DYNAMIC-STOCHASTIC MODELS OF RIVER RUNOFF GENERATION

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Summary

Achievements in development of dynamic-stochastic models of river runoff generation have been described. The models are considered as a natural basis for complimentary use of genetic (deterministic) and stochastic methods of river runoff hydrology. Evolution of dynamic-stochastic models with random inputs has been reviewed which combine a deterministic model describing runoff generation processes with stochastic models of the meteorological factors governing these processes. It has been shown that the dynamic-stochastic models open up fresh opportunities for hydrologists to refine understanding of runoff generation processes and extend the range of the traditional engineering applications of both deterministic and stochastic approaches. Examples of such an extension into the fields of extreme flood frequency assessment and probability flood forecasting have been demonstrated.

"In seeking to understand the behaviour of hydrologic systems of interest it is necessary to draw on standard results from both the statistical study of random systems and the deterministic analysis of classical fluid mechanics and hydraulics.…Progress in both areas [deterministic and stochastic hydrology] would benefit if they were considered as complementary rather than separate fields of investigation.

J. C. I. Dooge "Bringing it all together". Hydrology and Earth System Sciences, 9, 3–14, 2005

1. Introduction

River runoff generation is among the classic problems of the watershed hydrology. A large body of hydrometeorological observations and measurements has been accumulated for decades and allowed hydrologists to form general understanding of the main processes which control runoff generation under different physiographic and climatic conditions. However, because of baffling complexity and extreme spatial-temporal variability of these processes, it is not possible to investigate runoff generation and create reliable methods of runoff prediction and forecasting on the basis of the available observations only. Parallel with accumulating experimental data, the prospects are in developing models of runoff generation reflecting, as much as possible, the present-day knowledge on the hydrological systems and utilizing the available observations.

A large number of runoff generation models have been developed by hydrologists, teams of hydrologists and collaborating hydrological institutes during the last decades. Overwhelming majority of these models is deterministic, i.e. a single set of input and parameters is used to simulate a single set of the model output (e.g. runoff hydrograph). In compliance with the deterministic nature of the models, their developers associate capabilities for improvement of the models with including more and more detailed ***deterministic information*** into the model structure. Examples of such an information are, for instance, improved description of the main hydrological processes, accounting for peculiarities of the runoff generation mechanisms in the specific region, assimilating new experimental datasets (e.g. satellite data, detailed measurements in field experiments), etc. As a result, a substantial advance has been made, up to date, in development of the sophisticated physically based models and application of these models to the problems of water resources management, flood protection and other problems of the applied hydrology.

Whatever detailed deterministic, physically based model supplied with all needed data would be developed, the uncertainty still remains indescribable by the developed model over some spatial-temporal scales. To account for different sources of such an uncertainty, the appropriate *stochastic information* should be included in the model. Such an inclusion provides a framework to enhance capability of the runoff generation model in situations when the explicit spatial or temporal details of the model inputs, parameters, etc. are unknown but their relevant stochastic properties are known. Development of such a model which is based on the deterministic conceptualization of the hydrological processes and takes into account available stochastic information on the uncertainty in this conceptualization is the subject of the *dynamic-stochastic modeling*.

The following groups of the dynamic-stochastic hydrological models can be categorized which are distinguished in consideration of different sources of uncertainty:

1. Dynamic-stochastic models with random inputs accounting for stochasticity in *temporal* variations of meteorological factors (precipitation, surface air temperature, air humidity, etc.) controlling runoff generation processes.
2. Dynamic-stochastic models taking into account the stochasticity in *spatial* (within a basin) variations of factors of runoff generation, such as meteorological factors, soil properties, topography, etc.

3. Dynamic-stochastic models taking into account stochastic properties of the parameters which are used in the relationships describing runoff generation processes.
4. Dynamic-stochastic models accounting for stochastic character of measurements which are used under development of the runoff generation model.

The promising way in the dynamic-stochastic modeling is the development of the physically based hydrological model with random meteorological inputs. Such an approach allows one to account for the stochastic nature of the meteorological processes producing runoff generation. The resulting dynamic-stochastic model integrates two components: a physically based, deterministic model of runoff generation and stochastic models of the meteorological variables which are the inputs into the deterministic model (Figure 1).

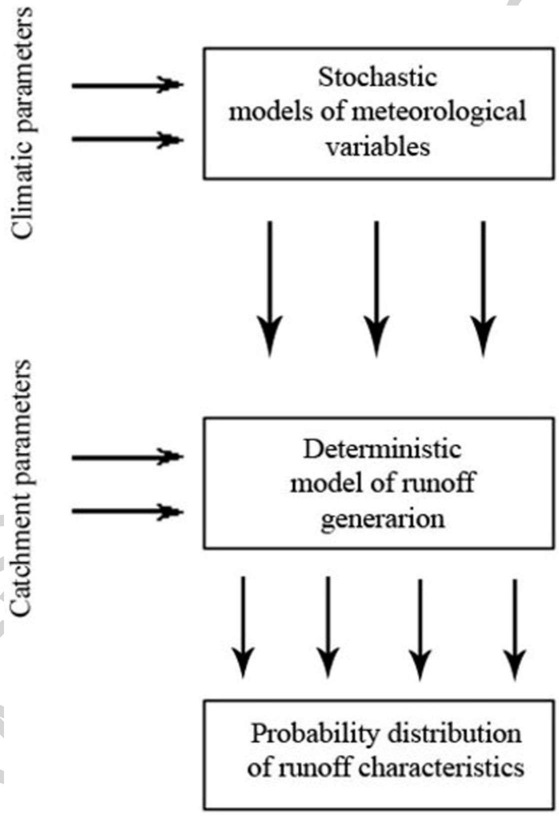

Figure 1. Schematic diagram of a dynamic-stochastic model with random inputs.

For hydrologists, such integration opens up fresh opportunities, which are hardly realizable, if at all, on the basis of either pure deterministic or pure stochastic approach. These opportunities cover all aspects of watershed hydrology from refining of understanding of runoff generation processes to improving the traditional methods of the applied hydrology. For instance, dynamic-stochastic models make possible to investigate hydrological processes under a great diversity of physically feasible meteorological conditions including conditions which have been not registered for the period of observations, to analyze critical meteorological conditions which can result in disastrous floods exceeding the observed ones. In the field of the applied hydrology, dynamic-stochastic models open up a new avenues for improving traditional methods of flood risk assessment (e.g. assessment of magnitude and probability of extreme floods, estimation of sensitivity of long-term flood statistics to different kinds of human activity in a river basin, such as deforestation, urbanization, land-reclamation, etc), flood forecasting (e.g. to change over the present-day deterministic hydrology forecasts to probabilistic ones), etc.

In the next sections, evolution of the dynamic-stochastic models with random hydrometeorological inputs is reviewed, as well as application of these models to solving some of the aforementioned hydrological problems is demonstrated

2. Evolution of the Dynamic-Stochastic Models of Runoff Generation and their Application to Flood Frequency Analysis

The problem of flood frequency estimation was, historically, the main motivation for the development of the dynamic-stochastic models with random hydrometeorological inputs and still remains the most common application of these models.

Planning and design of water resources systems, flood-plain management are fundamentally dependent upon having reliable estimates of flood frequency in terms of both peak discharge and flood volume. Increasing demands for the acceptable potential economic and environmental risk have necessitated improving reliability of the existing methods of estimation of extreme floods, especially floods of very low exceedance probabilities (or very large return period). This problem is becoming more and more complicated because of intensification of human activity in river watersheds and climate change. Most countries have used a set of empirical methods for flood estimation and among these methods the at-site flood frequency analysis (FFA) is the most commonly used method for over a century. Standard at-site flood frequency analysis is based on acquisition of data of flood extremes, computation of observed probabilities of occurrence, fitting of the appropriate probability distribution to the observed probabilities with use of an appropriate parameter estimation technique and, finally, estimation of flood quantiles of the desired probabilities. The fundamental weakness of the at-site FFA for estimating floods of large return period, that is much longer than the period of flood observations, is widely-known and arises from the facts that the recorded data of flood extremes are usually too scarce and statistically non-homogeneous because of man-induced changes of river basins and climate change.

These data deficiencies result in unreliability of estimations of the desired extreme floods. Much of the attempts in improving FFA so far have been focused on the statistical aspects, such as improvement of the parameter estimation techniques, seeking probability distribution for improving goodness of fit, etc. Prospect of these attempts has been clearly identified as long ago as half a century by the world renowned Australian statistician and probabilist P.A.P. Moran: “the form of the distribution is not known and any distribution must be guessed…since the part of the distribution we are interested in is well away from the part where observations provide some information…[this difficulty] cannot be overcome by mathematical sleight of hand” (citation from Klemeŝ, 1993). Lack of knowledge about the form of the parent distribution results in dramatic decreasing of reliability of the extreme floods estimations obtained by the standard FFA. For example, having 100-year series of observations of annual maximum flood discharges and assuming that their distribution form is unknown, one can find that the return period of the highest discharge estimated from this series lies in the interval from 28 to 4000 years (assuming the confidence level of 95%).

Opportunity for refinement of the extreme flood assessment is associated by the hydrological society with invoking new information in addition to statistical one extracted from the runoff observation series. Such an additional information may be both a posterior, empirical information containing in observations of factors affecting flood generation (e.g. meteorological factors, watershed conditions), and a priori information, reflecting accumulated knowledge on flood generation physics. In other words, in the case of lacking runoff data for the standard at-site FFA, a deficit of the information may be compensated, to some extent, by introducing deterministic information on physical processes and stochastic information on better-defined random variables, e.g. meteorological variables. Development of dynamic-stochastic models of runoff generation with random meteorological inputs is seemed to be a natural approach to increase informational content of the extreme flood assessment and allows one to assess magnitude and probability of extreme floods in the basins where flood observation series are short or non-homogeneous, i.e. where the standard at-site FFA turns out to be disabled. There are the following reasons for such a capability of the dynamic-stochastic models:

1. Statistical estimations of meteorological inputs are, mostly, more reliable than estimations of flood characteristics, because time series of meteorological observations are longer, as a rule, than flood observations, and weather depends on the local physiographic conditions to a smaller extent than runoff.
2. Manifold combinations of meteorological inputs varying within comparatively narrow ranges can lead to substantially larger variations of flood characteristics. It means that using the model, one can simulate floods of longer return periods than the available period of meteorological observations.
3. It is possible to separate floods of different mechanisms of generation (e.g. snowmelt and rainfall) for a river basin.
4. Physically based structure of the dynamic-stochastic model allows one to take into account man-induced changes of flood generation and to assess sensitivity of extreme floods to these changes.

The dynamic-stochastic models of runoff generation with random hydrometeorological inputs can be categorized into two groups distinguishing in technology of deriving flood frequency from probability properties of the caused meteorological processes. Such a categorization reflects historical evolution of the approaches to development of mathematical models, as a whole, in watershed hydrology.

The first group includes the models which are based on analytical approaches to transfer from probability properties of the model inputs to the corresponding runoff properties. The first attempt to develop such a model was made by famous Soviet hydrologist M.A. Velikanov in the end of 1940th but his work in this field and the works of his continuers are almost not known beyond the bounds of the former Soviet Union. Properly, the dynamic-stochastic models attract attention of the hydrological society in the beginning of 1970^{th} after the pioneer work of P.S. Eagleson. The models related to the first group are computationally efficient and allow one to obtain the desired probability properties of runoff in a closed analytical form. From the other side, the quest of the model developers to be in the framework of the analytical approach, leads to simplifications (sometimes excessive) of both the stochastic and the deterministic component of the model. These simplifications, in turn, can lead to restriction of applicability of the models under consideration.

Accumulation of knowledge about hydrological systems, development of sophisticated physically based models reflecting the accumulated knowledge, progress in the stochastic description of meteorological processes and development of stochastic weather generators, advance on numerical mathematics and mathematical physics, unexampled increasing of computer facilities all have contributed to an appearance, from the beginning of 1980s, of new generation of the dynamic-stochastic models. They are based on coupling a stochastic weather generator with a physically based model of runoff generation. Monte Carlo procedure is used for simulation of multi-year time-series of meteorological inputs into the physically based model and numerical methods are used for simulation of hydrological processes produced by these inputs. Development of such dynamic-stochastic models allows one to avoid excessive simplifications in the descriptions of hydrometeorological processes and extend applicability of the dynamic-stochastic approach in comparison with the potentialities of the models related to the first group. From the other side, the dynamic-stochastic models of the second group are incomparably more exigent to computer resources that result in appearance of the problems in numerical realization.

Considering the indicated distinctions between the models of two groups, they are very briefly reviewed in the separate subsections below.

2.1. Dynamic-Stochastic Models Based on Analytical Approaches

Overlapping majority of the dynamic-stochastic models related to this group is based on the derived distribution approach. The derived distribution approach is well established in probability theory and its essence is in the following. Let a variable $Y = Y(\boldsymbol{X})$ is functionally related to a random vector $\boldsymbol{X}$, whose components are random variables with joint probability density function f_{X}. Due to the randomness of $\boldsymbol{X}$, also Y is a random variable whose cumulative distribution function $F_Y(y)$ is derived by integration of the joint probability density function f_{X} over the region within which $Y(\boldsymbol{X})$ is less than y, i.e.:

$$F_Y(y) = \Pr\left[Y(\boldsymbol{X}) \le y\right] = \int\limits_{\{\boldsymbol{X}:Y(\boldsymbol{X})\le y\}} f_{\mathrm{X}} dx, \qquad (1)$$

For example, Y may represent the peak flood discharge and the components of $\boldsymbol{X}$ may include random characteristics of rainfall storm.

Velikanov (1949) have made the first attempt to use the derived distribution approach to determine the cumulative distribution of peak discharge of snowmelt flood. The authors used merely empirical formulae, rather than dynamic models, for calculation of peak discharge; the parameters of these formulae are not physically meaningful and can be assessed only from the available measurements of flood discharges. Below, the dynamic-stochastic models are reviewed which also use the derived distribution approach but whose deterministic components are dynamic models based, to a greater or lesser extent, on a priori physical principles.

P.S. Eagleson (1972) was the first who proposed a dynamic-stochastic model, based on the physically based description of hydrological processes of rainfall flood generation. The cumulative distribution function of flood peak discharge was derived by the author from joint probability density function for rainstorm climatic characteristics (intensity and duration) by using the kinematic wave equation to predict the flood hydrograph. The obtained closed form expression establishes dependence of peak discharge ($Q_{\max}$) of the assigned return period (T_{r}) on climatic and catchment parameters:

$$Q_{\max} = Q_{\max}\left(\lambda, \beta, P, \Theta, A_{\mathrm{c}}, A_{\mathrm{r}}, \alpha_{\mathrm{s}}, \alpha_{\mathrm{c}}, L_{\mathrm{s}}, \Phi\right) \qquad (2)$$

where the first 4 terms are climatic parameters, namely, two parameters of probability distributions of point storm rainfall intensity and duration, average annual rainfall, and average annual number of independent rainfall events; the next 5 terms are catchment parameters, namely, catchment area, area producing direct runoff, two parameter reflecting roughness of surface and channel bed, respectively, and channel length; the next term is runoff coefficient assumed to be a constant for a given catchment.

It was shown that the derived distribution obtained from (2) agrees well with runoff observations from three Connecticut catchments.

In the context of the flood frequency estimation, the Eagleson’s (1972) approach has been used and extended by many authors during the next two decades (R.L. Bras, M.A. Diaz-Granadoz, M. Sivapalan, J.B. Valdes, E. Wood among others).

In the middle 1980s, R.L. Bras with co-authors has published the papers (see, for instance, Bras et al., 1985) where three best-known dynamic-stochastic models developed by that time were tested. The objective of the test was to compare an ability of the models to derive flood frequency distribution for ungauged basins. Five river basins located in the different physiographic and climatic conditions with catchment areas from 100 to 1000 km^2 were selected. To find all the necessary parameters of the compared models, it was assumed that no streamflow data is available and only a few years of rainfall data exist. The return periods of flood peak discharges derived by each of the models for the five basins were compared with the return periods estimated from the observed floods. The comparison has shown that none of the models agreed well with the observations. The authors concluded that performance of the models could be significantly improved if the available observed floods would be used for calibration of the deterministic, rainfall-runoff models.

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Biography Sketch

Alexander N. Gelfan, Dr. of Sciences in Physics and Mathematics, leading researcher of the Hydrological Sciences Laboratory of the Water Problems Institute of Russian Academy of Sciences, Moscow, Russia. He has published 28 papers and two books in hydrology. His research interests include physically-based modeling processes of runoff generation, extreme flood modeling, flood-frequency prediction using dynamic-stochastic models, predicting the effects of land use and climate changes on floods. He has also dealt with modeling hydrological processes in permafrost regions. He can be contacted at the Water Problems Institute of Russian Academy of Sciences, Gubkina 3, 119333, Moscow, Russia.