the "mechanism method" and the "statical method." They are widely discussed in standard texts (Beedle 1961, Neal 1977, Salmon et al. 2009, Wong 2009), hence only a brief overview of the mechanism method will be provided herein for information.

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Biographical Sketches

Sutat Leelataviwat is Assistant Professor and Director of the Civil Engineering Technology Program in the Department of Civil Engineering at King Mongkut's University of Technology Thonburi, Thailand. He received his B.Eng. degree in Civil Engineering from Chulalongkorn University in Thailand and M.S. and Ph.D. in Structural Engineering from the University of Michigan at Ann Arbor, USA. After the completion of his Ph.D. degree, he worked as a structural engineer in the U.S. and later in Thailand before joining King Mongkut's University of Technology Thonburi. He currently serves as a committee member on Wind and Seismic Effects on Structures of the Engineering Institute of Thailand. His research has mainly focused on the modern applications of plastic design theory for the performance-based seismic design framework. He has also served as a consultant on various engineering projects ranging from structural and seismic design of high-rise buildings and seismic evaluation of industrial and oil refinery plants.

Subhash C. Goel is Professor Emeritus of Civil Engineering at the University of Michigan, Ann Arbor. His research activities have involved analytical and experimental studies of seismic behavior of steel and composite Steel-RC or masonry structures with the objectives to develop rational analytical models for use in analysis and design, as well as to formulate improved methods for earthquake resistant design of safer and more economical structures for new construction as well as upgrading of existing structures. More recently, he has been working on development of a new Performance-Based methodology for seismic design of structures which is based on energy and plastic design concepts. He is the recipient of the NOVA Award from Construction Innovation Forum in 1999 for development of Special Truss Moment Framing System for seismic resistance. He also received the ASCE Shortridge Hardesty Award in 2004 and the AISC Lifetime Achievement Award in 2007.

Shih-Ho Chao is Assistant Professor at the University of Texas, Arlington. He received his Ph.D. from the University of Michigan and was a post-doctoral research fellow and lecturer at the University of

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