SLIDING MODE CONTROL

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Contents

Summary

The paper presents the basic concepts, mathematical and design aspects of sliding mode control.It is shown that the main advantages of sliding mode control are order reduction, decoupling design procedures, disturbance rejection, insensitivity to parameter variations, simple implementation by means of conventional power converters. The methods of suppressing chattering, caused by discrete-time implementation and unmodeled dynamics, are given. The sliding mode control is demonstrated for linear time-invariant systems and for control of induction motors.

1. Introduction

The sliding mode control approach is recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic plant operating under uncertainty conditions. The research in this area were initiated in the former Soviet Union about 40 years ago, and then the sliding mode control methodology has been receiving much more attention from the international control community within the last two decades.

The major advantage of sliding mode is low sensitivity to plant parameter variations and disturbances which eliminates the necessity of exact modeling. Sliding mode control enables the decoupling of the overall system motion into independent partial components of lower dimension and, as a result, reduces the complexity of feedback design. Sliding mode control implies that control actions are discontinuous state functions which may easily be implemented by conventional power converters with “on-off” as the only admissible operation mode. Due to these properties the intensity of the research at many scientific centers of industry and universities is maintained at high level, and sliding mode control has been proved to be applicable to a wide range of problems in robotics, electric drives and generators, process control, vehicle and motion control.

2. Concept “Sliding Mode”.

The phenomenon “Sliding Mode” may appear in dynamic systems governed by ordinary differential equations with discontinuous state functions in the right-hand sides. The conventional example of sliding mode – a second order relay system - can be found in any text book on nonlinear control. The control input in the second order system

$$\ddot{x} + a_2\dot{x} + a_1 x = u,$$
$$u = -Msign(s), \quad s = cx + \dot{x}, \quad a_1, a_2, M, c \text{ - const}$$

may take only two values, M and $-M$, and undergoes discontinuities on the straight line $s = 0$ in the state plane $(x, \dot{x})$ (Fig.1 for the case $a_1 = a_2 = 0$). It follows from the analysis of the state plane that, in the neighborhood segment mn on the switching line $s = 0$, the trajectories run in opposite directions, which leads to the appearance of a sliding mode along this line. The equation of this line

$$\dot{x} + cx = 0$$

may be interpreted as the sliding mode equation. Note that

- the order of the equation is less than that of the original system
- the sliding mode does not depend on the plant dynamics, and is determined by parameter c only.

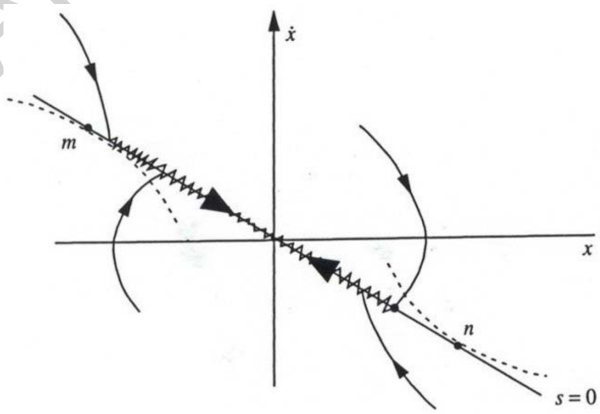

Figure 1. Sliding mode in a second relay system

Sliding mode became the principle operation mode in so-called variable structures systems. A variable structure system consists of a set of continuous subsystems with a proper switching logic and, as a result, control actions are discontinuous functions of the system state, disturbances (if they are accessible for measurement), and reference inputs. The previous example of the relay system with state dependent amplitude of the control variable may serve as an illustration of a variable structure system:

$u = -k \; |x| \; sign(s), \quad k$ is constant.

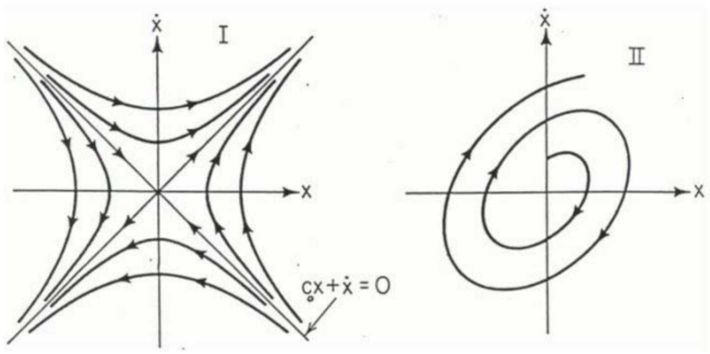

Figure 2. State planes of two unstable structures

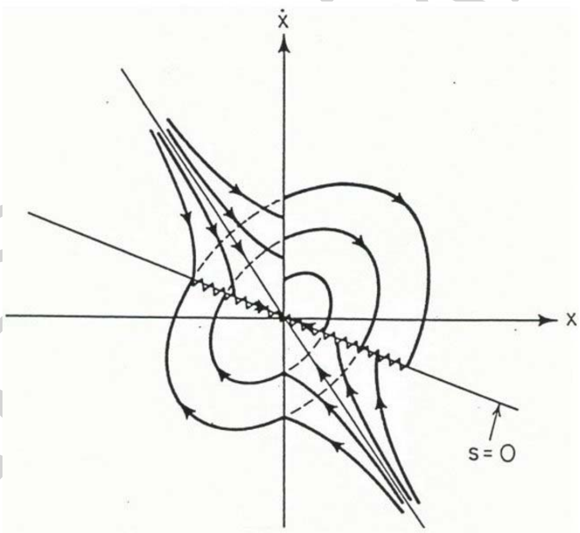

Figure 3. State plane of variable structure system

Now the system with $a_1 = 0$ and $a_2 < 0$ consists of two unstable linear structures ($u = kx$ and $u = -kx$, Fig.2) with $x = 0$ and $s = 0$ as switching lines. As it is clear from the system state plane, the state reaches the switching line $s = 0$ for any initial conditions. Then, the sliding mode occurs on this line (Fig.3) with the motion equation $\dot{x} + cx = 0$, while the state vector decays exponentially. Similarly to the relay system, after the start of the sliding mode, the motion is governed by a reduced order equation which does not depend on the plant parameters.

Now we demonstrate sliding modes in non-linear affine systems of general form

$$\dot{x} = f(x,t) + B(x,t)u \ , \qquad (1)$$

$$u_i = \begin{cases} u_i^+(x,t) & if \quad s_i(x) > 0 \\ u_i^-(x,t) & if \quad s_i(x) < 0 \end{cases} \quad i = 1,...,m, \qquad (2)$$

where $x \in R^n$ is a state vector, $u \in R^m$ is a control vector, $u_i^+(x,\mathrm{t})$, $u_i^-(x,t)$ and $s_i(x)$ are continuous functions of their arguments, $u_i^+(x,\mathrm{t}) \neq u_i^-(x,t)$. The control is designed as a discontinuous function of the state such that each component undergoes discontinuities in some surface in the system state space.

Similar to the above example, state velocity vectors may be directed towards one of the surfaces and sliding mode arises along it (arcs ab and cb in Fig.4). It may arise also along the intersection of two of surfaces (arc bd).

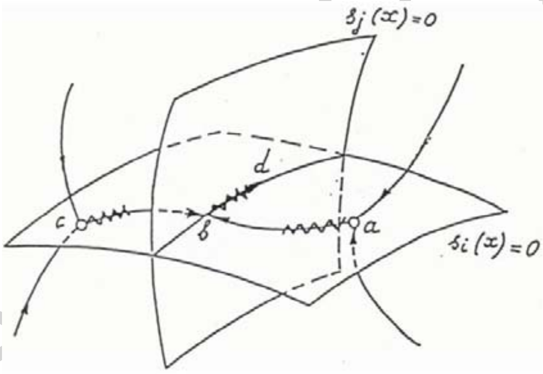

Figure 4. Sliding mode in discontinuouity surface and their intersection

Fig.5 illustrates the sliding mode in the intersection even if it does not exist at each of the surfaces taken separately.

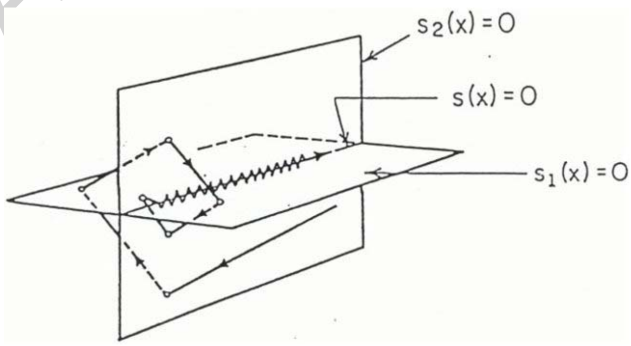

Figure 5. Sliding mode in intersection of discontinuity surfaces

For the general case (1) sliding mode may exist in the intersection of all discontinuity surfaces $s_i = 0,$ or in the manifold

$$s(x) = 0, \ \ s^T(x) = [s_1(x),..., s_m(x)] \tag{3}$$

of dimension $n - m.$

Let us discuss the benefits of sliding modes, if it would be enforced in the control system. First, in sliding mode the input s of the element implementing discontinuous control is close to zero, while its output (exactly speaking its average value u_{av}) takes finite values (Fig.6).

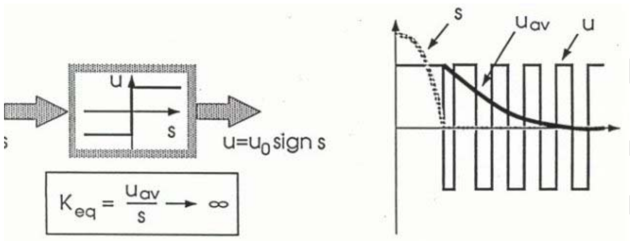

Figure 6. High gain implementation by sliding mode

Hence, the element implements high (theoretically infinite) gain, that is the conventional tool to reject disturbance and other uncertainties in the system behavior. Unlike to systems with continuous controls, this property called invariance is attained using finite control actions. Second, since sliding mode trajectories belong to a manifold of a dimension lower than that of the original system, the order of the system is reduced as well. This enables a designer to simplify and decouple the design procedure. Both order reduction and invariance are transparent for the above two second-order systems.

3. Sliding Mode Equations

So far the arguments in favor of employing sliding modes in control systems have been discussed at the qualitative level. To justify them strictly, the mathematical methods should be developed for describing this motion in the intersection of discontinuity surfaces and deriving the conditions for sliding mode to exist.

The first problem means deriving differential equations of sliding mode. Note that for our second-order example the equation of the switching line $\dot{x} + cx = 0$ was interpreted as the motion equation. But even for a time invariant second-order relay system

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1 u$$
$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + b_2 u, \ \ u = -M sign(s), \ \ s = cx_1 + x_2; \ M, a_{ij}, b_i, c \text{ are } const$$

the problem does not look trivial since in sliding mode $s = 0$ is not a motion equation.

The first problem arises due to discontinuities in control, since the relevant motion equations do not satisfy the conventional theorems on existence-uniqueness of solutions. In situations when conventional methods are not applicable, the usual approach is to employ regularization or replacing the initial problem by a closely similar one, for which familiar methods can be used. In particular, taking into account delay or hysteresis of a switching element, small time constants in an ideal model, replacing a discontinuous function by a continuous approximation are examples of regularization since discontinuity points (if they exist) are isolated. The universal approach to regularization consists of introducing a boundary layer $\|s\| < \Delta,\ \Delta - const$ around the manifold $s = 0,$ where an ideal discontinuous control is replaced by a real one such that the state trajectories are not confined to this manifold but run arbitrarily inside the layer (Fig. 7).

The only assumption for this motion is that the solution exists in the conventional sense. If, with the with of the boundary layer Δ tending to zero, the limit of the solution exists, it is taken as a solution to the system with ideal sliding mode. Otherwise we have to recognize that the equations beyond discontinuity surfaces do not derive unambiguously equations in their intersection, or equations of the sliding mode.

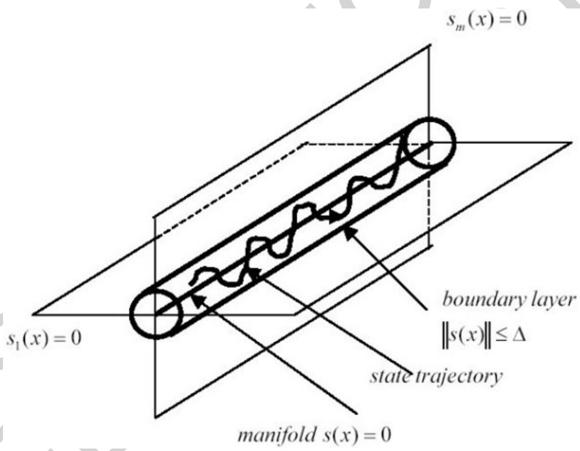

Figure 7. Boundary layer

The boundary layer regularization enables substantiation of so-called *Equivalent Control Method* intended for deriving sliding mode equations in manifold $s = 0$ in system (1). Following this method the sliding mode equation with a unique solution may derived for the nonsingular matrix

$$G(x)B(x),\ \ G(x) = \{\partial s / \partial x\},\quad \det(GB) \neq 0.$$

First, the *equivalent control* should be found for the system (1) as the solution to the equation $\dot{s} = 0$ on the system trajectories (G and $(GB)^{-1}$ are assumed to exist) :

$$\dot{s} = G\dot{x} = Gf + GBu_{eq} = 0,\quad u_{eq} = -(GB)^{-1}Gf\,.$$

Then the solution should be substituted into (1) for the control

$$\dot{x}=f-B(GB)^{-1}Gf \ . \qquad (4)$$

Equation (4) is the sliding mode equation with initial conditions $s(x(0),0)=0$.
Since $s(x)=0$ in sliding mode m components of the state vector may be found as a function of the rest $(n-m)$ ones: $x_2=s_0(x_1);\ x_2,s_0\in\Re^m;\ x_1\in\Re^{n-m}$ and, correspondingly, the order of the sliding mode equation may be reduced by m:

$$\dot{x}_2=f_1[x_1,t,s_0(x_1)],\ f_1\in\Re^{n-m}. \qquad (5)$$

The idea of the equivalent control method may be easily explained with the help of geometric consideration. Sliding mode trajectories lie in the manifold $s=0$ and the equivalent control u_{eq} being a solution to the equation $\dot{s}=0$ implies replacing the original discontinuous control by such continuous one that the state velocity vector lies in the tangential manifold and as a result the state trajectories are in this manifold. It will be important for control design that sliding mode equation

- Is of reduced order
- Does not depend on control
- Depends on the equation of switching surfaces.

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Biographical Sketch

Prof **V. Utkin** graduated from Moscow Power Institute (Dipl.Eng.) and received a Ph.D. from the Institute of Control Sciences (Moscow, Russia).

He was with the Institute of Control Sciences since 1960, as Head of the Discontinuous Control Systems Laboratory in 1973-1994. Currently he is Ford Chair of Electromechanical Systems at the Ohio State University.

Prof. Utkin is one of the originators of the concepts of Variable Structure Systems and Sliding Mode Control. He is an author of five books and more than 250 technical papers. In 1975-1978 he was in charge of an international project between his Institute and "Energoinvest" Sarajevo on the sliding mode control of induction motors. D.C., induction and synchronous drives with sliding mode control have been applied for metal-cutting machine tools, process control and electric cars.

His current research interests are control of infinite-dimensional plants including flexible manipulators, sliding modes in discrete time systems and microprocessor implementation of sliding mode control, control of electric drives and alternators, robotics and automotive control.

He is Honorary Doctor of University of Sarajevo, Yugoslavia, in 1972 was awarded Lenin Prize (the highest scientific award in the former USSR).

He held visiting positions at universities in the USA, Japan, Italy and Germany.

Prof. V.Utkin was IPC chairman of 199O IFAC Congress in Tallinn; now he is Associate Editor of "International Journal of Control", Chairman of Technical Committee of IEEE on Variable Structure and Sliding Mode Control, member of Administrative Committee of IEEE Industrial Electronics Society.