FLUID MECHANICS IN PIPELINES

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Contents

Summary

This article presents the fundamental theory of fluid flow through closed conduits and the engineering applications toward the design and operation of pipeline systems. It deals with head losses due to resistance of flow through tubes, valves, and other fittings; the optimization of pumping lines and flow networks; and water hammer and pump characteristics.

Optimization of pumping pipelines and reservoir sites is carried out by means of a spreadsheet program. A bibliography of the relevant publications dealing with the above aspects concludes the article.

1. Introduction

Water is conveyed over large and small distances by pipeline for irrigation, hydropower, and commercial or domestic use. In a typical water supply system, water is abstracted from a source, purified, and pumped to a storage reservoir. From there it is gravitated to consumers connected to a reticulation system. The pipeline is the main component in most systems.

Some of the older pipe materials are now seldom used, e.g., cast iron, and there are new materials, e.g., plastics, glass fiber, and composites. Steel, ductile iron, concrete, and fiber cement are used, and some require lining or coatings to ensure durability. The first record of the use of pipes leads back to the Chinese, who used bamboo pipes more than 3000 years ago. At a later stage in history, the Romans used lead pipes in Pompeii and stone pipes in Rome. Thus the idea of closed conduits was conceived, but perhaps not fully understood and consequently not fully exploited.

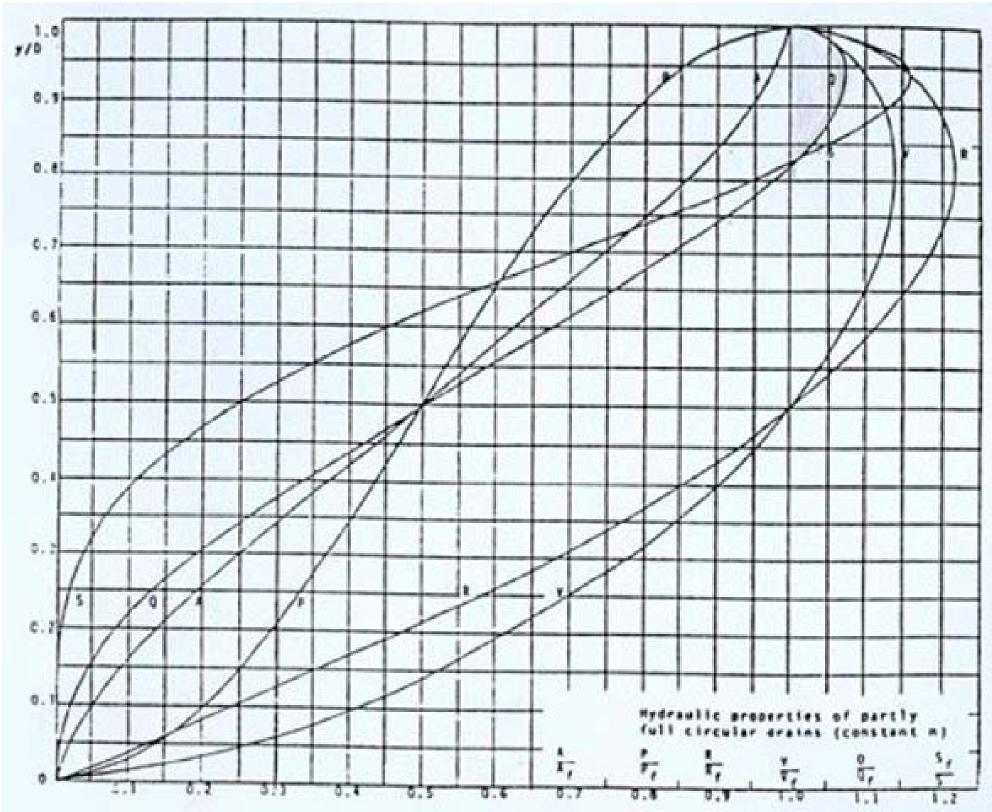

Figure 1. Hydraulic properties of partly-full circular drains

Early pipelines flowed part-full, the pipeline profile therefore being confined to the hydraulic grade line. Even though it is now realized that a pipe flowing partly full, at a water depth of about 95% of the diameter, has a larger hydraulic radius than a pipe flowing full, and consequently a part-full pipe can theoretically convey a greater flow than a full pipe, as shown in Figure 1, it is doubtful whether the ancient engineers realized this fact.

The reason for adopting a circular cross section was probably more a practical than a technical one. The fact that the circular section has a greater *hydraulic radius* (cross-sectional area divided by wetted perimeter) than any other shape, with its resultant beneficial effect on friction losses, would also probably not have been understood in ancient times. The structural advantage of circular pressurized conduits was probably also not realized at the time. A circular-section pressure pipe acts in tension, thus optimizing the material properties.

Pipelines are nowadays operated under pressure and flowing full-bore, as thus they convey greater flow than a part-full pipe. Sewers and drains, however, are often designed to flow part-full and are installed at the level of the hydraulic grade line to ensure that there are no dips that could block up. The increased flow when flow depth is about 95% full at any gradient, as can be seen in Figure 1, cannot be relied upon because of turbulence and obstructions, which could reduce the discharge capacity of a part-full pipe considerably.

Water supply engineers achieved a number of advances in pipeline technology during the nineteenth century. The most significant advances have perhaps been made in the science of fluid mechanics. Apart from a spate of empirical formulas early in the twentieth century, the science of boundary layers and roughness is the one that has advanced rapidly. The Darcy-Weisbach formula is gaining acceptance as the most reliable method for estimating friction losses (see *Fluid Mechanics*).

The analysis of unsteady flow in pipelines progressed further with the advent of computers. The theory of the water hammer was understood and developed at the beginning of the twentieth century, but advances were the most rapid in the last three decades of the twentieth century. The accessibility of computers to engineers has also facilitated the analysis of complex pipe networks, and the computer-oriented techniques of system analysis have been adapted to the optimization of pipe layouts and sizes.

2. The Fundamental Equations of Fluid Flow

The three basic equations in fluid mechanics are the continuity equation, the momentum equation, and the energy equation. For steady, incompressible, one-dimensional flow, the continuity equation is simply obtained by equating the flow rate at any section to the flow rate at another section along the stream tube. By "steady flow" is meant that there is no variation in velocity at any point with time. "One-dimensional" flow implies that the flow is along a stream tube and there is no lateral flow across the boundaries of stream tubes. It also implies that the flow is irrotational.

The *momentum equation* stems from Newton's basic law of motion and states that the change in momentum flux between two sections equals the sum of the forces on the fluid causing the change. For steady, one-dimensional flow, the momentum equation is given by:

$$\Delta F_x = \rho Q \Delta V_x \qquad (1)$$

where F is the force, ρ the fluid mass density, Q the volumetric flow rate, V the velocity, and subscript x the "x" direction.

The *energy equation* is basically derived by equating the work done on an element of fluid by gravitational and pressure forces to the change in energy. Mechanical and heat energy transfer is excluded from the equation. In most systems, there is energy loss due to friction and turbulence, and a term is included in the energy equation to account for this.

The resulting energy equation for steady flow of incompressible fluids is termed the Bernoulli equation and is written as follows:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 + h_l \qquad (2)$$

where

V = mean velocity at sections 1 and 2
P/γ = pressure head (units of length)
$V^2/2g$ = velocity head (units of length)
γ = unit weight of fluid
g = gravitational acceleration
Z = elevation above an arbitrary datum
P = pressure
h_l = head loss, due to friction or turbulence, between sections 1 and 2

The sum of the velocity head, the pressure head, and the elevation head is termed the *total head*.

Strictly speaking, the velocity head should be multiplied by a coefficient to account for the variation in velocity over the cross section of the conduit. The average value of this coefficient for turbulent flow is 1.06 and for laminar flow it is 2.0.

For the Bernoulli equation to apply, the flow should be *steady*, i.e. there should be no change with time in the velocity at any point in the conduit. The flow is assumed to be one-dimensional and irrotational. The fluid should be incompressible, although the equation may also be applied, with reservations, to gases.

3. Flow Head-Loss Relationships

Flow formulas relating head loss to either velocity or discharge have been derived over a long period of time, originally based on empirical or experimental concepts, but more recently supplanted by rational formulas. These two sets of relationships are now examined in more detail in the following section.

3.1. Empirical Flow Formulas

The throughput or discharge capacity of a pipe of fixed dimensions depends on the total head difference between the ends. This head is consumed by friction and other (minor) losses.

The first friction head-loss/flow relationships were derived from field observations. These empirical relationships are still popular in waterworks practice, although more rational formulas have been developed.

The head-loss/flow formulas thus established are termed *conventional* formulas and are usually in an exponential form of the following type:

$$V = K R^x S^y \quad \text{or} \quad S = K'Q^n / D^m \qquad (3)$$

where V is the mean velocity of flow, K and K' are coefficients, R is the hydraulic radius (cross-sectional area of flow divided by the wetted perimeter, and, for a circular pipe flowing full, equaling one quarter of the diameter), and S is the hydraulic total-head gradient (in meters of head loss per meter length of pipe), Q is the discharge, D is the pipe's internal diameter, and x, y, n, and m are numerical valued exponents.

Some of the equations that are more frequently applied are listed in Table 1 (where the Ks, the Cs, n, and λ are numerical coefficients), as presented by Stephenson in *Pipeline Design for Water Engineers*:

Name of Originator(s)	Basic equation	SI units
Hazen-Williams	$S = K_1(V/C_w)^{1.85}/D^{1.167}$	$K_1 = 6.84$
Manning	$S = K_2(nV)^2/D^{1.33}$	$K_2 = 6.32$
Chezy	$S = K_3(V/C_z)^2/D$	$K_3 = 13.13$
Darcy-Weisbach	$S = \lambda V^2/2gD$	Nondimensional

Table 1. Pipe-friction equations

The above equations are not universal, except for the Darcy-Weisbach formula, and the form of the equations depends on the units. It should be borne in mind that these formulas were derived for normal waterworks practice and take no account of variations in gravity, temperature, or type of liquid. They are intended for application to turbulent flow in pipes that are more than 50 mm in diameter. The friction coefficients vary with pipe diameter, type of interior finish, and the age of the pipe.

The conventional formulas are comparatively simple to use, as they do not involve the fluid viscosity. They may be solved directly, as they do not require an initial estimate of the Reynolds Number for determining the friction factor, as shown in the next section. The rational equations cannot be solved directly for the flow. Solution of the formulas for velocity, diameter, or friction-head gradient is a simple matter with the aid of a slide rule, calculator, computer, nomograph, or graphs plotted on log-log paper. The equations are particularly useful for analyzing flows in pipe networks where the flow/head-loss equations have to be iteratively solved many times over.

The most popular flow formula used in waterworks practice is the Hazen-Williams formula. If this formula is to be used frequently, its solution with the aid of a chart is the most efficient way to proceed. Many waterworks organizations use graphs of head-loss gradient plotted against flow for various pipe diameters and various discharge and friction-loss coefficients (C, n, and λ values in Table 1). As the value of C, for instance, decreases with age, type of pipe, and properties of water, field tests are desirable for a more accurate assessment of C.

3.2. Rational Flow Formulas

Although the *conventional* flow formulas are likely to remain in use for many years, more *rational* formulas are gradually gaining acceptance among engineers. These new formulas have a sound scientific basis backed by numerous measurements, and they are universally applicable. Any consistent units of measurement may be used, and liquids of various viscosities and at different temperatures conform to the proposed formulas.

The rational flow formulas for flow in pipes are similar to those for flow past immersed bodies or over flat plates, as described by Schlichting. The original research was done on small-bore pipes with artificial roughness. Lack of data on roughness for large pipes has been one deterrent to the use of these relationships in waterworks practice.

The velocity in a pipe flowing full varies from zero on the internal boundary to a maximum in the center. Shear forces on the pipe wall oppose the flow, and a *boundary layer* is established, with each annulus of fluid imparting a shear force onto an inner, neighboring concentric annulus. The resistance to relative motion of the fluid is termed the *kinematic viscosity*. In turbulent flow, resistance is imparted by turbulent mixing, with the transfer of particles of different momentum between one annular layer and the next. A boundary layer is established at the entrance to a conduit, and this layer gradually expands with distance along the conduit until it reaches the center (see *Turbulent Flow Modeling*).

The *Reynolds Number*, Re = VD/ν, is a nondimensional number incorporating the fluid's kinematic viscosity, ν, which is absent in the conventional flow formulas. Flow in a pipe is laminar for low Re values (less than 2000) and becomes turbulent for higher Re values (which is normally the case in water supply practice). The basic head-loss equation of Darcy-Weisbach is derived by setting the boundary shear force (over a length of the pipe) equal to the loss in the pressure, multiplied by the area (over the same length), as given below:

$$\tau\pi DL = \gamma h_f \cdot \pi D^2, \qquad \therefore h_f = \frac{4\tau/\gamma}{V^2/2g}\frac{L}{D}\frac{V^2}{2g} = \lambda\frac{L}{D}\frac{V^2}{2g} \qquad (4)$$

where $\lambda = (4\tau/\gamma)/(V^2/2g)$ (referred to as the Darcy friction factor), τ is the shear stress, D is the pipe diameter, and h_f is the friction head loss over a length, L, of pipe. The friction factor, λ, is a function of the value of *Re* and the relative roughness, e/D, where e is the absolute roughness. For laminar flow, Poiseuille found that λ = 64/Re, i.e. λ is independent of the relative roughness. Laminar flow will not occur in normal civil-engineering waterworks practice. The transition zone between laminar and turbulent flow is complex and undefined, but is also of little interest in hydraulic engineering practice.

Turbulent flow conditions may occur with either a smooth or a rough boundary. The equations for the friction factor (for both laminar and turbulent flow conditions) are derived from the general equation for the velocity distribution in a turbulent boundary layer, which is derived from the *mixing-length* theory:

$$\tau = \rho k^2 l^2 \left(\frac{dv}{dy}\right)^2 \qquad (5)$$

Integrating the above over the pipe cross section with Von Karman's constant k = 0.4, yields:

$$v / \sqrt{(\tau / \rho)} = 5.75 \log y / y' \qquad (6)$$

where *v* is the velocity at a distance *y* from the boundary, and *y′* is the thickness of the laminar boundary sublayer. For a *hydrodynamically smooth* boundary, there is such a *laminar sublayer*, which Nikuradse found to be expressed by the proportionality $y' \propto \nu/\sqrt{\tau/\rho}$, thus resulting in:

$$\frac{v}{\sqrt{\tau/\rho}} = 5.75 \log y \frac{\sqrt{\tau/\rho}}{\nu} + 5.5 \qquad (7)$$

The constant 5.5 was determined experimentally.

where the boundary is rough, the laminar sublayer is affected, and it was established by Nikuradse that y' = e/30 where *e* is the boundary absolute roughness. Thus

$$\frac{v}{\sqrt{\tau/\rho}} = 5.75 \log \frac{y}{e} + 8.5 \qquad (8)$$

By rearranging Eqs. (7) and (8) and expressing *v* in terms of the average velocity *V* by means of the equation $Q = \int v dA$, it is obtained that:

$$\frac{1}{\sqrt{\lambda}} = 2 \log \mathrm{Re} \sqrt{\lambda} - 0.8 \qquad (9)$$

(turbulent boundary layer for *smooth* boundary)

and

$$\frac{1}{\sqrt{\lambda}} = 2 \log \frac{D}{e} + 1.14 \qquad (10)$$

(turbulent boundary layer for *rough* boundary)

It is noticed that for a smooth boundary, λ is independent of the relative roughness, *e/D*, and for a very rough boundary, it is independent of the Reynolds Number, *Re*, for all practical purposes.

Colebrook and White combined Eqs. (9) and (10) to produce an equation covering both smooth and rough boundaries, as well as the *transition zone* between these two cases. Following the research on turbulence carried out by Reynolds, Von Karman, and other investigators, the *boundary-layer theory* was developed to yield a relationship between flow and head loss for turbulent flow conditions in pipes. Colebrook and White fitted an equation to the data, yielding the so-called Darcy-Weisbach *friction factor* λ (for which the symbol *f* is used in the US).

$$\frac{1}{\sqrt{\lambda}} = 2 \log \left(\frac{k}{3.7D} + \frac{2.5}{\mathrm{Re}\sqrt{\lambda}} \right) \qquad (11)$$

In view of the complex relationship existing between the Darcy-Weisbach friction factor, λ, the Reynolds Number, Re, and the relative roughness, *k*/*D*, explicit head-loss charts have been prepared by Ackers at Wallingford. The symbol *k* used here is a measure of the absolute boundary roughness and is the same as *e* used before in Equation (8).

By comparing the Hazen-Williams equation with the Darcy-Weisbach equation, it is deduced that

$$C_w = 422.4/(\lambda^{0.54}\ \mathrm{Re}^{0.08}) \qquad (12)$$

The Hazen-Williams coefficient, C_w, is therefore a function of λ and Re, and values may be plotted on what is known as a *Moody diagram*, which is shown in Figure 2.

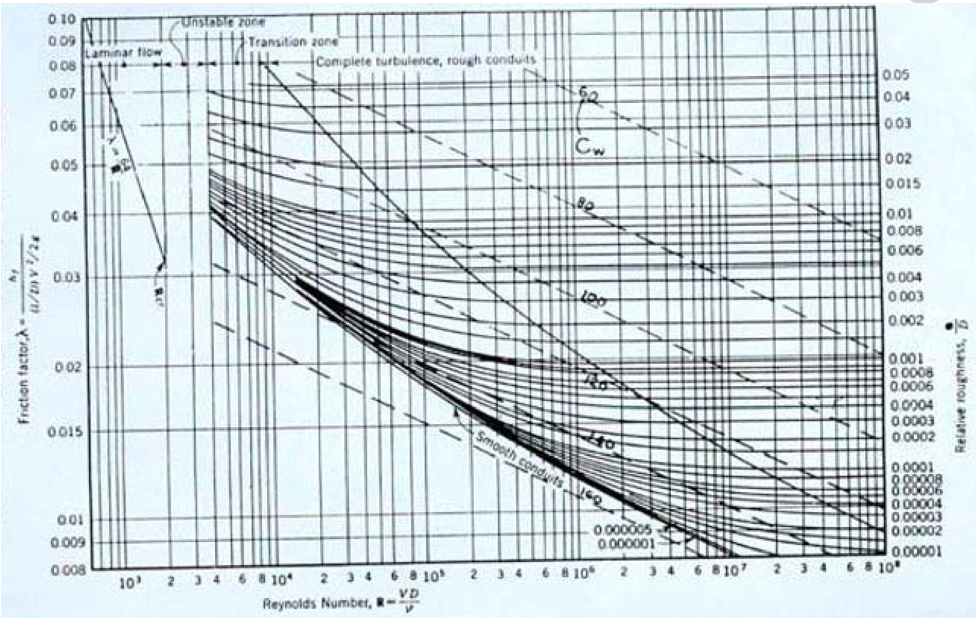

Figure 2. Moody resistance diagram for uniform flow in conduits

It will also be observed from Figure 2 that lines for constant Hazen-Williams coefficient coincide with the Colebrook-White lines only in the transition zone between *partially turbulent smooth boundary* and *completely turbulent rough boundary* flow. In the completely turbulent zone with a particular pipe, the Hazen-Williams coefficient declines with an increasing flow rate, and with increasing roughness. The Hazen-Williams equation should therefore be used with caution for high Reynolds Numbers and rough pipes. It will also be noted that values of C_w above about 155 are impossible to be attained in waterworks practice. The corresponding C_w lines fall below the smooth line on the Moody diagram.

Table 2 gives a summary of the accepted friction factors for different pipe materials and lining conditions.

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Biographical Sketch

D. Stephenson is professor of hydraulic engineering in the Department of Civil Engineering, University of the Witwatersrand, Johannesburg, South Africa, since 1977, where he is holder of Rand Water Chair of Water Engineering. He has the following qualifications: Pr. Eng., D.Sc. (Eng.) (Wits), FSAICE, FICE, FASCE, Member and Council Member of IAHR. He is an authority on water-hammer analysis, pipe flow, and pipeline engineering and has taught many postgraduate courses on these subjects. He is chairman of the African division of the International Association of Hydraulic Engineering and Research (IAHR) and is actively attending and participating in international conferences on hydraulics and water engineering.

His main activity is teaching and research, heading the Water Systems Research Group at the University of the Witwatersrand since 1982. He also is a guest visiting professor at the University of Stuttgart in Germany on a part-time annual basis and does consulting work, too. He is the author of more than 150 professional papers and of 8 technical books.